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Linear Model Representation for Robust Leak Location in Water Distribution Networks

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Abstract: In this paper, we introduce a new scheme for leak location in water distribution networks. It is based on a new representation that we propose to call the Leak Signature Space (LSS). In that space, a specific signature can be associated to each leak location that does not depend on the leak magnitude. The LSS relies on an analysis of the linear dependencies between pressure residuals. Besides, its dimension depends on the number of sensors installed in the network. This new approach, whose robustness can be improved through a time horizon analysis, allows to infer the location of a given leak by comparing the position of its signature in the LSS with other leak signatures references. The efficiency of the method is highlighted on a real network with various scenarios involving different number of sensors and the potential presence of noise in the measurement.

Keywords: Water networks, Leak location, Linear dependency, Linear transformation, Leak signature space.

1. INTRODUCTION

Water Distribution Networks (WDN) are used in everyday life, whether being employed for domestic or for industrial use. They are usually large scale systems which demand continuous improvements in water loss management and new technologies to achieve higher levels of efficiency. Water losses due to leaks in distribution networks have been estimated to account up to 30% of the extracted water. Thus, the development of efficient leak detection and location strategies has become an important research issue in recent years.

A substantial amount of work has been published on leak detection methods for WDN. Colombo et al. (2009) offer a review of transient-based leak detection methods that summarizes current and past contributions. However, the performance obtained until now is still far from allowing the detection of WDN leaks with only a few sensors in a robust and fast way.

The traditional approach to leakage control in real life is a passive one, where the leak is repaired only when it becomes visible. Thus, methods that reports leaks at an earlier stage are important to be developed. As an alternative, Xia and Guo-jin (2010) proposed a leak detection model based on the cluster-analysis and fuzzy pattern recognition theorem. Here, simulations of pipe leaks are clustered in virtual partitions. Then, fuzzy recognition identifies the leak region reducing the scope for the leak detection. However, this theory presents some problems with virtual partition size that requires further improvement. Recently Khulief et al. (2012), developed acoustic instruments that allow the location of invisible leaks, but unfortunately, their application on a large-scale water network is very expensive and time-consuming. Rosich and Puig (2013) have proposed a leak detection methodology based on the generation of a new class of structured residuals which has been satisfactory applied to a water distribution network. This approach allows to detect leaks in an efficient way but it may be time-consuming due to the numerical algorithm used to compute the residuals.

Model based leak detection and isolation techniques, based on pressure measurements and sensitivity analysis have been studied for two decades since the paper of Pudar and Liggett (1992) which formulates the leak detection and isolation problem as a least-squares estimation problem. However, when using non-linear models as it is the case in the WDN, the estimation of the parameters describing models is a difficult task. In Pérez et al. (2011), a model-based method that relies on pressure measurements and leak sensitivity analysis is proposed. Although this approach has good efficiency under ideal conditions, its performance decreases due to the nodal demand uncertainty and noise in the measurements. This methodology has been extended in Casillas et al. (2014) where an analysis along a time horizon has been taken into account and a comparison of several leak isolation methods is offered. In this case, the efficiency was improved but the leak magnitude uncertainty was still affecting the performance. Aiming to improve the efficiency of these methods, we propose a new scheme where the leaks are identified through a specific signature that is independent of the leak magnitude. This signature can be represented in what we call the Leak Signature Space (LSS). The signature can also incorporate a time horizon representation which increases the robustness of the method regarding leak detection.

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The organization of the paper is as follows. Section 2 presents the WDN model and highlights how, under given conditions, pressures depend linearly on leaks magnitudes. Section 3 introduces the LSS, an original spatial representation that discriminates the leak location between all possible locations. Section 4 shows how this leak location is actually performed from the LSS. A complete application of the method in a real network is shown in Section 5. Finally, Section 6 summarizes and presents potential extensions of this method.

2. MODEL REPRESENTATION

In this work, we assume that only one leak may occur at a time in the WDN. With such hypothesis, we will show that under some equilibrium conditions, there is a linear dependency between the leak magnitude and the pressure variations deduced from the sensors measurements. Then, we will use such property to build a leak signature model that can be used for leak detection. In this section, we show from the equations governing the behavior of the pressure and the water flow how a variation in the leak magnitude affects linearly the pressure variations recorded by sensors. Such behavior is then illustrated on a small example.

2.1 Water network model solution

In the following, we will assume that the behavior of the WDN is similar to that described in Todini and Pilati (1988). Let us consider a WDN with m nodes, f pipe flows and n pressure sensors located at the nodes (usually $n < m$). Let us also define the vectors $\mathbf{p}$, $\mathbf{p}^*$, $\mathbf{q}$, $\mathbf{d}$ which are respectively the vectors of pressure in the junction nodes, pressure in reservoirs, flows trough the pipes and demands:

$$\begin{aligned} \mathbf{p} &= (p_1, \ldots, p_m)^{\mathbf{T}} \\ \mathbf{p}^* &= (p_1^*, \ldots, p_u^*)^{\mathbf{T}} \\ \mathbf{q} &= (q_1, \ldots, q_f)^{\mathbf{T}} \\ \mathbf{d} &= (d_1, \ldots, d_m)^{\mathbf{T}} \end{aligned} \tag{1}$$

with u corresponding to the number of reservoirs supplying the WDN. Then, the water network model can be formulated through the following matrix representation:

$$\begin{pmatrix} A_{11}(\mathbf{q}) & A_{12} \\ A_{21} & 0 \end{pmatrix} \begin{pmatrix} \mathbf{q} \\ \mathbf{p} \end{pmatrix} = \begin{pmatrix} -A_{10}\mathbf{p}^* \\ \mathbf{d} \end{pmatrix} \tag{2}$$

where $A_{12} = A_{21}^T$, $A_{10} = A_{01}^T$ and with A_{21}, A_{01} incidence matrices obtained when only junction and reservoir nodes are considered respectively. $A_{11}(\mathbf{q})$ is a diagonal matrix where diagonal terms represent the head losses in each pipe according to the flow and it is computed as:

$$A_{11}(\mathbf{q}) = \mathrm{diag}(c_i|q_i|^{\gamma_i}), i \in [1, f]. \tag{3}$$

Here, $|q_i|$ is the absolute value of the flow q_i, c_i is a constant parameter which depends on the diameter, the roughness and the length of the pipe and γ_f is the flow exponent parameter.

The resulting water network model can then be solved numerically using a Newton-Raphson iterative scheme where the iteration $k+1$ is given by the two following equations:

$$\begin{aligned} \mathbf{p}^{k+1} = &-(A_{21}N^{-1}A_{11}^{-1}(\mathbf{q}^k)A_{12})^{-1} \\ &\cdot \left(A_{21}N^{-1}(\mathbf{q}^k + A_{11}^{-1}(\mathbf{q}^k)A_{10}\mathbf{p}^*) + (\mathbf{d} - A_{21}\mathbf{q}^k)\right) \end{aligned} \tag{4}$$

$$\mathbf{q}^{k+1} = (I - N^{-1})\mathbf{q}^k - N^{-1}A_{11}^{-1}(\mathbf{q}^k)(A_{12}\mathbf{p}^k + A_{10}\mathbf{p}^*) \tag{5}$$

where N is a diagonal matrix such that $N = \mathrm{diag}(\gamma_i)$, $i \in [1, f]$. It is important to note that this resolution approach is commonly employed, as e.g. in the EPANET simulator (See Rossman (2000)) where large WDN can be simulated efficiently.

The solution of the system of equations (4) and (5) corresponds to the case where an equilibrium point has been reached for the network, i.e. the flows and pressures are constant along the time which means $\mathbf{p}^{k+1} = \mathbf{p^k} = \mathbf{p}$ and $\mathbf{q}^{k+1} = \mathbf{q^k} = \mathbf{q}$.

A thorough representation of a WDN would theoretically involve a graph structure where each possible leak could be represented by a graph node. However, a leak could possibly appear at any point of any network pipe. For this reason, the exact modeling of any possible leak becomes unfeasible in practice. To mitigate this issue, it is usually sufficient to assume that leaks only appear at existing nodes (Pudar and Liggett (1992)). With such assumption, we can write the leak as a vector of extra demand $\mathbf{\Delta d}$ and express the new demand $\mathbf{d'}$ such as:

$$\mathbf{d'} = \mathbf{d} + \mathbf{\Delta d}. \tag{6}$$

where $\mathbf{\Delta d}$ is a m dimensional vector with zeros everywhere except at the node's index where the leak occurs. Now, assuming the network flow equilibrium has also been reached in presence of leak, we can express the new pressure as:

$$\begin{aligned} \mathbf{p'} = &-(A_{21}N^{-1}A_{11}^{-1}(\mathbf{q})A_{12})^{-1} \\ &\cdot \left(A_{21}N^{-1}(\mathbf{q} + A_{11}^{-1}(\mathbf{q})A_{10}\mathbf{p}^*) + (\mathbf{d} + \mathbf{\Delta d} - A_{21}\mathbf{q})\right). \end{aligned} \tag{7}$$

Then, it is possible to compute the residual $\mathbf{r}$ (Pérez et al. (2011)) as the difference between the nominal pressure and the pressure in case of leak:

$$\begin{aligned} \mathbf{r} &= \mathbf{p} - \mathbf{p'} \\ &= (A_{21}N^{-1}A_{11}^{-1}(\mathbf{q})A_{12})^{-1}\mathbf{\Delta d} \\ &= \mathbf{K} \cdot \mathbf{\Delta d}. \end{aligned} \tag{8}$$

As one can see, there is a proportional relation between the residual (and consequently with the pressure measurement) and the leak through a $\mathbf{K}$ matrix factor under our equilibrium assumptions.

The pressure in a network at a given instant of time can be represented by a point in the m-dimensional space of the pressure measurement. In case of a leak, (8) tells us that the position of this point will be on a line passing through the origin and whose direction depends on the node where the leak occurs. Moreover, the position of this point on the line will depend on the leak magnitude. However, in practice, we only have access to measure the pressure in some limited number of nodes n where the sensors are placed. Fortunately, the n-dimensional subspace of the sensors is a subspace (a projection) of the m-dimensional space of the pressure measurement. Thus, this linear dependency is also valid in this projected space. We will now illustrates such linear dependencies on a small network.

2.2 Small WDN example: model solution

In the following, we propose to apply the analysis developed in Section 2.1 to a small WDN example. We use the network shown in Figure 1 containing 1 reservoir, 3 demand nodes and 4 flow pipes. Then, the matrices describing the network model (according to (2)) are given by:

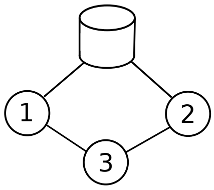


Fig. 1. Small WDN example. A reservoir is supplying a network made of 3 demand nodes and 4 pipes.

$$A_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}, A_{10} = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}. \tag{9}$$

Assuming that the pressure in the reservoir is known and according to the the pipes characteristics, matrices A_{11}, N and finally K (according to (8)) can be computed such as:

$$A_{11} = \begin{bmatrix} 0.81 & 0 & 0 & 0 \\ 0 & 0.19 & 0 & 0 \\ 0 & 0 & 1.16 & 0 \\ 0 & 0 & 0 & 2.79 \end{bmatrix}, N = 1.852 \cdot I$$
$$K = \begin{bmatrix} 1.26 & 0.06 & 0.91 \\ 0.06 & 0.35 & 0.14 \\ 0.91 & 0.14 & 2.21 \end{bmatrix} \tag{10}$$

where I is the identity matrix. Then, it is possible to compute the residuals for each possible leak in the network. Assuming in this example, the space of the pressure measurement is 3-dimensional (i.e. all pressure nodes are measured $m = n = 3$). Figure 2 presents this space where we represent the differences of pressures for leaks appearing in each of the 3 nodes and for leaks magnitudes varying from 1 to 5 liters per second (lps). Figure 2 illustrates how each node can be associated to a residual that varies linearly, according to the leak magnitude.

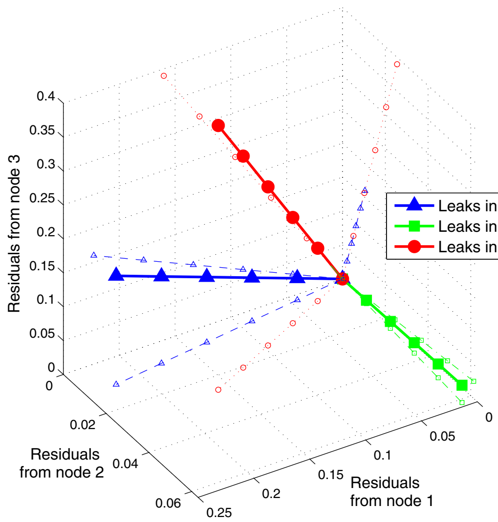


Fig. 2. 3-dimensional space of the pressure measurements. We see a linear residual variation respect to the leak magnitude for leaks simulated in each of the three network nodes.

Figure 3 shows the case where sensors are only installed in nodes 1 and 2 ($n = 2$). Thus, observations can be made only in the projected 2-dimensional space corresponding to these 2 sensors. In this space, the linear dependency remains. However, it is interesting to note that the projection could lead to superpose two lines of two distinct leak nodes, whereas they had distinct representation in the full m-dimensional space.

In the following, we explain how we can exploit these linear dependencies to discriminate leaks through their representation in what we call the Leak Signature Space.

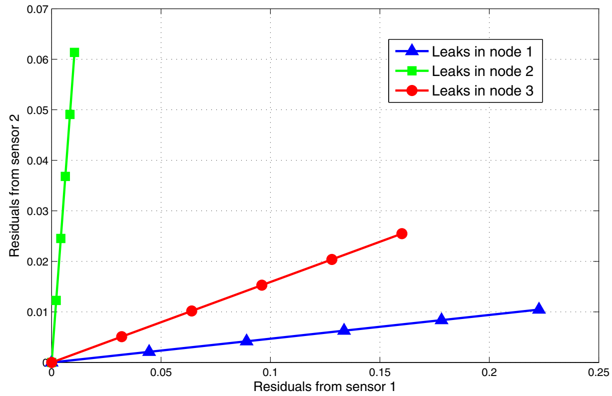


Fig. 3. The linear dependency respect to the leak magnitude also appears for the 3 types of leak location in the projected 2-dimensional space of 2 sensors installed at node 1 and node 2.

3. LEAK SIGNATURE SPACE

We now present the Leak Signature Space, an original spatial representation that allows to discriminate the possible leak location. The linear dependency presented above is such that for any pair of residual vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ corresponding to different leak magnitudes but occurring in the same node j, we have:

$$\mathbf{r}_2^j = \lambda \mathbf{r}_1^j \tag{11}$$

with λ proportional to the leak magnitude. Thus, any residual corresponds to a direction vector of the line representing the leak at a specific node. Based on the sensor representation, we propose to use the normalized direction vector where the last coordinate r_n^j [1] is equal to 1. Thus, from a given residual $\mathbf{r}^j = [r_1^j, \cdots, r_n^j]^{\mathbf{T}}$ we can compute the vector $\bar{\mathbf{r}}^j$:

$$\bar{\mathbf{r}}^j = [\frac{r_1^j}{r_n^j}, \cdots, \frac{r_{n-1}^j}{r_n^j}, 1]^{\mathbf{T}} \tag{12}$$

and there is a unique expression of such normalized vector for a linear representation of the residuals. Thus, it is possible to associate to the leak in node j and independently of its magnitude a unique point $\tilde{\mathbf{r}}^j = [\bar{r}_1^j, \ldots, \bar{r}_{n-1}^j]$ within a $(n-1)$-dimensional space that we call the Leak Signature Space.

3.1 Dealing with realistic cases

In a real scenario, the ideal linear dependency presented in the previous section may be partially affected. First, the model representation we used is always imperfect/incomplete respect to what is happening in a real network and our simulation based on the Newton-Raphson scheme, may introduce some numerical errors. Also, some noise typically appears with the sensor measurements. Finally, the demand may vary along the time, whereas the leak in our model appears as an extra demand which assumes the reference demand is fixed. The problem of

[1] r_n^j is assumed to be non null i.e. the pressure at node n is affected by the presence of the leak at node j, which is typically the case when j and n are in the same connected component of the network.

changes in the water demand will be considered later on. For now, we will only explain how to deal with general noise and imperfect linear dependencies.

To represent the leak associated to a given node j in the network with n sensors, we simulate s different leaks magnitudes ${}^k l$, with $k \in [1, s]$. Then, we compute the associated s residual vectors ${}^k\mathbf{r}^j = [{}^k r_1^j, \dots, {}^k r_n^j]^{\mathrm{T}}$ and their $(n-1)$-dimensional representations in the LSS ${}^k\tilde{\mathbf{r}}^j = [\frac{{}^k r_1^j}{{}^k r_n^j}, \dots, \frac{{}^k r_{n-1}^j}{{}^k r_n^j}]^{\mathrm{T}}$. Then, the point $\tilde{\mathbf{r}}^j$, signature of leak j is taken as the barycenter of these s partial signatures ${}^k\tilde{\mathbf{r}}^j$ built from the different leak magnitudes:

$$\tilde{\mathbf{r}}^j = \left[\frac{1}{s} \sum_{k=1}^{s} \frac{{}^k r_1^j}{{}^k r_n^j}, \dots, \frac{1}{s} \sum_{k=1}^{s} \frac{{}^k r_{n-1}^j}{{}^k r_n^j} \right] \tag{13}$$

Computing such barycenters for the m possible leak nodes, we obtain m leak signatures $\tilde{\mathbf{r}}^j$, with $j \in [1, m]$, independent of the leak magnitude and that can be used to perform leak location. Let us also remark that when the number of sensors increases, it increases the dimension of the LSS and thus increases the chances to discriminate between the signatures of each leak location.

3.2 Small WDN example: leak signature

Following with the small WDN example of Section 2.2, and considering that the 3 nodes are equipped with sensors ($n = 3$), we can represent the leak signature of each node in the LSS. We simulate for each of the 5 leak magnitudes considered a white noise on the pressure measurements which affects the residuals values. Figure 4 shows for a leak in each node j the partial leak signatures ${}^k\tilde{\mathbf{r}}^j$ for each magnitude and the final leak signatures $\tilde{\mathbf{r}}^j$ obtained from the barycenter of these points.

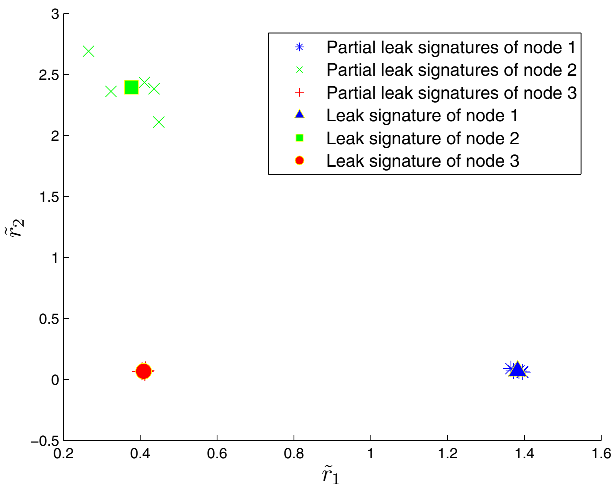


Fig. 4. Representation of leak signatures in the LSS of the small WDN, when it is equipped with 3 sensors. The signatures are computed as the barycenter of the partial leak signatures corresponding to given leak magnitudes.

4. LEAK LOCATION METHOD

In this section, we explain how to estimate the leak location based on the LSS representation. First, the m leak signatures of the network's nodes are computed thanks to the method presented in the previous section. Then, when a leak occurs, we also look at its signature in the LSS, i.e. we compute the $(n-1)$-dimensional point $\tilde{\mathbf{r}}^*$ representative of this leak from real residual measurements. Finally, we compute the Euclidean distance d between $\tilde{\mathbf{r}}^*$ and the various leak signatures $\tilde{\mathbf{r}}^j$ in the LSS. The leak node is then estimated as the one whose signature is the closest to the current leak signature i.e the index id of the leak is such that:

$$id = \arg \min_{j \in [1,m]} d(\tilde{\mathbf{r}}^*, \tilde{\mathbf{r}}^j)) \tag{14}$$

where $\arg\min$ is the argument of the minimal distance evaluated.

4.1 Time horizon analysis

In practice, for a given network, the demand usually varies along the time and it is important to make an analysis taking into account a given time horizon (Casillas et al. (2014)). To address this point, we propose to record for each potential leak node j, the various signatures it has along a time horizon corresponding to one cycle of demand pattern. Since the demand varies along this pattern, the position of the leak signatures changes accordingly. Then, in presence of a leak, we compare the positions of its signatures in the LSS along the day with the positions of the reference leak signatures. This comparison is performed by summing the Euclidean distances for each instant of time considered. Thus, if we consider T instants of time, we will have T signatures ${}_t\tilde{\mathbf{r}}^*$, $t \in [1, T]$ from the measured pressures. The index id of the leak node is then estimated as the one for which the sum of the distances between the node signature and the current leak signature along the time horizon is minimal:

$$id = \arg \min_{j \in [1,m]} \left(\sum_{t \in [1,T]} d({}_t\tilde{\mathbf{r}}^*, {}_t\tilde{\mathbf{r}}^j) \right). \tag{15}$$

In the following section, we apply the leak location method based on the LSS on a real network and compare the performance with and without time horizon analysis.

5. REAL NETWORK APPLICATION

The proposed method is applied to a real network (WDN of Limassol city in Cyprus) simulated in EPANET. This network consists of 1 reservoir, 197 junction nodes and 239 pipes as shown in Figure 5.

Leaks are simulated in a range going from 2 lps to 6 lps. To simulate individual leaks in EPANET, according to Rossman (2000), it is necessary to specify an emitter coefficient [2] (EC in $lps/m^{1/2}$) based in the equation:

$$EC = \frac{q}{p^{P_{exp}}} \tag{16}$$

where q is the flow rate, p is the fluid pressure and P_{exp} is the pressure exponent. Then, the EC will vary in a range going from 0.3 to 0.9 $lps/m^{1/2}$.

First, we propose to analyze how the number of sensors used in the network impacts the quality of the leak location. The sensors are distributed such that they try to cover as much as possible the various network areas and we assume they are

2 Note that whereas in our analysis we represent the leak as an extra demand, here a different formulation is used. In practice, these two representations are very similar and a more thorough analysis will be kept as future work.

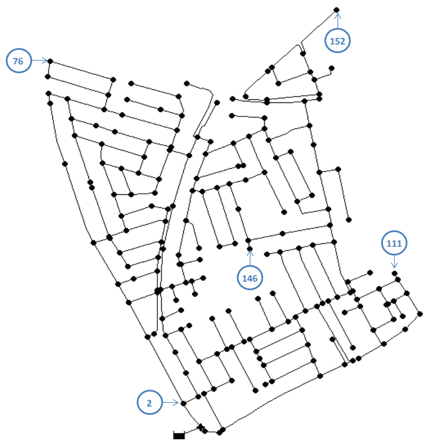


Fig. 5. Limassol water distribution network.

sensitive to any pressure change at their location. By experience, we have noticed that sensors located at the edges of a network are more sensitive to changes in pressure. Thus, we select border nodes trying to cover all the pressure changes (note that an optimal sensor placement has not been performed, but it will be part of the future work). The possible locations of the sensors are summed up in Table 1 and shown in Figure 5.

Table 1. Indexes of the nodes where the sensors are located according to the total number of sensors.

Number of sensors	Sensor node locations
2	2, 152
3	2, 146, 152
4	2, 76, 111, 152
5	2, 76, 111, 146, 152

5.1 Analysis for a single node

Leak location in a single instant time. First, we consider the case of a leak in the node of index 69 with a magnitude of 3.5 lps (simulated with $EC = 0.54$). Figure 6 represents the leaks in leak signature spaces in case of 2, 3, and 4 sensors which have a dimension of 1, 2, and 3 respectively. Pictures of the right (b,d,f) are zooms of the left pictures (a,c,e).

When 2 sensors are installed in the network (Figure 6.(a,b)), the closest leak signature corresponds to the node of index 194 which is at a topological distance of 4 nodes from the node 69 we are looking for. In the case of 3 sensors, the leak is located at node 70 (Figure 6.(c,d)) which is at a topological distance of 1 node from the node 69, the result comes from the fact that nodes 69 and 70 have almost a superposed leak signature in case of this 2-dimensional space. Finally, when 4 sensors are installed (Figure 6.(e,f)), the leak can now be discriminated from any other node and it is correctly located at node 69.

Leak location with time horizon analysis. In this case, we simulate a leak in the node of index 150 with a magnitude of 3 lps (simulated with an $EC = 0.46$) with a time horizon of 24 hours. Figure 7 represents the leak in case of 2, 3, and 4 sensors in the region of the LSS where the real leak is measured.

When 2 sensors are installed in the network (Figure 7.a), the node with the minimum distance accumulated along the time horizon is the node of index 149 which is at a topological distance of 2 nodes from the node 150 we are looking for.

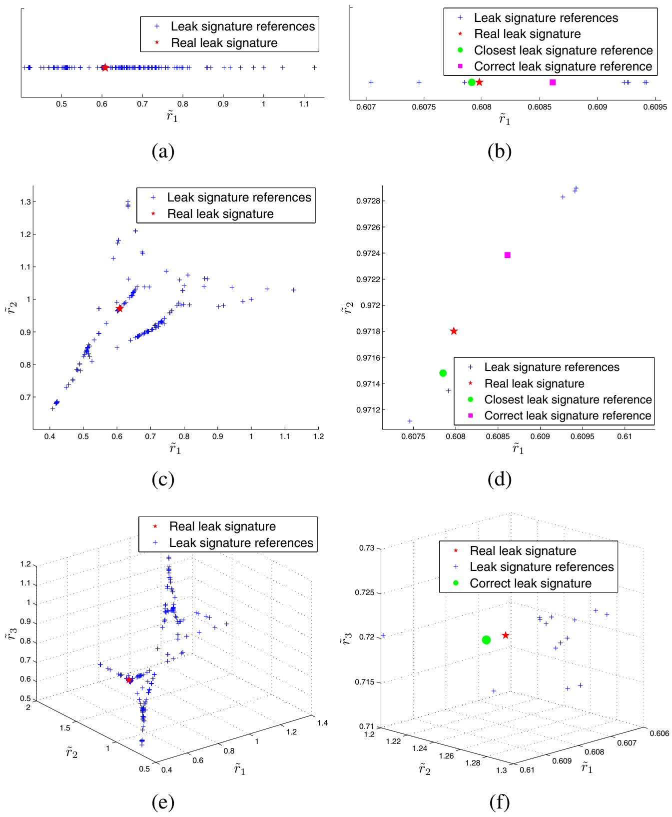


Fig. 6. View of a leak at the node 69 in the LSS in case of: 2 sensors (a,b), 3 sensors (c,d), and 4 sensors (e,f). Right pictures are zooms of the left pictures. It is only in the case of 4 sensors that the leak is correctly located.

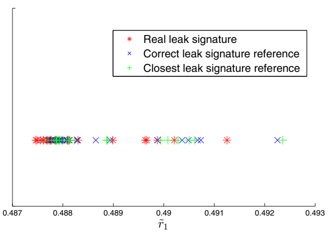
However, in the case of 3 sensors, the leak is correctly located at node 150 (Figure 7.b). It means that with 3 sensors we are able to discriminate this leak from any other when we are analyzing the behavior along the time horizon proposed. Finally, when 4 sensors are installed (Figure 7.c), the leak can also be discriminated from any other node and it is correctly located at node 150.

5.2 Analysis for the full WDN

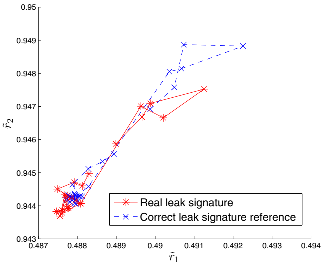
A global analysis of the capacity to locate leaks in the WDN is performed by successively simulating leaks in each of the network nodes. Here, the number of sensors may vary from 2 to 5 (see Table 1). In order to perform more realistic tests, some scenarios may include measurements noise. Here, we add Gaussian white noise to the measurements with a mean amplitude corresponding to 0.5% of the expected measurements.

Tests in a single instant time. Table 2 shows the efficiency achieved in the test for each group of sensors with and without noise in the measurements affecting the network. This table indicates the number of sensors present (column 1), the percentage of correct leak locations (column 2 and 4) and the average topological distances in case of incorrect location (column 3 and 5). In the cases where the noise is not considered, leak magnitude tested is of 4 lps ($EC = 0.6$). In cases with noise the leak magnitudes are randomly taken between 2 and 6 lps.

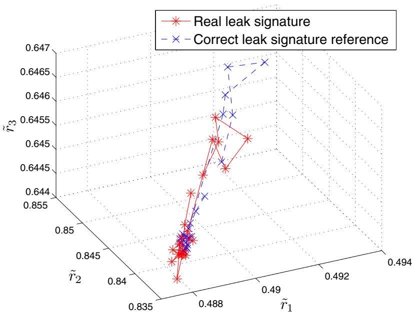
Tests with time horizon analysis. Table 3 summarizes the results for a time horizon of 24 hours. First we see a high



(a)



(b)



(c)

Fig. 7. View of a leak at the node 150 in the LSS for a 24-hour time horizon analysis in case of: 2 sensors (a), 3 sensors (b), and 4 sensors (c). Lines correspond to connections between successive time samples. The leak is correctly located when 3 and 4 sensors are present.

Table 2. Leak location efficiency when considering only one instant of time

	Without noise		With noise	
Sensors	% correct	Av. topo. dist.	% correct	Av. topo. dist.
2	61.92	2.79	55.32	3.18
3	75.63	1.76	64.97	1.78
4	85.78	1.32	74.61	1.44
5	86.29	1.30	74.61	1.44

% of correct locations without noise. Moreover, we see that even if the % of correct locations decreases when the noise is present, it remains higher than 75% with 4 and 5 sensors. Also, the average topological distance is still very low in case of incorrect location. As we have seen in Tables 2 and 3, the

Table 3. Leak location efficiency with time horizon analysis.

	Without noise		With noise	
Sensors	% correct	Av. topo. dist.	% correct	Av. topo. dist.
2	87.31	1.56	68.52	2.16
3	91.37	1.29	69.54	1.70
4	95.29	1.31	75.63	1.30
5	96.44	1.14	78.17	1.42

proposed method offers a highly efficient performance even in presence of noise and demand changes. By comparison, using the structured residuals proposed by Rosich and Puig (2013) in the Limassol, Cyprus network, it is possible to discriminate 136 of the 197 possible leaks in the exact node using 3 sensors (i.e. 69%). We also perform some preliminary comparisons with our angle method recently proposed in Casillas et al. (2014). It seems to indicate that the LSS approach can be more efficient in real scenarios because it has a lower dependency on the leak magnitude. These comparison results illustrates how the LSS representation allows to improve over other reference methods.

6. CONCLUSION

In this paper, we have presented a new scheme for leak location in water distribution networks. This scheme is based on an original representation of the leaks in a space that we call Leak Signature Space. The distance between the measured leak and the reference leaks in this space gives an estimation of the node where the leak may occur. The robustness of the method can also be improved by considering a time horizon analysis.

Experiments involving simulations on a real network have shown that such method allows an interesting ratio of correct leak detection when at least 3 sensors are present and this, even in presence of measurements noise. Also, when the node is incorrectly located, the distance from the node found and the leak node is usually small. Another advantage of our approach is that with such representation, it is easy to evaluate which of the signatures are more similar. i.e. in which cases two nodes have a high risk to be interchanged during the leak location.

For future work, a direction will be to use the LSS representation in order to optimize the sensor placement. Then, the objective would be to find the best combination of sensors to optimize the distances between the leak signatures in the leak signature space. Also, an analysis about differentiation between leak presence and sensor failure could be an interesting topic.

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