

Formation and Trajectory Tracking of Discrete-time Multi-agent Systems with Unknown Disturbances

Carlos López-Limón* Javier Ruiz-León*
Antonio Ramírez-Treviño*

* *Centro de Investigación y de Estudios Avanzados del Instituto
Politécnico Nacional (CINVESTAV-IPN) unidad Guadalajara, Av. del
Bosque 1145, Col. el Bajío, C.P. 45019, Zapopan, Jalisco, México.
(e-mail: {lopez, jrui, art }@gdl.cinvestav.mx)*

Abstract: This work proposes a design of a decentralized discrete-time block control scheme, for a multi-agent system with a fixed topology. The control scheme achieves both formation and trajectory tracking. Each agent dynamics is represented either by a first-order or second-order discrete-time system with unknown disturbances.

The position is the only measured variable, the velocity and acceleration are unknown, and only the agents that are connected with the virtual agent have access to the velocity and acceleration of the reference. Each agent is provided with discrete-time state observers to calculate the velocity and acceleration of its neighbors. Also, an estimator for the perturbations is proposed in order to attenuate them. It is proved that formation and trajectory tracking can be achieved applying the proposed block control with a consensus scheme.

Keywords: Discrete-time systems, decentralized control, multi-agent systems, state observers, perturbations estimation.

1. INTRODUCTION

A multi-agent system can be defined as a group of dynamical systems which interact with each other via the states or the outputs of their neighbors. Besides, this group has a common goal. Each element of a multi-agent system is provided with a control and position of its neighbors, in order to execute common tasks.

Multi-agent systems (*MAS*) have attracted considerable attention in Computer Science and Control Theory in the last years. The *MAS* are suitable for cooperative unmanned vehicles, formation, flocking and traffic control, transportation systems, power systems networks, and military applications, among other areas. An important topic concerning *MAS* is the problem where a group of autonomous agents must follow a predefined trajectory while maintaining a required separation (or formation) with each other. In multi-agent networks systems, the consensus means to achieve an agreement on a certain quantity of interest that depends on the state of all agents.

In the following, a brief description of some works related to the topic of this work is presented. A theoretical framework for the analysis of consensus algorithms in *MAS* is provided in Olfati-Saber et al. (2007) and Ren and Beard (2008), where an overview of methods and conditions for convergence and analysis of consensus algorithms is presented.

The finite-time consensus tracking control for multi-robot systems or *MAS* is presented in Hernández-Mendoza et al. (2011), Khoo et al. (2010) and Khoo et al. (2009). In these works, Lyapunov functions are proposed to guarantee the convergence of the sliding surfaces, and the convergence of the consensus error.

The simultaneous problem of formation and trajectory tracking of linear *MAS* is analyzed in Lopez-Limon et al. (2013a), where a distributed asymptotical control strategy is proposed.

In Wang et al. (2011), a neighbor-based local control law and a state-observer rule for discrete-time autonomous agents are presented. In that work, however, the acceleration is supposed to be known for all the agents and it is focused on first-order agent dynamics.

The design of a control law, based on observation of the agent's states, to solve the consensus problem is proposed in Liu et al. (2009), where the consensus of a *MAS* discrete-time model with second-order dynamics is considered. However, in that work the accelerations must be known by all the agents.

Protocols for discrete-time consensus tracking considering time-delays are presented in Yang et al. (2010). That work focuses on a constant reference state and first-order dynamics.

Novel protocols and conditions to achieve consensus in discrete-time *MAS* with switched and fixed topologies are introduced in Fanti et al. (2012), Wen and Xiaofan (2012)

* Carlos López-Limón is supported by CONACYT, grant No. 219300.

and Han and Chesi (2012). Nevertheless, the reference tracking problem is not addressed.

All the previously mentioned contributions have important results but most of the existing results related to MAS consensus are restricted to agents whose dynamical model are of the same dimension.

In the present work we focus on the design of a decentralized discrete-time control scheme for MAS, where each agent dynamics is represented as a first-order or second-order discrete system. The proposed control law ensures trajectory tracking, while maintaining a group formation.

The main difference with respect to previous contributions, and important advantage of our results, is that we consider the consensus and trajectory tracking of MAS, where each agent dynamical model can have different state space dimension. This overcomes previous limitations in existing results, where all the agents have the same dynamics and/or the same state space dimension. Furthermore, unknown perturbations are considered.

The proposed control scheme, based on *Discrete-time Block Control*, adds the benefits of the sliding modes to the tracking and observation problem such as finite time convergence, Lopez-Limon et al. (2013b). In this work, an observer based on sliding modes techniques, to observe the velocity and acceleration of each agent using only the output of its neighbors is also developed. The observer presents a significant improvement over traditional methods (Liu et al. (2009), Wang et al. (2011)).

This work is organized as follows. Section 2 presents the notation and some preliminaries about multi-agent systems and the discrete-time block control. In Section 3, the problem statement and the control law for the consensus and trajectory tracking are presented. In Section 4, a simulation example is shown in order to illustrate the application of the proposed control strategy. Finally, the conclusions and future work are presented in Section 5.

2. NOTATION AND PRELIMINARIES

This section introduces some basic concepts and notation about graph theory, communication topologies, consensus and discrete block control.

2.1 Graph Theory and Communication Topologies

The communication of a group of N mobile agents is commonly described by a directed graph, $G = (\mathcal{V}, \xi)$, where $\mathcal{V} = \{0, 1, 2, \dots, N\}$ is a set of nodes, $\xi \in \{\mathcal{V} \times \mathcal{V}\}$ is a set of edges connecting nodes (self edges are not allowed). The adjacency matrix associated to G is denoted by $\mathcal{A} = [\alpha_{i,j}] \in \mathbb{R}^{N \times N}$. An edge $(\nu_i, \nu_j) \in \xi$ means that node ν_j can get information from node ν_i but not necessarily *vice versa*, in the edge (ν_i, ν_j) the element ν_i is the *parent* node and ν_j is the *child* node. For this work, the node ν_0 represents the reference or virtual agent. If an edge (ν_i, ν_j) is contained in ξ , then $\alpha_{j,i} = 1$ and 0 otherwise. The set of neighbors of node i at a time t will be denoted by $\mathcal{N}_i = \{\nu_j : (\nu_j, \nu_i) \in \xi, j = 1, \dots, N\}$.

A *directed path* is a sequence of edges in a directed graph of the form $(\nu_0, \nu_1), (\nu_1, \nu_2), \dots, (\nu_i, \nu_j)$. A *cycle* is a directed

path that starts and ends at the same node. A *directed tree* is a directed graph in which every node has exactly one parent except for the root node which has no parent and has a directed path to every other node. A directed tree has no cycles because every edge is oriented away from the root. A directed graph G has a *directed spanning tree* if and only if G has at least one node with a directed path to all other nodes.

Note that at least a directed tree is required to achieve consensus and tracking in a MAS because a directed path is the only way that all the agents can obtain the information for the reference, Lafferriere et al. (2005); Ren and Beard (2008).

The next assumptions are considered in this work:

Hypothesis 1. The graph G incorporates a directed spanning tree with root on ν_0 (reference) for any communication topology.

Hypothesis 2. The nodes connected to ν_0 know the position, velocity and acceleration of the reference.

The consensus is achieved or reached by the states of the agents if all of them converge to the same value. In this work the position (x) is the variable for the consensus, i.e., for all the agents it is intended that $\lim_{t \rightarrow \infty} |x_i - x_j| = 0$ and consequently also the velocity (v) achieves consensus $\lim_{t \rightarrow \infty} |v_i - v_j| = 0 \forall i, j = 0, 1, \dots, N$.

2.2 Discrete Block Control

The sliding mode control has proved its high accuracy and robustness with respect to various internal and external disturbances. The above produces the so called chattering effect, i.e., dangerous high-frequency vibrations in the controlled system.

In order to avoid the chattering, there exist many techniques Utkin et al. (1999), and the one that we choose in this work is the discrete-time block control Sanchez et al. (2008).

Consider the following discrete-time system that with a nonsingular transformation, Loukianov et al. (2002); Utkin and Chang (2002), can be taken into the block controllable form, which in the particular case of single-input single-output systems is represented as

$$\begin{aligned} x_1[k+1] &= a_{1,1}x_1[k] + b_1x_2[k] + d_1[k], \\ x_2[k+1] &= a_{2,1}x_1[k] + a_{2,2}x_2[k] + b_2x_3[k] + d_2[k], \\ &\vdots \\ x_{n-1}[k+1] &= \sum_{i=1}^{n-1} a_{n-1,i}x_i[k] + b_{n-1}x_n[k] + d_{n-1}[k], \\ x_n[k+1] &= \sum_{i=1}^n a_{n,i}x_i[k] + b_nu[k] + d_n[k], \\ y[k] &= x_1[k], \quad i = 1, 2, \dots, r-1, \end{aligned} \quad (1)$$

where $x[k] = [x_1[k] \dots x_n[k]]^T$ is the state vector, $u[k]$ is the input, a_\bullet and b_\bullet are constants. The vector function $d[k] = [d_1[k] \dots d_n[k]]^T$ is a disturbance vector which satisfies $|d[k] - d[k-1]| \leq c$ where c is a constant vector.

Using the transformation $z_i[k] = x_i[k] - x_i^d[k]$, system (1) can be rewritten as

$$\begin{aligned} z_1[k+1] &= L_1 z_1[k] - b_1 z_2[k], \\ &\vdots \\ z_{n-1}[k+1] &= L_{n-1} z_{n-1}[k] - b_{n-1} z_n[k], \\ z_n[k+1] &= \sum_{i=1}^n a_{n,i} x_i[k] - b_n u[k] + d_n[k] - x_n^d[k+1], \end{aligned} \quad (2)$$

where $y_0[k] = x_1^r[k]$ is the desired trajectory to be tracked, x_i^d is the desired value for x_i ($i = 1, 2, \dots, n$), and L_i is a constant where $|L_i| < 1$. Using a sliding mode block control technique, the sliding manifold is $S_D[k] = z_n[k] = 0$. Thus, $S_D[k+1] = \sum_{i=1}^n a_{n,i} x_i[k] - b_n u[k] + d_n[k] - x_n^d[k+1]$. Once the sliding manifold is chosen, the next step is to define the input $u[k]$ as

$$u[k] = \begin{cases} u_{eq}[k] & \text{for } \|u_{eq}[k]\| \leq u_0, \\ u_0 \frac{u_{eq}[k]}{\|u_{eq}[k]\|} & \text{for } \|u_{eq}[k]\| > u_0, \end{cases} \quad (3)$$

where the equivalent control is calculated from $S_D[k+1] = 0$ as

$$u_{eq}[k] = [b_n]^{-1} \left(- \sum_{i=1}^n a_{n,i} x_i[k] - d_n[k] + x_n^d[k+1] \right), \quad (4)$$

and $u_0 > |[b_n]^{-1}| \left(-x_n[k] + x_n^d[k] + \sum_{i=1}^n a_{n,i} x_i[k] + d_n[k] - x_n^d[k+1] \right)|$.

From Sanchez et al. (2008), we have the next result

Theorem 3. The control law (3) ensures that the sliding manifold $S_D[k] = z_n[k] = 0$ is stable for system (1).

3. PROBLEM DEFINITION

In this work, each agent is represented by a linear discrete-time system, the dynamics of the second-order agents is given by

$$\begin{aligned} x_i[k+1] &= x_i[k] + T_s v_i[k], \\ v_i[k+1] &= v_i[k] + u_i[k] + d_i[k], \\ y_i[k] &= x_i[k], \end{aligned} \quad (5)$$

where, for convenience instead of using $x_{i,1}$ and $x_{i,2}$ for the states, x_i is the state of position, v_i is the state of velocity, u_i is the input, y_i is the output, d_i is an unknown perturbation and T_s is the sample time. For the function d_i it is supposed that $|d_i[k] - d_i[k-1]| \leq d_{M_i} \forall k$, where d_{M_i} is a constant.

The first-order agents are also considered for the control law, and are represented as

$$\begin{aligned} x_i[k+1] &= x_i[k] + u_i[k] + d_i[k], \\ y_i[k] &= x_i[k], \end{aligned} \quad (6)$$

and the virtual leader dynamics is given by

$$\begin{aligned} x_0[k+1] &= x_0[k] + T_s v_0[k], \\ v_0[k+1] &= v_0[k] + T_s a_0[k], \\ y_0[k] &= x_0[k], \end{aligned} \quad (7)$$

where a_i is the acceleration. Note that the systems (5) and (6) are in the block controllable form (1).

We define the tracking and formation error for the position as

$$e_{i,1}[k] = \sum_{j=0}^N \alpha_{i,j} \left((x_i[k] - x_j[k]) - \Delta_{i,j} \right), \quad (8)$$

where $\Delta_{i,j}$ is the separation between the outputs of the agents i and j .

Thus, it is required to design a control law such that a MAS composed by N agents with system dynamics (5) and/or (6) achieve formation and trajectory tracking. Considering the defined error (8) it is required that

$$\lim_{t \rightarrow \infty} e_{i,1}[k] = \lim_{t \rightarrow \infty} \sum_{j=0}^N \alpha_{i,j} (x_i[k] - x_j[j] - \Delta_{i,j}) = 0, \quad (9)$$

$$\forall i = 1, 2, \dots, N.$$

3.1 Observer for the neighbors states

The velocity and the acceleration of the neighbors of each agent are required to calculate an appropriate control law for trajectory tracking and formation, therefore we need to design an observer in order to calculate them, the measurements from the other agents is only the position.

Then, each agent is provided with an observer for each neighbor. The proposed observer is given by

$$\begin{aligned} \hat{x}_i[k+1] &= \hat{x}_i[k] + T_s \hat{v}_i[k], \\ \hat{v}_i[k+1] &= \hat{v}_i[k] + T_s \hat{a}_i[k] + w_i[k], \\ \hat{y}_i[k] &= \hat{x}_i[k], \end{aligned} \quad (10)$$

where \hat{x}_i , \hat{v}_i and \hat{a}_i are the position, velocity and acceleration, respectively, of the observers. The term $w_i[k]$ is a function introduced to reduce the observer error. In order to obtain the acceleration $\hat{a}_i[k]$ we define the equation $\hat{a}_i[k+1] = \hat{a}_i[k] + (\hat{a}_i[k] - \hat{a}_i[k-1])$ where $\hat{a}_i[k] = (1/T_s)(\hat{v}_i[k+1] - \hat{v}_i[k])$, and finally we have $\hat{a}_i[k+1] = (1/T_s)(2\hat{v}_i[k+1] - 3\hat{v}_i[k] + \hat{v}_i[k-1])$.

Now, using the following transformation

$$\begin{aligned} z_{o(i,1)}[k] &= \hat{x}_i[k] - x_i[k], \\ z_{o(i,2)}[k] &= \hat{v}_i[k] - v_i^d[k], \end{aligned} \quad (11)$$

to determine the value of $v_i^d[k]$ we use

$$\begin{aligned} z_{o(i,1)}[k+1] &= (\hat{x}_i[k] + T_s \hat{v}_i[k]) - (x_i[k] + T_s v_i[k]), \\ z_{o(i,2)}[k+1] &= (\hat{v}_i[k] + T_s \hat{a}_i[k]) - v_i^d[k+1] + w_i[k], \end{aligned} \quad (12)$$

where the velocity can be approximated by $\bar{v}_i[k] = v_i[k-1] + (v_i[k-1] - v_i[k-2])$ and represented in position changes $\bar{v}_i[k] = (1/T_s)(3x_i[k] - 6x_i[k-1] + 4x_i[k-2] - x_i[k-3])$.

It is not possible to calculate $v_i^d[k]$ and $v_i^d[k+1]$ since we have no access to $v_i[k]$, then an approximation $\hat{v}_i^d[k]$ is required. Now, we select a $\hat{v}_i^d[k]$ such that when $\hat{v}_i[k] \rightarrow \hat{v}_i^d[k]$ in order to obtain $z_{o(i,1)}[k+1] = 0$,

$$\begin{aligned} \hat{v}_i^d[k] &= (1/T_s)(-\hat{x}_i[k] + x_i[k] + T_s \bar{v}_i[k]), \\ \hat{v}_i^d[k+1] &= (1/T_s)(-\hat{x}_i[k] + T_s \hat{v}_i[k] \\ &\quad + (x_i[k] + T_s \bar{v}_i[k]) + T_s(\bar{v}_i[k] + T_s \bar{a}_i[k])), \end{aligned} \quad (13)$$

where $\bar{a}_i[k] = a_i[k-2] + (a_i[k-2] - a_i[k-3])$ and represented in position changes $\bar{a}_i[k] = (1/T_s^2)(2x_i[k] - 5x_i[k-1] + 4x_i[k-2] - x_i[k-3])$.

It is intended that $z_{o(i,2)}[k+1] = 0$, then solving for $w_i[k]$ from (12) we have that

$$w_i[k] = -(\hat{v}_i[k] + T_s \hat{a}_i[k]) + \hat{v}_i^d[k+1]. \quad (14)$$

Notice that the previous observer is also valid for the first-order agents.

3.2 Observation error computation

Now, we calculate the error of the observer for the position $e_{\hat{x}_i}$ when $z_{o(i,1)}[k+1] = 0$. The position error obtained once the observer converges is given by

$$\begin{aligned} e_{\hat{x}_i}[k] &= x_i[k] - \hat{x}_i[k], \\ &= T_s(v_i[k-1] - \bar{v}_i[k-1]), \\ &= T_s((v_i[k-2] + (v_i[k-1] - v_i[k-2])) \\ &\quad - (v_i[k-2] - (v_i[k-2] - v_i[k-3]))), \\ &= T_s((v_i[k-1] - v_i[k-2]) - (v_i[k-2] - v_i[k-3])), \\ e_{\hat{x}_i}[k] &= T_s^2 \delta_{ai}[k-2], \end{aligned} \quad (15)$$

where $\delta_{ai}[k-2] = a_i[k-2] - a_i[k-3]$, and the position error is bounded by $|e_{\hat{x}_i}| \leq T_s^2 \max(|\delta_{ai}|)$.

In order to compute the error of the observer for the velocity by $e_{\hat{v}_i}$ we consider $z_{o(i,2)}[k+1] = 0$, when the observer converges and the velocity is given by $\hat{v}_i^d[k]$, then

$$\begin{aligned} e_{\hat{v}_i}[k] &= v_i[k] - \hat{v}_i^d[k], \\ &= v_i[k] - (1/T_s)(x_i[k] - \hat{x}_i[k] + T_s \bar{v}_i[k]), \\ &= v_i[k] - \bar{v}_i[k] - (1/T_s)(x_i[k] - \hat{x}_i[k]), \\ &= (1/T_s)(e_{\hat{x}_i}[k+1] - e_{\hat{x}_i}[k]), \\ e_{\hat{v}_i}[k] &= T_s \gamma_{ai}[k-1], \end{aligned} \quad (16)$$

where $\gamma_{ai}[k-1] = \delta_{ai}[k-1] - \delta_{ai}[k-2]$, and the velocity error is bounded by $|e_{\hat{v}_i}| \leq T_s \max(|\gamma_{ai}|)$.

Finally, to calculate the error of the observer for the acceleration we have

$$\begin{aligned} e_{\hat{a}_i}[k] &= a_i[k] - \hat{a}_i[k], \\ &= (2a_i[k-1] - a_i[k-2] + \beta_{ai}[k]) - (1/T_s)(2\hat{v}_i[k] \\ &\quad - 3\hat{v}_i[k-1] + \hat{v}_i[k-2]), \\ &= (1/T_s)((2v_i[k] - 3v_i[k-1] + v_i[k-2]) - (2\hat{v}_i[k] \\ &\quad - 3\hat{v}_i[k-1] + \hat{v}_i[k-2])) + \beta_{ai}[k], \\ e_{\hat{a}_i}[k] &= (1/T_s)(2e_{\hat{v}_i}[k] - 3e_{\hat{v}_i}[k-1] + e_{\hat{v}_i}[k-2]) + \beta_{ai}[k], \end{aligned} \quad (17)$$

where $\beta_{ai}[k] = a_i[k] - (2a_i[k-1] - a_i[k-2])$, and the acceleration error is bounded by $|e_{\hat{a}_i}| \leq \max(|2\gamma_{ai}[k-1] - 3\gamma_{ai}[k-2] + \gamma_{ai}[k-3] + \beta_{ai}[k]|)$.

3.3 Control design

In order to design the block control for a second-order dynamics we use the following transformation

$$z_{i,1}[k] = \sum_{j=0}^N \alpha_{i,j}((x_i[k] - \hat{x}_j[k]) - \Delta_{i,j}), \quad (18)$$

$$z_{i,2}[k] = v_i[k] - v_i^d[k],$$

where $\Delta_{i,j}$ is the desired separation between the outputs of the agents i and j , and

$$z_{i,1}[k+1] = \sum_{j=0}^N \alpha_{i,j}((x_i[k] + T_s v_i[k] - \Delta_{i,j}) - \hat{x}_j[k+1]).$$

Now we calculate $v_i^d[k]$, which is the desired value of $v_i[k]$, to get dynamics as $z_{i,1}[k+1] = L_{i,1} z_{i,1}[k] + z_{i,2}[k]$ and $z_{i,2}[k+1] = 0$ with $|L_{i,1}| < 1$.

Define $\ell_i = \sum_{j=0}^N \alpha_{i,j}$, then

$$\begin{aligned} v_i^d[k] &= (-T_s \ell_i)^{-1} \left(\sum_{j=0}^N \alpha_{i,j}((x_i[k] - \hat{x}_i[k+1]) \right. \\ &\quad \left. - \Delta_{i,j}) - L_{i,1} z_{i,1}[k] - z_{i,2}[k] \right), \\ v_i^d[k+1] &= (-T_s \ell_i)^{-1} \left(\sum_{j=0}^N \alpha_{i,j}(x_i[k+1] - \hat{x}_i[k+1] \right. \\ &\quad \left. - T_s \hat{v}_i[k+1] - \Delta_{i,j} - L_{i,1} z_{i,1}[k+1]) \right). \end{aligned} \quad (19)$$

where, according to Sanchez et al. (2008), $z_{i,2}[k] = v_i[k] - (L_{i,1} z_{i,1}[k] - x_i[k])$.

The following is to determine u_i from $z_{i,2}[k]$ on (18), where we get

$$z_{i,2}[k+1] = (v_i[k] + u_i[k] + d_i[k]) - v_i^d[k+1], \quad (20)$$

and solving for u_i from the previous equation we have the equivalent input

$$u_{i,eq}[k] = v_i^d[k+1] - v_i[k] - \bar{d}_i[k], \quad (21)$$

where $\bar{d}_i[k] = d_i[k-1] + (d_i[k-1] - d_i[k-2])$ which can be rewritten as

$$\bar{d}_i[k] = 2(v_i[k] - v_i[k-1] - u_i[k-1]) - (v_i[k-1] - v_i[k-2] - u_i[k-2]). \quad (22)$$

Notice that the input (21) is to be applied to agents with second-order dynamics. Now, for the agents with first-order dynamics the $u_{i,eq}$ can be obtained from (19), and the control law results as

$$\begin{aligned} u_{i,eq}[k] &= -(\ell_i)^{-1} \left(\sum_{j=0}^N \alpha_{i,j}((x_i[k] - \Delta_{i,j}) \right. \\ &\quad \left. - \hat{x}_i[k+1]) + \ell_i \bar{d}_i[k] - L_{i,1} z_{i,1}[k] \right), \end{aligned} \quad (23)$$

where $\bar{d}_i[k]$ is given by

$$\bar{d}_i[k] = 2(x_i[k] - x_i[k-1] - u_i[k-1]) - (x_i[k-1] - x_i[k-2] - u_i[k-2]). \quad (24)$$

In general

$$u_i[k] = \begin{cases} u_{i,eq}[k] & \text{for } |u_{i,eq}[k]| \leq u_{i,0}, \\ u_{i,0} \text{sign}(u_{i,eq}[k]) & \text{for } |u_{i,eq}[k]| > u_{i,0}, \end{cases} \quad (25)$$

where $u_{i,eq}[k]$ is (21) or (23), corresponding to the dynamics of the i -th agent.

The next result presents the conditions to achieve formation and trajectory tracking for a MAS.

Theorem 4. Consider a set of N discrete agents with first-order (6) and/or second-order (5) dynamics, and suppose that the Hypothesis 1 and Hypothesis 2 hold for the topology communication of the MAS. Then, formation and trajectory tracking of the MAS is achieved with the control law u_i given by (25).

Proof. The proof is presented in Appendix A at the end of the paper.

4. EXAMPLE

The following example illustrates the results presented in the previous section. Consider four agents of second-order and first-order dynamics. The sampling time is $T_s = 0.1$ seconds, the bound of the inputs is $u_{0(i)} = 2$ and the bound for the acceleration $a_{\bullet} < 2$, are bounds of the system

The first three agents are taken as second-order systems and the fourth agent is a first-order system

The reference function to be tracked is given by

$$\begin{aligned} x_0[k+1] &= x_0[k] + 0.1v_0[k], \\ v_0[k+1] &= v_0[k] + 0.1a_0[k], \\ y_0[k] &= x_0[k], \\ a_0[k] &= -\sin(0.1 * k), \end{aligned} \quad (26)$$

and the unknown perturbations are taken as $d_1[k] = -0.1 \cos(0.1 * k)$, $d_2[k] = -0.1 \sin(0.1 * k)$, $d_3[k] = 0.1 \sin(0.1 * k)$ and $d_4[k] = 0.1 \cos(0.1 * k)$.

The agent initial conditions are $x_1[0] = [-1 \ 0]^T$, $x_2[k] = [2 \ 0]^T$, $x_3[k] = [1 \ 0]^T$ and $x_4[k] = 4$.

Let us define a separation from each agent to the output of its neighbors, given by the distances $\Delta_{1,0} = 0$, $\Delta_{2,3} = 2$, $\Delta_{2,4} = -1$, $\Delta_{3,1} = 1$, $\Delta_{4,1} = 2$ and $\Delta_{4,2} = 1$. The separation of each agent is determined by the initial conditions and the required formation.

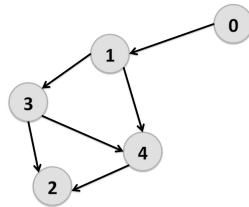


Fig. 1. Topology graph that represents the communication topology.

We choose the constants $L_{1,1} = 0.8$, $L_{2,1} = 0.7$, $L_{3,1} = 0.7$ and $L_{4,1} = 0.6$ to impose the desired error dynamics.

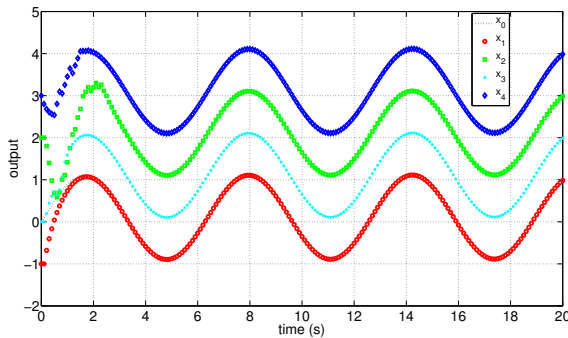


Fig. 2. Output path of each agent with respect to the reference signal.

5. CONCLUSIONS

In this work, we presented a discrete-time control law strategy for the formation and trajectory tracking of MAS with fixed topologies, using a block control scheme.

Each agent can have a discrete-time dynamics representation with different dimension from the other agents (first-order or second-order).

The design of an observer to estimate the velocity and acceleration reduces the communication requirements to the knowledge of only the position of the agent's neighbors.

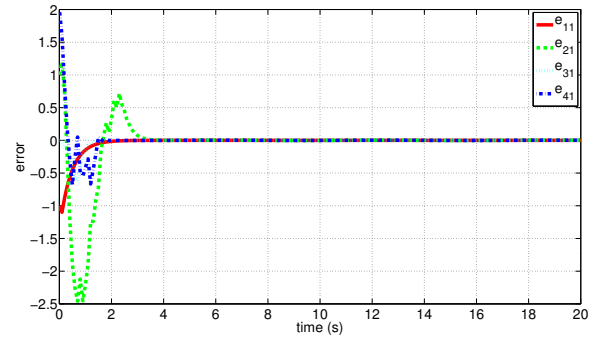


Fig. 3. Formation and trajectory tracking error of each agent.

An estimation of the perturbation in the agent dynamics is designed in order to attenuate the disturbances.

With the proposed control, the error dynamics and the dynamics of each agent are ensured to be stable.

As future work, the results presented in this paper can be extended to consider the case of coupled *multiple-input multiple-output* discrete-time systems and including the use of potential fields in order to avoid collisions. Also, it can be addressed the study of other control techniques in order to consider a discrete communication with delays.

REFERENCES

- Fanti, M., Mangini, A., Ukovich, W., and Boschian, V. (2012). New consensus protocols for networks with discrete time dynamics. In *American Control Conference (ACC), 2012*, 38–43.
- Han, D. and Chesi, G. (2012). Robust discrete-time consensus of multi-agent systems with uncertain interaction. In *Control Applications (CCA), 2012 IEEE International Conference on*, 1136–1141.
- Hernández-Mendoza, D., Peñalosa Mendoza, G.R., and Aranda-Bricaire, E. (2011). Discrete-time formation and marching control of multi-agent robots systems. In *Electrical Engineering Computing Science and Automatic Control (CCE), 2011 8th International Conference on*, 1–6.
- Khoo, S., Xie, L., and Man, Z. (2010). Leader-follower consensus control of a class of nonholonomic systems. In *Proc. 11th Int Control Automation Robotics & Vision (ICARCV) Conf*, 1381–1386.
- Khoo, S., Xie, L., Man, Z., and Zhao, S. (2009). Observer-based robust finite-time cooperative consensus control for multi-agent networks. In *Proc. 4th IEEE Conf. Industrial Electronics and Applications ICIEA 2009*, 1883–1888.
- Lafferriere, G., Williams, A., Caughman, J., and Veerman, J. (2005). Decentralized control of vehicle formations. *Systems and control letters*, 54(9), 899–910.
- Liu, B., Liang, H., Zhang, J., and Wang, J. (2009). Leader-following control of multi-agent consensus for a discrete-time case. In *Artificial Intelligence and Computational Intelligence, 2009. AICI '09. International Conference on*, volume 4, 11–15.
- Lopez-Limon, C., Ruiz-Leon, J., Cervantes-Herrera, A., and Ramirez-Trevino, A. (2013a). Consensus and trajectory tracking of siso linear multi-agent systems un-

der switching communication topologies and formation changes. *Kybernetika*, 49(2), 375–390.

Lopez-Limon, C., Ruiz-Leon, J., Cervantes-Herrera, A., and Ramirez-Trevino, A. (2013b). Formation and trajectory tracking of discrete-time multi-agent systems using block control. In *Emerging Technologies Factory Automation (ETFA), 2013 IEEE 18th Conference on*, 1–4.

Loukianov, A.G., Rivera, J., and Cañedo, J.M. (2002). Discrete-time sliding mode control of an induction motor. In *2002 IFAC 15th Triennial World Congress*, 1074–1079.

Olfati-Saber, R., Fax, J.A., and Murray, R.M. (2007). Consensus and cooperation in networked multi-agent systems. *Proceedings of the IEEE*, 95(1), 215–233.

Ren, W. and Beard, R.W. (2008). *Distributed Consensus in Multi-vehicle Cooperative Control*. Springer-Verlag.

Sanchez, E.N., Alanis, A.Y., and Loukianov, A.G. (2008). *Discrete-Time Block Control*. Springer-Verlag.

Utkin, V., Guldner, J., and Shi, J. (1999). *Sliding Mode Control in Electromechanical Systems*. Taylor & Francis.

Utkin, V.I. and Chang, H.C. (2002). Sliding mode control on electro-mechanical systems. *Mathematical Problems in Engineering*, 8(4-5), 451–473.

Wang, F., Gao, L., and Wang, L. (2011). Tracking control for multi-agent consensus for discrete-time case. In *Control and Decision Conference (CCDC), 2011 Chinese*, 923–928.

Wen, Y. and Xiaofan, W. (2012). Controllability of discrete-time consensus protocol under fixed and switched leaders. In *Control Conference (CCC), 2012 31st Chinese*, 6047–6052.

Yang, Y., Wu, X., and Gui, Z. (2010). Consensus tracking with a reference state using discrete-time algorithms. In *Intelligent Computing and Intelligent Systems (ICIS), 2010 IEEE International Conference on*, volume 2, 157–160.

Appendix A. PROOF OF THEOREM 4

First, it is necessary to analyze the influence of the connections in the control applied to the agents of the MAS. Suppose we have N second-order agents and a connection represented for an adjacency matrix as

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & 1 & 0 & \dots & 0 & 0 \\ 1 & 1 & 1 & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & 1 & \dots & 1 & 0 \end{bmatrix}, \quad (\text{A.1})$$

where the triangular form represents the communication topology satisfying $(\mathcal{H}1)$ and has the highest possible number of edges.

Now, for a MAS with an adjacency matrix as (A.1), inputs (25), and for simplicity $\Delta_{i,j} = 0 \forall i, j$, we have

$$\begin{aligned} z_{1,1}[k] &= x_1[k] - x_0[k], \\ z_{1,1}[k+1] &= L_{1,1}z_{1,1}[k] + z_{1,2}[k], \end{aligned} \quad (\text{A.2})$$

$$\begin{aligned} z_{1,2}[k] &= v_1[k] - v_1^d[k], \\ z_{1,2}[k+1] &= 0. \end{aligned}$$

Carrying out the same analysis for the next agent, then

$$\begin{aligned} z_{2,1}[k] &= (x_2[k] - x_0[k]) + (x_2[k] - x_1[k]), \\ z_{2,1}[k+1] &= (x_2[k] + T_s v_2[k] - x_0[k+1]) \\ &\quad + (x_2[k] + T_s v_2[k] - x_1[k] - T_s v_1^d[k]), \\ z_{2,1}[k+1] &= L_{2,1}z_{2,1}[k] + z_{2,2}[k] + T_s(\hat{v}_1[k] - v_1^d[k]), \end{aligned} \quad (\text{A.3})$$

which can be rewritten as

$$\begin{aligned} z_{2,1}[k+1] &= L_{2,1}z_{2,1}[k] + z_{2,2}[k] + m_{2,1}z_{1,1}[k] + m_{2,2}z_{1,2}[k] \\ &\quad + h_{2,0}(x_0, v_0) + h_{2,1}(\hat{x}_1, \hat{v}_1), \end{aligned} \quad (\text{A.4})$$

where $m_{2,1}, m_{2,2}$ are constants and $h_{2,0}$ and $h_{2,1}$ are bounded functions. Continuing with $z_{2,2}[k]$ we have

$$\begin{aligned} z_{2,2}[k] &= v_2[k] - v_2^d[k], \\ z_{2,2}[k+1] &= (d_2[k] - \bar{d}_2[k]) + h'_{2,0}(v_0, a_0) + h'_{2,1}(\hat{v}_1, \hat{a}_1), \end{aligned} \quad (\text{A.5})$$

where $h'_{2,0}$ and $h'_{2,1}$ are bounded functions. Carrying out the procedure with all the remaining elements of z we can construct a triangular matrix as

$$Fz[k] = \begin{bmatrix} L_{1,1} & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ m_{2,1} & 0 & L_{2,1} & 0 & \dots & 0 & 0 \\ 0 & m'_{2,1} & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ m_{N,1} & 0 & m_{N,2} & 0 & \dots & L_{N,1} & 1 \\ 0 & m'_{N,1} & 0 & m'_{N,2} & \dots & 0 & 0 \end{bmatrix} \mathbf{z}[k], \quad (\text{A.6})$$

where $\mathbf{z} = [z_{1,1}[k], z_{1,2}[k], \dots, z_{N,1}[k], z_{N,2}[k]]^T$ and the complete system can be rewritten as

$$\mathbf{z}[k+1] = F\mathbf{z}[k] + H(x, v, a) + D(d), \quad (\text{A.7})$$

where H and D are ultimately bounded vector functions composed by all the elements h and h' and the differences $(d_i[k] - \bar{d}_i[k])$, respectively. From (A.7) it can be seen that the stability of the system depends exclusively on the elements of the diagonal, since F is a lower triangular matrix. Then, if $|L_{i,1}| < 1 \forall i$, the whole discrete-time system is stable and the formation and trajectory tracking is achieved.

For the general case, any adjacency matrix \mathcal{A} that represents a communication topology which satisfies Hypotesys 1 can be taken to a lower triangular form because there are no cycles in G .

The adjacency matrix \mathcal{A} can be taken to a lower triangular matrix \mathcal{A}' with a transformation that interchange rows and columns, and the stability analysis holds.

The analysis including agents of first-order can be done in a similar way.