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Behavior-Based Formation Control Through Adaptive Super Twisting Algorithm

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Abstract: In this paper, a formation control for a group of unicycle mobile robots is presented. According to this approach, complex maneuvers are decomposed into a sequence of formation patterns. With the aim of driving the movements of the robots, a decentralized control strategy using the Adaptive Super Twisting Algorithm (ASTA) is introduced. The proposed control scheme increases robustness against unknown dynamics and disturbance, as not all the parameters of the system nor the bound of the perturbation are required to be known.

Keywords: Adaptive Control, Mobile Robots, Multi-Robot System, Behaviour-based Approach, Formation Control.

1. INTRODUCTION

Applications of robotics in daily life are being increased enormously, for example service robots for cleaning (Yuan et al. (2011)) or transporting (Tsay et al. (2003)). Furthermore, there are tasks such as surveillance, exploration, search and rescue, which can be performed more efficiently by a group of robots instead of a single one. Moving large objects represents another example of coordinated tasks, where the robots move holding a rigid formation to displace the oversize object. These kinds of cooperative works involve robots moving with a defined formation to maximize detection capabilities.

In order to tackle the formation control, several methods have been proposed. For example, in the virtual structure approach (Ren et al. (2004)), the whole system is consi -dered as a single rigid structure or entity and the desired path is assigned to the structure, while maintaining a rigid formation. Potential field methods (Ge (2002)), focus on filling the workspace of the robots with an artificial potential field, where the robot is attracted to the target position and then is repelled from obstacles. According to the leader-follower method (Alexandre et al. (2009)), a robot is defined as the responsible leader for guiding all the other robots involved in the formation, in such a way that they reach desired positions and keep the composed formation while moving. Behavior-based methods represent a different approach, where the desired behaviors for each robot are prescribed, while the final action is derived from a weighting of the relative importance of each behavior (Lawton et al. (2002)).

On the other hand, a control based on sliding modes technique is the Super-Twisting Algorithm (Levant (2003)), which is used in many applications due to its robustness properties. Furthermore, it is designed to converge in a finite-time. However, under this approach, the bounds of uncertainties and perturbations present on the system are required to be known. Adaptive Super-Twisting Algorithm (Shtessel et al. (2012)) represents an interesting alternative as it is not necessary to known the bounds of the perturbations.

The layout of this paper is as follows: In Section 2, a system description and a mathematical model of a mobile robot are provided. The formation control problem, as motion between a sequence of formation patterns and the corresponding coupled dynamics control, is introduced in section 3. The design of an Adaptive Super-Twisting Control for driving positions of the mobile robots is addressed in section 4. With the aim of implementing the proposed controller, necessary information is estimated through a robust differentiator introduced in section 5. In order to illustrate the feasibility and performance of the proposed scheme, simulation results are given in section 6. Finally, conclusions of this work are drawn.

2. SYSTEM DESCRIPTION: UNICYCLE MOBILE ROBOT DYNAMICAL MODEL

Unicycle mobile robots represent an attractive combination of a simple wheel configuration with a high traction through pneumatic tires (see Fig. 1). In this work, slip uncertainties vector and moments of inertia of the motor, gear and wheel, are assumed as null. Moreover, it is considered that the center of mass coincide with the geometric center. Consider a system composed by a group of N robots, where the mathematical model (Zhang et al. (1998); Lawton et al. (2002)) of the i-th robot for $i = 1, 2, \ldots, N$; is defined as

$$\begin{pmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \\ \dot{v}_i \\ \dot{\phi}_i \end{pmatrix} = \begin{pmatrix} v_i cos(\theta_i) - L_i\phi_i sin(\theta_i) \\ v_i sin(\theta_i) + L_i\phi_i cos(\theta_i) \\ \phi_i \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{m_i} & 0 \\ 0 & \frac{1}{I_i} \end{pmatrix} \begin{pmatrix} F_i \\ \tau_i \end{pmatrix}, \quad (1)$$

where θ_i denotes the orientation, v_i the linear speed, ϕ_i the angular speed, F_i and τ_i correspond to the force and torque applied at the geometric center respectively. m_i correspond to the mass and I_i to the moment of inertia. The system output is described as

$$\Pi_i = \begin{pmatrix} x_i & y_i \end{pmatrix}^T \in \Re^b, \qquad i = 1, 2, ..., N. \quad (2)$$

The equation (2) describes the position of the i-th robot in the Cartesian space, and is defined as a point located a distance L_i along the line that is perpendicular to the wheel axis and intersects the geometric center.

Taking the derivative of (2) with respect to time, it follows that

$$\dot{\Pi}_i = \begin{pmatrix} cos(\theta_i) & -L_i sin(\theta_i) \\ sin(\theta_i) & L_i cos(\theta_i) \end{pmatrix} \begin{pmatrix} v_i \\ \phi_i \end{pmatrix}. \quad (3)$$

From the second derivative of (2) we have

$$\ddot{\Pi}_i = \begin{pmatrix} -v_i\phi_i sin(\theta_i) - L_i\phi_i^2 cos(\theta_i) \\ v_i\phi_i cos(\theta_i) - L_i\phi_i^2 sin(\theta_i) \end{pmatrix} + \begin{pmatrix} \frac{1}{m_i} cos(\theta_i) & -\frac{L_i}{I_i} sin(\theta_i) \\ \frac{1}{m_i} sin(\theta_i) & \frac{L_i}{I_i} cos(\theta_i) \end{pmatrix} \begin{pmatrix} F_i \\ \tau_i \end{pmatrix}. \quad (4)$$

Taking into account

$$det \begin{pmatrix} \frac{1}{m_i} cos(\theta_i) & -\frac{L_i}{I_i} sin(\theta_i) \\ \frac{1}{m_i} sin(\theta_i) & \frac{L_i}{I_i} cos(\theta_i) \end{pmatrix} \neq 0, \quad i = 1, 2, ..., N.$$

The system (1)-(2) has constant relative degree equals vector $\{2, 2\}$, and can be output feedback linearized about the position Π_i (Lawton et al. (2002)). By defining the map $\varphi : \Re^n \to \Re^n$ as

$$\zeta_i = \varphi(X_i) = \begin{pmatrix} x_i \\ y_i \\ v_i cos(\theta_i) - L_i\phi_i sin(\theta_i) \\ v_i sin(\theta_i) + L_i\phi_i cos(\theta_i) \\ \theta_i \end{pmatrix}, \quad (5)$$

where $X_i = (x_i, y_i, \theta_i, v_i, \phi_i)^T$ is the vector state of the system (1). The inverse of (5) is given by

$$X_i = \varphi^{-1}(\zeta_i) = \begin{pmatrix} \zeta_{1i} \\ \zeta_{2i} \\ \zeta_{5i} \\ \zeta_{3i} cos(\zeta_{5i}) + \zeta_{4i} sin(\zeta_{5i}) \\ -\frac{1}{L_i}\zeta_{3i} sin(\zeta_{5i}) + \frac{1}{L_i}\zeta_{4i} cos(\zeta_{5i}) \end{pmatrix}, \quad (6)$$

for $i = 1, 2, \ldots, N$. Then, equations (1) and (2) can be written as

$$\begin{aligned} \begin{pmatrix} \dot{\zeta}_{1i} \\ \dot{\zeta}_{2i} \end{pmatrix} &= \begin{pmatrix} \zeta_{3i} \\ \zeta_{4i} \end{pmatrix} \\ \begin{pmatrix} \dot{\zeta}_{3i} \\ \dot{\zeta}_{4i} \end{pmatrix} &= \alpha_i(\zeta_i) + \beta_i(\zeta_i)u_i \\ \dot{\zeta}_{5i} &= -\frac{1}{L_i}\zeta_{3i} sin(\zeta_{5i}) + \frac{1}{L_i}\zeta_{4i} cos(\zeta_{5i}). \end{aligned} \quad (7)$$

The functions $\alpha_i(\zeta_i) \in \Re^b$ and $\beta_i(\zeta_i) \in \Re^{b\times m}$ can be deduced from (4) and (6). Now, due to (6), the system (7) can be written in the transformated coordinates as

$$\dot{\zeta}_i = f_i(\zeta_i) + g_i(\zeta_i)u_i, \quad (8)$$

where $\zeta_i = (\zeta_{1i}, \zeta_{2i}, ..., \zeta_{5i})^T = (\Pi_i^T, \dot{\Pi}_i^T, \zeta_{5i})^T \in \Re^n$, and $u_i = (F_i, \tau_i)^T \in \Re^m$ is the system input. On the other hand, dynamics of $\dot{\zeta}_{5i}$ from equation (7) denote the internal dynamics, which are rendered non-observable and uncontrollable by the transformation (5). By setting $\zeta_{1i}, \zeta_{2i}, ..., \zeta_{4i} = 0$, thus $\dot{\zeta}_{5i} = 0$, having stable zero dynamics. As $\zeta_{5i} = \theta_i$, where $(\zeta_{3i}, \zeta_{4i})^T$ represent the velocity of the hand position Π, then the angle θ_i will stop moving when the hand position stops moving.

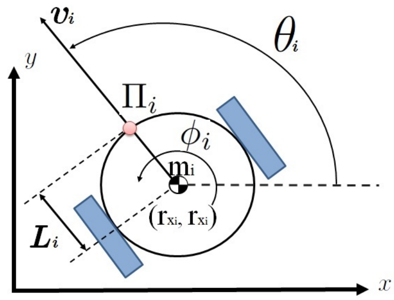

Fig. 1. Differential-drive Mobile Robot.

3. FORMATION CONTROL PROBLEM

Let us define a set of N nonlinear systems written in transformated coordinates identical to (8), given by

$$\begin{aligned} \dot{\zeta} &= f(\zeta) + g(\zeta)u, \\ s(\zeta) &= \lambda\left((I_N \otimes I_b) + (c \otimes \psi)\right)\tilde{\Pi} + \left((I_N \otimes I_b) + (c \otimes \psi)\right)\dot{\Pi}, \end{aligned} \quad (9)$$

where $\zeta = (\zeta_1^T, \ldots, \zeta_N^T)^T$, $u = (u_1^T, \ldots, u_N^T)^T$ denotes the control input vector, $f(\zeta) = (f_1(\zeta_1)^T, \ldots, f_N(\zeta_N)^T)^T$ correspond to a vector of nonlinear terms and $g(\zeta) = diag\{g_i(\zeta_i)\}$. Moreover, $\dot{\Pi} = (\dot{\Pi}_1^T, \ldots, \dot{\Pi}_N^T)^T$, where $\dot{\Pi}_i \in$

$\Re^b$ is the estimated speed vector in the cartesian space. $\tilde{\Pi} = (\tilde{\Pi}_1^T, \ldots, \tilde{\Pi}_N^T)^T$ describe the position error vector. By taking Π_{id} as a constant reference, the i-th position error can be written as $\tilde{\Pi}_i = \Pi_i - \Pi_{id}$. $s(\zeta) = (s_1(\zeta_1)^T, \ldots, s_N(\zeta_N)^T)^T$, where $s_i(\zeta_i) \in \Re^b$ represents the sliding variable of the i-th robot defined as

$$s_i(\zeta_i) = \lambda_i\left(\tilde{\Pi}_i + \psi(2\tilde{\Pi}_i - \tilde{\Pi}_{i-1} - \tilde{\Pi}_{i+1})\right) + \left(\dot{\Pi}_i + \psi(2\dot{\Pi}_i - \dot{\Pi}_{i-1} - \dot{\Pi}_{i+1})\right), \quad i = 1, 2, \ldots, N. \tag{10}$$

The term $\psi \in \Re^{b\times b}$ represents an interconnection weight matrix. Relationship among robots is given by Hankel matrix $c = \hat{c}^T\hat{c}$, where $\hat{c} = [\hat{c}_{i\iota}]$ is defined by

$$\hat{c}_{i\iota} = \begin{cases} 1, & \iota = i \\ -1, & \iota = i+1 \\ 0, & \text{otherwise.} \end{cases}$$

Then, the system (9) is formed by unicycle robots group in the formation, which is defined as

$$P = \{\Pi_{1d}, ..., \Pi_{Nd}\} \tag{11}$$

where Π_{id} is the desired location of the hand position Π_i. In this work, the class of formation control problems where the group of robots require to commute through a sequence of formation patterns $P_j, j = 1, ..., \mathcal{J}$; will be considered. It is assumed that the sequence of formation patterns are designed in such a way to avoid robot collisions. Besides, it is desirable to maintain the robots formation similar to the destination pattern.

There are two competing objectives. The first objective it to move the robots to their final destination as specified in the formation pattern. The second objective is to maintain formation during the transition (Lawton et al. (2002)).

Concerning to system (9), and defining $\mathbf{0}_\mathrm{b} \in \Re^b$ as a null vector of dimension b, the following assumptions are introduced

Assumption B1. The sliding variable $s_i(\zeta_i) \in \Re^b$ is designed so that the desired compensated dynamics of the system (9) are achieved in the sliding mode $s_i = s_i(\zeta_i) = \mathbf{0}_b$, then $s = s(\zeta) = \mathbf{0}_{bN}$.

Assumption B2. The relative degree of the system (9) is equal to 1, and the internal dynamics are stable.

Then, the dynamics of the sliding variable s are given by

$$\dot{s} = a(\zeta, t) + Jb(\zeta, t)u. \tag{12}$$

where $J = (I_N \otimes I_b) + (C \otimes \psi)$ is the matrix interconnection robots, $a(\zeta, t) = \lambda J\dot{\Pi} + J\alpha(\zeta)$, where $\alpha(\zeta, t) = \left(\alpha_1(\zeta_1, t)^T, \ldots, \alpha_N(\zeta_N, t)^T\right)^T$, and $b(\zeta, t) = diag\{\beta_i(\zeta_i, t)\}$, for $i = 1, 2, \ldots, N$.

Assumption B3. The function $b(\zeta, t)$ is unknown and different to zero $\forall \quad \zeta, t \in [0, \infty)$. Furthermore, $b(\zeta, t) = b_0(\zeta, t) + \Delta b(\zeta, t)$, where $b_0(\zeta, t)$ is the nominal part of $b(\zeta, t)$ which is known, and there exists δ_1 an unknown positive constant such that $\Delta b(\zeta, t)$ satisfies

$$\sigma_{\mathrm{Max}}\left(\Delta b(\zeta, t) b_0^{-1}(\zeta, t)\right) \le \sigma_{\mathrm{Max}}(\Delta b(\zeta, t))\sigma_{\mathrm{Max}}(b_0^{-1}(\zeta, t)) \le \delta_1.$$

where the maximum singular value of some square matrix is denoted by $\sigma_{\mathrm{Max}}(\cdot)$. Then, there exists locally $\gamma_{1i} \in \gamma_1$ so that

$$\sigma_{\mathrm{Max}}(\Delta\beta_i(\zeta_i, t))\sigma_{\mathrm{Max}}(\beta_{0i}^{-1}(\zeta_i, t)) \le \gamma_{1i}, \quad i = 1, 2, \ldots, N.$$

Assumption B4. There exists δ_2 an unknown positive constant such that the derivative of vector function $a(\zeta, t)$ is bounded

$$||\dot{a}(\zeta, t)|| \le \delta_2 \tag{13}$$

and exist $\gamma_{2i} \in \gamma_2$ so that $\dot{\alpha}_i(\zeta_i, t) \in \dot{a}(\zeta, t)$ satisfies

$$|\dot{\alpha}_i(\zeta_i, t)| \le \gamma_{2i}, \quad i = 1, 2, \ldots, N.$$

In order to incorporate both competing objectives, the global control input is defined as

$$u = b_0^{-1}(\zeta, t)\omega, \tag{14}$$

where $u = (u_1^T, \ldots, u_N^T)^T$, the decoupling matrix $b_0^{-1} = diag\{\beta_{0i}^{-1}\}$ for $i = 1, 2, \ldots, N$; includes nominal values and information about angular position θ_i, the vector $\omega = (\omega_1^T, \ldots, \omega_N^T)^T$, where ω_i denotes the control law for the i-th subsystem. Then, the system (12) can be written as follow

$$\dot{s} = a(\zeta, t) + Jb_1(\zeta, t)\omega. \tag{15}$$

From B3, $b_1(\zeta, t) = (I_\mathrm{N} \otimes I_{\mathrm{b}\times\mathrm{m}}) + \Delta b(\zeta, t) b_0^{-1}(\zeta, t)$, being $b_1(\zeta, t) \in \Re^{bN\times mN}$, it follows that

Assumption B5. $1 - \delta_1 \le \sigma_{\mathrm{Max}}\left(b_1(\zeta, t)\right) \le 1 + \delta_1$

Then, the problem is to drive the sliding variable s and $\dot{s}$ to zero in finite time in the presence of the bounded perturbations with the unknown boundaries $\delta_1, \delta_2 > 0$ by means of continuous control without overestimated the gain. Furthermore, owing to parameters variations and non-modeled dynamics, $b_0(\zeta, t)$ is an approximation from the real parameters system $b(\zeta, t)$. Thus, the design of a robust control based on adaptive super-twisting control algorithm (see Shtessel et al. (2012)), is introduced in the following section.

4. ADAPTIVE SUPER TWISTING ALGORITHM

In this section, the synthesis of a control law based on a super-twisting adaptive control algorithm, proposed in (Shtessel et al. (2012)), is presented. Under this approach, the bounds of uncertainties and perturbations present on the system are not required to be known. Thus, the designed controller ensures its convergence in a finite-time, increasing the robustness of the system under uncertainties. From the ASTA, whose equations are given by

$$\omega = -K_1\varsigma_1 + \nu, \qquad \dot{\nu} = -\frac{1}{2}K_2 S^{-1}\varsigma_1, \tag{16}$$

where $\omega = (\omega_1^T, \ldots, \omega_N^T)^T$ and $\omega_i \in \Re^b$ represents the control signal for i-th robot; the matrix $S = diag\{S_i\}$, where $S_i = diag\{|s_{li}|^{\frac{1}{2}}\}, \quad l = 1, 2, \ldots, b$; $\varsigma_1 = Ssign(s)$, where $sign(s)$ returns a vector with the signs of the corresponding elements of s; $K_1 = diag\{K_{1i}\}, K_2 = diag\{K_{2i}\}$ are the matrix control gains i-th robot, $i = 1, 2, \ldots, N$. From the ASTA, the gains K_{1i}, $K_{2i} \in \Re^{m\times m}$ are diagonal matrices chosen such that they are functions of the sliding surface dynamics as follows

$$K_{1i} = K_{1i}(t, s_i, \dot{s_i}) \ , \ K_{2i} = K_{2i}(t, s_i, \dot{s_i}). \tag{17}$$

Then, from the system (15) and (16), the closed loop system is given by

$$\begin{aligned} \dot{s} &= a(\zeta, t) - Jb_1(\zeta, t)K_1\varsigma_1 + Jb_1(\zeta, t)\nu, \\ \dot{\nu} &= -\frac{1}{2}K_2 S^{-1}\varsigma_1. \end{aligned} \tag{18}$$

Let us assume that the terms of $\kappa = Jb_1(\zeta, t)\nu \in \Re^{bN}$ are bounded with unknown boundary $\gamma_{3li} > 0$ i.e.

$$|\kappa_{li}| \leq \gamma_{3li} \in \gamma_3, \tag{19}$$

where $\gamma_3 \in \Re^{bN}$ and $\kappa_{li} \in \kappa$, $i = 1, 2, \ldots, N$.

Theorem 1. *Consider the system (9) in closed-loop with the control (14), expressed in terms of the sliding variable dynamics (15). Furthermore, the assumptions $B3 - B5$ for unknown gains $\lambda_1, \delta_1, \delta_2 > 0$ and (19) are satisfied. Then, for given initial conditions $\varsigma(0)$ and $s(0)$, there exists a finite time $t_F > 0$ is established $\forall t \geq t_F$, under the action of ASTA (16) with the adaptive gains $K_1(t, s, \dot{s}), K_2(t, s, \dot{s}) > 0$.* ⋄

Proof. Consider the following change of variable

$$\varsigma = (\varsigma_1^T, \varsigma_2^T)^T = (sign(s)^T S^T, a(\zeta, t)^T + \nu^T b_1(\zeta, t)^T J^T)^T. \tag{20}$$

Then, the system (18) can be written as

$$\dot{\varsigma} = \frac{1}{2}\tilde{A}(\varsigma_1)\varsigma + \tilde{g}(\varsigma_1) \otimes \bar{\varrho}(x, t), \tag{21}$$

where

$$\tilde{A}(\varsigma_1) = \begin{pmatrix} -S^{-1}Jb_1(\zeta, t)K_1 & S^{-1} \\ -Jb_1(\zeta, t)K_2S^{-1} & \mathbf{0}_{\mathrm{bN}} \end{pmatrix}, \qquad \tilde{g}(\varsigma_1) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Furthermore, due to the Assumption B.4 and (19), the boundary of the uncertain function $\bar{\varrho}(\zeta, t) = \dot{a}(\zeta, t) + \dot{b}_1(\zeta, t)\upsilon$ exists, but is unknown. Then, we can write

$$\bar{\varrho}(\zeta, t) = 2\varrho(\zeta, t)S^{-1}\varsigma_1,$$

where S^{-1} is a diagonal matrix. Then, the system (22) can be rewritten as follows

$$\dot{\varsigma} = \frac{1}{2}\bar{A}(\varsigma_1)\varsigma, \tag{22}$$

where

$$\tilde{A}(\varsigma_1) = \begin{pmatrix} -S^{-1}Jb_1(\zeta, t)K_1 & S^{-1} \\ -(Jb_1(\zeta, t)K_2 - 2\varrho(\zeta, t))S^{-1} & \mathbf{0}_{\mathrm{bN}} \end{pmatrix},$$

where $|\varsigma_{1li}| = |s_{li}|^{1/2} \in S$, $\quad l = 1, 2, \ldots, b$; it is appealing to consider the quadratic function

$$V_0 = \varsigma^T \tilde{P}\varsigma, \tag{23}$$

where $\tilde{P}$ is a constant, symmetric and positive matrix, as a strict Lyapunov candidate function for (16). Taking its derivative along the trajectories of (16), we have

$$\dot{V}_0 = -\frac{1}{2}\varsigma^T \tilde{Q}\varsigma, \tag{24}$$

almost everywhere, where $\tilde{P}$ and $\tilde{Q}$ are related by the Algebraic Lyapunov Equation

$$\bar{A}^T\tilde{P} + \tilde{P}\bar{A} = -\tilde{Q}. \tag{25}$$

Since $\bar{A}$ is Hurwitz for $Jb_1(\zeta, t)K_1 > 0$, and the matrix $Jb_1(\zeta, t)K_2 - 2\varrho(\zeta, t) > 0$, for every $\tilde{Q} = \tilde{Q}^T > 0$, there exist a unique solution $\tilde{P} = \tilde{P}^T > 0$ of the (25), so that V_0 is a strict Lyapunov function.

Notice that, from (24), as ς_1 and ς_2 converge to 0 in finite time, it follows that s and $\dot{s}$ converge to 0 in finite time too (Shtessel et al. (2012)). ⋄

Remark 1. The stability of the equilibrium $\varsigma = 0$ of (22) is completely determined by the stability of the matrix $\bar{A}$. However, since they require a continuously differentiable, classical versions of Lyapunov 's theorem (Filippov (1988)) cannot be used, or at least locally Lipschitz continuous Lyapunov function, though V_0 (23) is continuous but not locally Lipschitz. Nonetheless, as it is explained in Theorem 1 in (Moreno et al. (2012)), it is possible to show the convergence properties by means of Zubov 's theorem (Pozniak (2009)), that requires only continuous Lyapunov functions. This argument is valid in all the proofs of the present paper, so that no further discussion of these issues will be required.

However, to implement the proposed controller, it is ne -cessary to know the vector speed $\dot{\Pi}_i = (\zeta_{3i} \quad \zeta_{4i})^T$. Then, to overcome this difficulty, the estimation of unmeasurable terms will be addressed in next section.

5. HIGH-ORDER SLIDING MODE DIFFERENTIATOR

In this section, some previous results are introduced in order to design a differentiator for computing the real-time derivative of output function with finite-time convergence.

Let $f(t)$ be a function defined in $[0, \infty)$, consisting of a bounded Lebesgue-measurable noise with unknown features and $f_0(t)$ an unknown basic signal, whose k-th derivative has a known Lipschitz constant $\tilde{L} > 0$. Thus, the problem of finding real-time robust estimations of $f_0^{(k)}(t)$, for $k = 0, ..., r$; being exact in the absence of measurement noises, is known to be solved by the robust exact differentiator (see Levant (2003) for more details), which is given by

$$O_i : \begin{cases} \dot{z_{0li}} &= \nu_{0li} \\ \nu_{0li} &= -\lambda_3 \tilde{L}^{1/3} |z_{0li} - \zeta_{li}|^{2/3} sign\,(z_{0li} - \zeta_{li}) + z_1 \\ \dot{z}_{1li} &= \nu_{1li} \\ \nu_{1li} &= -\lambda_2 \tilde{L}^{1/2} |z_{1li} - \nu_{0li}|^{1/2} sign\,(z_{1li} - \nu_{0li}) + z_2 \\ \dot{z}_{2li} &= -\lambda_1 \tilde{L} sign\,(z_2 - \nu_{1li}), \quad l = 1, 2. \end{cases} \tag{26}$$

where ζ_{li} for $l = 1, 2$; and $i = 1, \ldots, N$; is the output measurable, $\nu_{0li} = \zeta_{li}$ for $l = 3, 4$; is the estimation speed.

Proposition 1. Consider the system (9) in closed-loop with the controller (14)-(16), using the estimates obtained by the differentiator (26). Then, the trajectories of the system (22) converge in finite-time to the reference signal Π_{id}.

Remark 2: Since the observer converges in finite-time, the control law and the observer can be designed sepa -rately, *i.e.*, the separation principle is satisfied. Thus, if the controller is known to stabilize the process then the stabilization of the system in closed-loop is assured whenever the differentiator dynamics are fast enough to provide an exact calculation of $\dot{\Pi}$.

On the other hand, from (14), notice that b_0 depends of θ_i for $i = 1, 2, \ldots, N$; given (6), the i-th orientation angle θ_i can be calculated from the estimated values $\dot{\hat{\Pi}}_i = (\hat{\zeta}_{3i} \quad \hat{\zeta}_{4i})^T$ as $\theta_i = \hat{\zeta}_{5i}$, where

$$\dot{\hat{\zeta}}_{5i} = -\frac{1}{L_i}\hat{\zeta}_{3i} sin(\hat{\zeta}_{5i}) + \frac{1}{L_i}\hat{\zeta}_{4i} cos(\hat{\zeta}_{5i}), \quad i = 1, 2, \ldots, N; \qquad (27)$$

and $\hat{\zeta}_{3i}$ and $\hat{\zeta}_{4i}$ are estimate by (26).

6. SIMULATION RESULTS

In this section, simulation results are provided to illustrate the effectiveness of the proposed methodology. The systems and the control scheme where developed in the MAT-LAB/Simulink environment, using Runge-Kutta solver with an integration step of $0.001s$. The multi-robot system are composed by three unicycle robots, were the distance between wheels of i-th robot is given by $l_i = 0.0525m$, the diameter of the wheels $d_i = 0.041m$, maximum dia -meter of the robot as $d_{Ai} = 0.09m$, the mass of the robot $m_i = 0.2Kg$, and $L_i = 0.09m$ is the distance from geometric center. Furthermore, the maximum linear speed and maximum angular speed are $\nu_{i\max} = 0.13m/s$ and $\phi_{i\max} = 4.96rad/s$, respectively.

The Controller parameters are displayed in the Table 1. The differentiator parameters for the i-th robot are defined as $\tilde{L}$, $\lambda_{1i,j} = 1$, $\lambda_{2i,j} = 14$ and $\lambda_{3i,j} = 17$, for $i = 1, 2, 3$; and $j \in \{x, y\}$, taking x and y as axis in a coordinate space.

In order to verify the robustness of the proposed scheme, the mass of the robots have a variation of 80% percent on the nominal value. Furthermore, disturbances of 0.06m in positions for measures in y and x axes are introduced at 40s and 150s, respectively.

i	ω_j	λ_j	μ_j	γ_x	γ_y	ϵ_{*j}
1	0.1160	0.2	0.001	0.08	0.045	$4{\times}10^{-5}$
2	0.1160	0.2	0.001	0.08	0.045	$4{\times}10^{-5}$
3	0.1160	0.2	0.001	0.08	0.045	$4{\times}10^{-5}$

Table 1. Control parameters for axis x and y, taking $j \in \{x, y\}$

Furthermore, the robots are commanded to transition through the series of formation patterns given by

$$\mathcal{P}_0 = \left\{ \begin{pmatrix} 0.0 & 0.0 \end{pmatrix}^T, \begin{pmatrix} 0.5 & 0.0 \end{pmatrix}^T, \begin{pmatrix} 1.0 & 0.0 \end{pmatrix}^T \right\}$$
$$\mathcal{P}_1 = \left\{ \begin{pmatrix} 0.5 & 0.5 \end{pmatrix}^T, \begin{pmatrix} 1.0 & 0.5 \end{pmatrix}^T, \begin{pmatrix} 1.5 & 0.5 \end{pmatrix}^T \right\}$$
$$\mathcal{P}_2 = \left\{ \begin{pmatrix} 0.5 & 1.0 \end{pmatrix}^T, \begin{pmatrix} 1.0 & 0.5 \end{pmatrix}^T, \begin{pmatrix} 1.5 & 1.0 \end{pmatrix}^T \right\}$$
$$\mathcal{P}_3 = \left\{ \begin{pmatrix} 0.8 & 1.3 \end{pmatrix}^T, \begin{pmatrix} 1.3 & 0.8 \end{pmatrix}^T, \begin{pmatrix} 1.8 & 1.3 \end{pmatrix}^T \right\}$$
$$\mathcal{P}_4 = \left\{ \begin{pmatrix} 1.1 & 1.3 \end{pmatrix}^T, \begin{pmatrix} 1.6 & 0.8 \end{pmatrix}^T, \begin{pmatrix} 2.1 & 1.3 \end{pmatrix}^T \right\}$$

Figure 2 shows the corresponding control input for each robot. The robots positions related to the competing objectives are shown in the Figure 3, where can be seen that the robots move to desired positions holding the formation pattern along the transition from point to point.

The behavior of the adaptive gain K_1 can be observed in the Figure 4, and K_2, due that it is depend of K_1, is assumed bounded as K_1. Moreover, in the Figure 5, the convergence of the sliding variable s to 0 is shown.

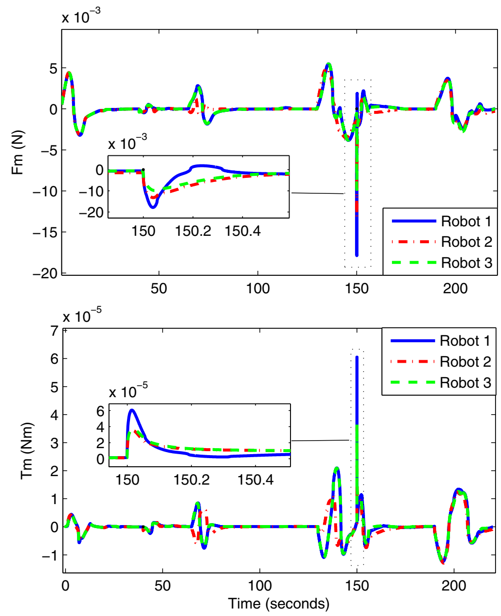

Fig. 2. Control signals input for linear and angular motion.

7. CONCLUSION

In this work, the problem of transitioning a group of robots through a sequence of formation patterns has been considered. The group objective is to maintain the desired formation pattern during the transition. With this aim, a control strategy that combines behavior-based approach with an adaptive super-twisting control algorithm has been proposed. Simulation results demonstrated the effectiveness of the proposed scheme.

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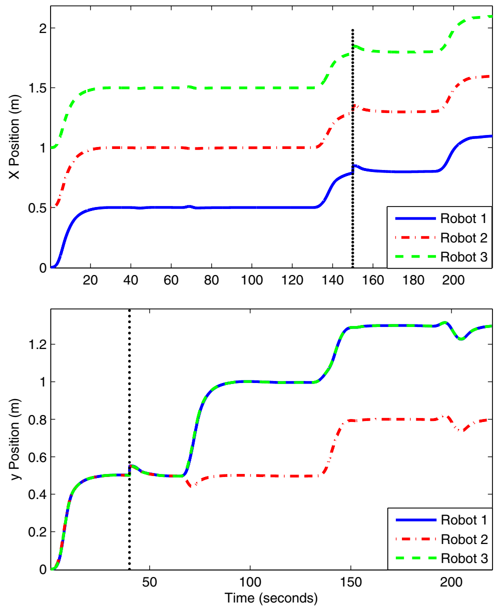

Fig. 3. Robots position hand in coordinate space.

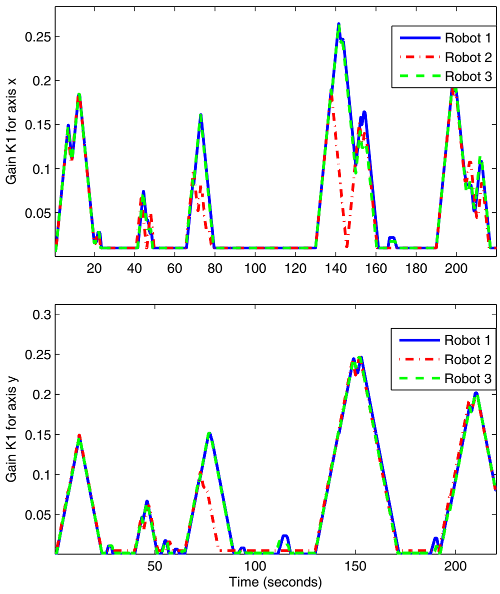

Fig. 4. Adaptive gain K_1 of multi robot system.

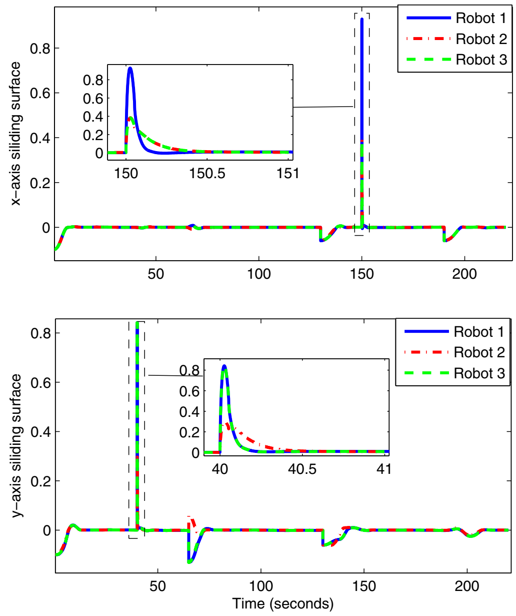

Fig. 5. Evolution of sliding variables.

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