

# Hybrid Modeling and Predictive Control for an Agglomeration Process<sup>\*</sup>

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**Abstract:** Process simulation is an increasingly useful tool for the analysis of processes in the mineral industry. The ability to simulate process behavior without having to disturb plant operation prevents the loss of man-hours and production. Additionally, simulation tools provide a platform for the development of control and optimization strategies. Model predictive control, in particular, relies heavily on precise models to make accurate predictions. In mineral processing, due to complexities such as strong nonlinearities, variable coupling, time varying parameters, etc., the development of accurate process models becomes an increasingly difficult task.

Furthermore, current control solutions mostly use linearized models and rely on expert systems to handle discrete variables such as discrete valves, signaling lights and tripper cars. Controllers that do not take into account the operation of these discrete variables may exhibit a suboptimal performance. For the previous reason, it is of special interest the design and development of hybrid controllers, capable of incorporating both continuous and discrete variables.

This paper firstly describes the modeling and simulation of the main processes of a hydrometallurgical plant, followed by the hybrid model identification of the agglomeration process. Simulation results of the identified models are compared to real plant data to qualitatively validate the models developed. The hybrid models identified in this manner are used for the design and development of hybrid controllers for the agglomeration process. Simulation results show that the hybrid controllers exhibit a better performance when compared to an expert control scheme.

*Keywords:* Process control, hybrid systems, hybrid model predictive control, hydrometallurgy, agglomeration.

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## 1. INTRODUCTION

Hydrometallurgical processing of copper accounts for 20% of the primary copper production, with 4.5 million tonnes of cathode copper per year as of 2011. Most of this is produced by heap leaching. This process is usually preceded by a crushing stage and an agglomeration stage, which improves the efficiency and speed of copper recovery (Schlesinger et al., 2011).

Extensive research has been conducted on the development of mathematical models to simulate the processes involved in a hydrometallurgical plant. Despite this efforts, models based on physical principles for agglomeration are scarcely found in industrial applications. Empirical models, on the other hand, do not give a good fit to plant data. This mismatch between plant and model can be attributed to different factors, but the main reason is that the oversimplified population balance model fails to incorporate all the mechanism present in an agglomeration process, and also fails to reflect that different sections of the drum are dominated by different mechanisms.

Different schemes of model based control of granulation processes have been proposed in literature. A linear MPC (Model Predictive Control) scheme has been developed by Adetayo et al. (1997) and applied to a pan granulation process, using black-box, input-output models. In Zhang et al. (2000), a control scheme for granulation circuits based on PI controllers was implemented. A dynamic model based on physical principles was developed to simulate the process. Although the control scheme is not model based, simulation results show that the solution is robust.

Most model predictive control strategies rely on linearized models of the processes. These strategies will prove to be effective as long as the system remains near its nominal operating point, but its performance will worsen when the system drifts from said point. Furthermore, these strategies are not designed for systems that exhibit multiple modes of operation or discrete process variables, such as the opening and closing of a valve.

To address these issues, we propose a methodology to represent the agglomeration process of a hydrometallurgical plant by means of hybrid models, as described in Bemporad and Morari (1999). This hybrid model will serve as the basis for the design and development of hybrid control strategies. Finally, the hybrid controller will be

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compared with a conventional expert controller to evaluate its performance.

The organization of this article is as follows. Section 2 describes the hydrometallurgical plant studied. The structure of the models used in the dynamic simulator are outlined in Section 3. The identification of the hybrid models for the agglomeration process is performed in Section 4. The design of the expert controller and hybrid controller based on the previous identified models is detailed in Section 5. The performance of the controllers is compared in Section 6. Finally, conclusions are presented in Section 7.

## 2. CASE STUDY

As previously stated, the hydrometallurgical plant studied in this article is constituted by a crushing circuit followed by agglomeration drums and heap leaching. Fig. 1 shows a simplified flow sheet from primary crushing to heap leaching.

The crushing circuit is composed of primary crushing and fine crushing. The primary cone crusher is fed by trucks carrying 90 tons of ore and the product is transported by conveyor belts to a stockpile. The fine crushing plant includes the secondary and tertiary crushing circuits. Ore is extracted from the stockpile and transported to the primary screen, where the oversize is fed into the secondary cone crusher. The tertiary crushing circuit includes four vibratory screens, where the oversize is fed back into three cone crushers. In both the secondary screens and tertiary crusher, tripper cars are used to distribute the material into the multiple bins. Finally, the undersized mineral of the secondary screens is conveyed into a silo.

The ore from the silo is conveyed through two belts, which feed two agglomerating drums operating in parallel. The curing process begins adding sulphuric acid and raffinate solution, depending on the amount of carbonate present in the ore.

Finally, the agglomerated ore is stacked to form heaps. The leaching process uses seven areas for the assembly of heaps, where each heap is divided into modules and each module is irrigated independently. The leaching time for each heap is typically 170 days.

## 3. DYNAMIC SIMULATOR

A dynamic simulator was developed to implement the models for the hydrometallurgical plant described in previous sections. The models used in the simulator include both static and dynamic, as well as empirical and phenomenological models. The simulator was calibrated with actual plant data, so its behavior is both qualitatively and quantitatively similar to that of a real hydrometallurgical plant.

The following section describes the models used in each process and the simulated variables. A thorough description of the simulator can be found in Reyes et al. (2013) and Tejeda et al. (2013). The latter presents a decision support system designed for plants in which fully automated control strategies cannot be implemented, due to insufficient instrumentation.

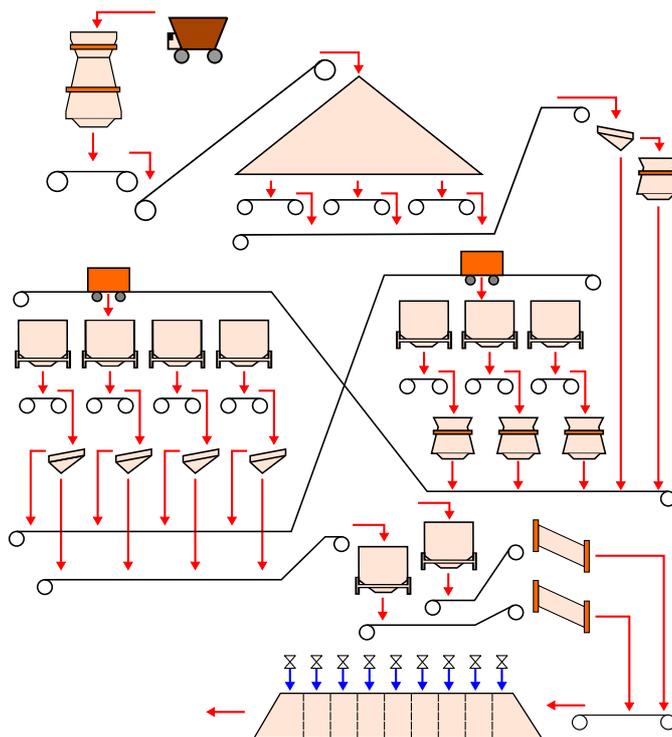


Fig. 1. Simulated plant flow sheet.

### 3.1 Crushing circuit and transport

The units modeled in the crushing circuit are cone crushers, vibratory screens, conveyor belts and storage units (stockpile, silo and bins). Units are connected with each other through flows of mineral, which is modeled as a particle size distribution.

The model for the cone crusher, described in (1), is based on the model developed by Whiten (1972), which simplifies the crusher to a single breakage zone and performs a mass balance to express the crusher product  $\mathbf{p}$  in terms of the feed  $\mathbf{f}$ :

$$\mathbf{p} = (\mathbf{I} - \mathbf{C})(\mathbf{I} - \mathbf{BC})^{-1} \mathbf{f} \quad (1)$$

The lower triangular matrix  $\mathbf{B}$  gives the relative distribution of each size fraction after it is broken and the diagonal matrix  $\mathbf{C}$  gives the proportion of particles entering the breakage region. Both these matrices are calibrated from plant operation data.

The vibratory screen model is based in the model developed by Hatch (1977), which is derived from statistical and mechanical considerations. The general form of the efficiency equation is expressed by (2):

$$Y_i = \frac{1}{1 + e^{\frac{x_{50}^3 - x_i^3}{a}}}, \quad (2)$$

where  $Y_i$  is the fraction of feed in size  $x_i$  reporting to the oversize product. The model parameters  $x_{50}$  (ore size at which 50% of the feed reports to the oversize) and  $a > 0$  are calibrated from plant operation data.

The conveyor belt model, described by (3), is basically a one dimension transport equation:

$$\frac{\partial}{\partial t}\rho(x, t) + v\frac{\partial}{\partial x}\rho(x, t) = 0, \quad (3)$$

where  $\rho$  is the linear density of the ore in the belt and  $v$  is the speed of the belt.

The stockpile, silo and bins are modeled as simple integrator plants.

For the cone crushers and vibratory screens in the crushing circuit simulator, calibration was performed using least squares with operation data and particle size distribution profiles. For silos, bins and conveyor belts, calibration was performed considering physical and geometric parameters.

### 3.2 Agglomeration

The agglomeration process is characterized through dynamic population balance equations, based on the model proposed by Wang and Cameron (2007) and described by (4):

$$\frac{d}{dt}n_i = -\frac{\partial}{\partial L}(Gn_i) + B_i - D_i + F^{in}\frac{n_i^{in}}{N^{in}} - F^{out}\frac{n_i}{N}, \quad (4)$$

where  $n_i$  is the number of agglomerates of size  $x_i$ ;  $L$  is the characteristic length of the particle;  $N$  and  $N^{in}$  are the total number of agglomerates in and out of the agglomeration drum;  $G$ ,  $B_i$  and  $D_i$  are the growth, birth and death rate of agglomerates, respectively; and  $F^{in}$  and  $F^{out}$  represent the flow rate in and out the drum.

Due to the lack of instrumentation in the agglomeration process, specially in its output, calibration was performed by a qualitative analysis validated by an expert.

### 3.3 Heap leaching

The leaching process, as opposed to crushing and agglomeration, has spatial and temporal scales of much greater extension. Partial differential equations, based on Cariaga et al. (2005), are used to model water saturation and solute transport of the heap, as described in (5) and (6):

$$\frac{\partial}{\partial t}\theta = \nabla \cdot (D\nabla\theta + K\hat{k}) \quad (5)$$

$$\frac{\partial}{\partial t}C_i = \nabla \cdot (D\nabla C_i - q\nabla C_i) + \phi_i, \quad (6)$$

where  $\theta$  represents the water content of the heap and  $C_i$  represents the concentration of solute (copper or acid) in the leaching solution. Additionally,  $D$  is the diffusion tensor,  $K$  is the hydraulic conductivity,  $q$  is the Darcy's velocity of the solution,  $\phi_i$  is the rate of solute added or removed from the solution and  $\hat{k}$  is the unit vector in the  $z$  direction.

The hydraulic parameters of the leaching model were calibrated by qualitative analysis validated by an experts. Chemical parameters were calibrated using least squares with operation data, copper recovery curves and irrigation information.

### 3.4 Simulator behavior

To qualitatively evaluate the simulator behavior, Fig. 2 shows the evolution of the product humidity in the agglomeration process, in response to changes in mineral feed and water addition with disturbances in feed humidity.

When the water valve is open, it can be seen that an increase or decrease in mineral feed produces the opposite effect in product humidity. This is explained because water addition is held constant and is not proportional to the mineral feed, thus any change in the latter will have and inverse effect on the water-to-ore ratio of the feed and the product humidity.

On the other hand, when the water valve is closed, it can be seen that changes in mineral feed have no effect on product humidity. In this case, as there is no water addition, product humidity depends on the addition of sulphuric acid and raffinate solution, as well as the mineral feed humidity. Both sulphuric acid and raffinate solution addition are proportional to mineral feed, so any change in the last one will not have an effect on the water-to-ore ratio of the feed.

## 4. HYBRID MODEL IDENTIFICATION

The aim of hybrid identification is to reconstruct a PWA (Piece-Wise Affine) map from a finite collection of regressors and output samples. A PWA map  $f : \mathcal{X} \mapsto \mathbb{R}$  is defined by (7):

$$y(k) = \begin{cases} [\mathbf{x}(k)^T \ 1] \boldsymbol{\theta}_1 & \text{if } \mathbf{x}(k) \in \mathcal{X}_1 \\ \vdots & \\ [\mathbf{x}(k)^T \ 1] \boldsymbol{\theta}_s & \text{if } \mathbf{x}(k) \in \mathcal{X}_s \end{cases} \quad (7)$$

where  $\{\mathcal{X}_i\}_{i=1}^s$  is a polyhedral partition of the regressor set  $\mathcal{X}$ . Thus, the aim of the PWA identification is to estimate  $\boldsymbol{\theta}_i$  using the input and output samples generated by a given system (Ferrari-Trecate et al., 2003).

When dealing with dynamic systems, such as the agglomeration process, the regressor vector  $x(k)$  must include

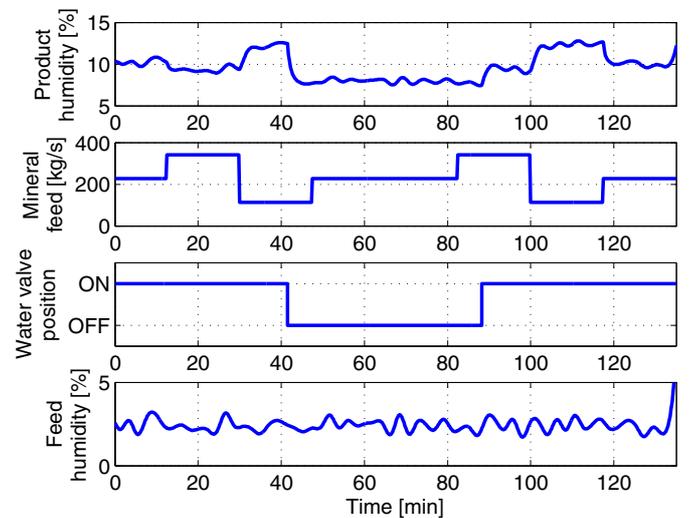


Fig. 2. Dynamic simulator behavior.

the  $n_a$  past samples of the outputs  $y(k)$  and the  $n_b$  past samples of the inputs  $u(k)$ :

$$\mathbf{x}(k) = [y(k-1) \ y(k-2) \ \dots \ y(k-n_a) \ u(k-1) \ u(k-2) \ \dots \ u(k-n_b)]. \quad (8)$$

The resulting model using the regressor vector (8) is termed a PWARX (Piece-Wise Auto Regressive eXogenous) model.

The agglomeration process considered includes two manipulated variables: mineral feed and water addition; one controlled variable: product humidity; and one measured disturbance: feed humidity. The mineral feed can be manipulated continuously by varying the speed of the conveyor belt. On the other hand, water addition is controlled by an on/off valve and it must be modeled as a binary variable. Therefore, the agglomeration process may be characterized as a hybrid dynamic system with two operating modes given by the state of the water valve.

For the identification procedure, a training set of four hours of operation was generated through simulation. PRBS signals were used for the mineral feed with a variation of 25%. An affine model is identified using least squares for each operating mode, with  $n_a = n_b = 1$ . The identified PWARX model is described in (9):

$$X_w(k+1) = \begin{cases} 0.72X_w(k) - 0.003F(k) + 0.23X_w^{\text{in}}(k) + 2.92 & \text{if } v(k) = 1 \\ 0.71X_w(k) + 0.3X_w^{\text{in}}(k) + 1.62 & \text{if } v(k) = 0 \end{cases} \quad (9)$$

where  $X_w(k)$  is the product humidity,  $F(k)$  is the mineral feed,  $X_w^{\text{in}}(k)$  is the feed humidity and  $v(k)$  is the binary position of the water valve.

The identified model was then validated with a different set of data of the same duration. Fig. 3 shows a comparison between the simulator output and the identified PWARX models for the validation set. The results show that the identified model is able to reproduce accurately the behavior of the product humidity in both operating modes (given by the alternating state of the water valve). To measure the predictive performance of the PWARX models, the RMS (Root Mean Squared) error between the simulator output and the model prediction of the product humidity is calculated. The resulting absolute RMS error is 0.08%.

## 5. CONTROLLER DESIGN

### 5.1 Expert controller

The design of the expert controller used in this study is based on the observation that if the product humidity reference is below certain value, then the mineral feed can be kept at its maximum while product humidity can be controlled by switching the state of the water valve. On the other hand, if the product humidity reference is above this value, mineral feed must be decreased. This threshold value for product humidity is a function of the mineral

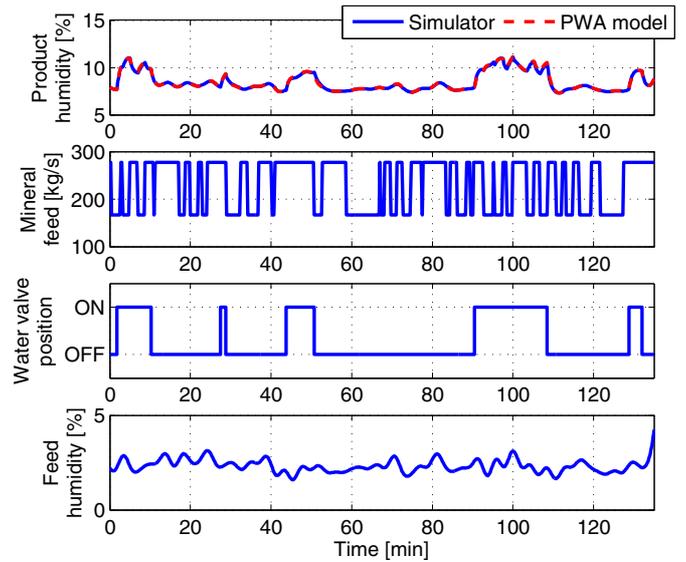


Fig. 3. Identified PWARX model for product humidity.

feed; water, sulphuric acid and raffinate solution addition; and feed humidity.

Fig. 4 shows the results of the expert controller. The controller was tested on a simulation of the agglomeration process over a period of one hour, with a sample time of 15 seconds. Changes in the reference of the product humidity are added over the course of the simulation. Feed humidity is considered a normally distributed random variable with a mean value of 2.3% and a standard deviation of 0.6%.

### 5.2 Hybrid controller

To develop the hybrid controller used in this study, an HMPC (Hybrid Model Predictive Control) framework is adopted. In this framework, a MLD (Mixed Logical Dynamical) model is used to predict the future behavior of the hybrid system. The MLD system is described by

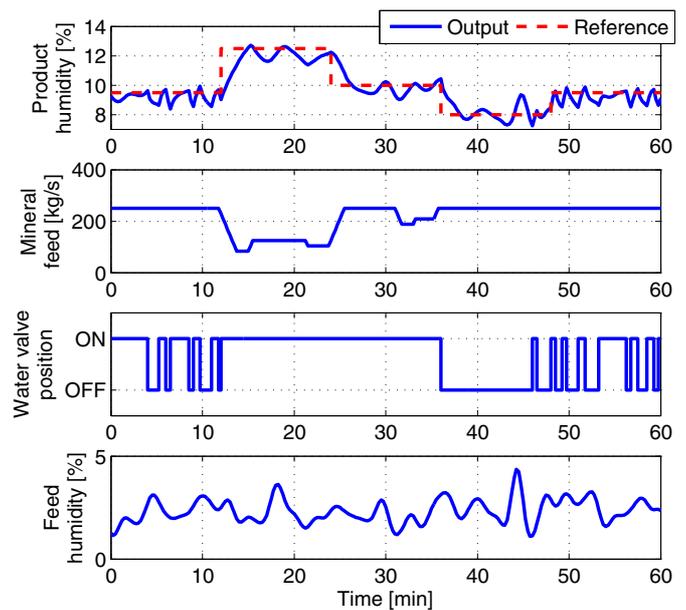


Fig. 4. Expert controller simulation results.

linear dynamic equations subject to linear mixed-integer inequalities through the linear relations (see Bemporad and Morari, 1999)

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}_1\mathbf{u}(k) + \mathbf{B}_2\delta(k) + \mathbf{B}_3\mathbf{z}(k) \quad (10a)$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}_1\mathbf{u}(k) + \mathbf{D}_2\delta(k) + \mathbf{D}_3\mathbf{z}(k) \quad (10b)$$

$$\mathbf{E}_2\delta(k) + \mathbf{E}_3\mathbf{z}(k) \leq \mathbf{E}_1\mathbf{u}(k) + \mathbf{E}_4\mathbf{x}(k) + \mathbf{E}_5, \quad (10c)$$

where  $\mathbf{x}(k)$  is the state of the system,  $\mathbf{u}(k)$  is the input vector and  $\mathbf{y}(k)$  is the output vector. Any one of these vectors can include both continuous and discrete variables. Additionally,  $\delta(k)$  and  $\mathbf{z}(k)$  are auxiliary logical and continuous variables, respectively. The MLD representation of the hybrid system is obtained directly from the PWARX models described previously using the HYbrid System Description Language (HYSDEL; see Torrisi and Bemporad, 2004).

For the control implementation, a predictive control scheme based on a receding horizon strategy is used. A sequence of control actions is chosen to optimize an objective function. The optimal control sequence must satisfy the constraints imposed by (10c) and any operational constraint that may apply to the process variables. Due to the presence of integer variables, the optimization problem described above is a MIQP (Mixed-Integer Quadratic Programming) problem.

The objective function is defined with the goal of minimizing the error between the product humidity and its reference value, minimizing the change in manipulated variables, minimizing water consumption and maximizing the processed mineral tonnage. The resulting optimization problem is described by (11):

$$\min \sum_{i=0}^{N-1} \|\mathbf{y}(k+i+1) - \mathbf{r}_y(k+i+1)\|_{\mathbf{Q}_y}^2 + \|\Delta\mathbf{u}(k+i)\|_{\mathbf{R}}^2 + \mathbf{L}_u^T \mathbf{u}(k+i), \quad (11)$$

subject to the MLD model described by (10) and the operational constraints

$$\mathbf{u}_{\min} \leq \mathbf{u}(k+i) \leq \mathbf{u}_{\max} \quad \forall i \in \{0, \dots, N-1\}, \quad (12)$$

where  $N$  is the prediction horizon,  $\mathbf{Q}_y$  is the weight matrix that penalizes reference error in controlled variables,  $\mathbf{R}$  is the weight matrix that penalizes changes in manipulated variables and  $\mathbf{L}_u$  is the weight vector that allows the controller to minimize or maximize the value of the manipulated variables. The controller parameters, shown in (13), are tuned experimentally through simulation.

$$\mathbf{Q}_y = 1, \quad \mathbf{R} = \begin{bmatrix} 10^{-4} & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{L}_u = \begin{bmatrix} -0.01 \\ 1 \end{bmatrix} \quad (13)$$

$$N = 3, \quad F_{\min} = 50 \text{ [ton/h]}, \quad F_{\max} = 900 \text{ [ton/h]}$$

Fig. 5 shows the results of the hybrid controller. The scenario simulated is the same as the one used for the test of the expert controller. It can be seen that the controller is able to keep the product humidity around its reference value in the presence of disturbances on the feed humidity. Additionally, the controller maximizes mineral tonnage when possible.

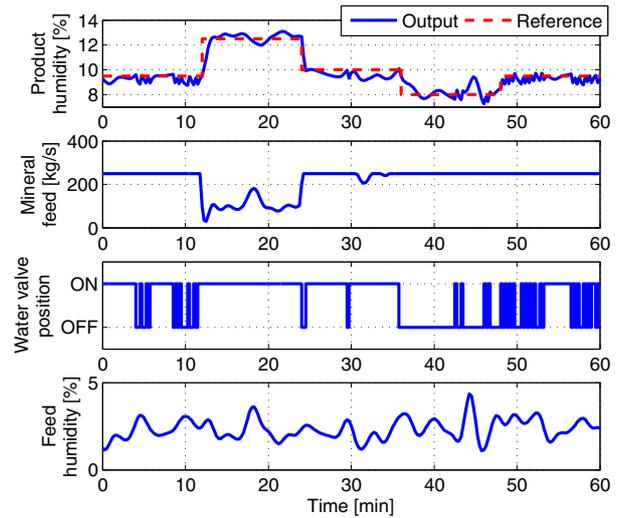


Fig. 5. Hybrid controller simulation results.

## 6. CONTROLLER COMPARISON

The performance of both the expert controller and hybrid controller is tested through simulation. The results, displayed in Fig. 4 and Fig. 5, show that the performance of both schemes are similar in terms of the actions on the manipulated variables. In both cases, mineral feed is kept at a maximum except when the reference for product humidity is 12.5%. While the performance of the controllers are similar, the deviation of the product humidity in the expert scheme is larger than in the hybrid scheme.

Table 1 shows performance indices for both controllers. While total mineral processed and water consumption are similar in both case, the hybrid controller manages to increase total mineral processed in 0.25% and decrease water consumption in 1.27%, when compared to the expert controller. The biggest difference is in the RMS error in product humidity, where the hybrid controller achieves a decrease of 33.3%.

Table 1. Performance indices for the tested controllers

	Expert	HMPC	Change
RMS error in product humidity [%]	0.78	0.52	33.3%
Mineral processed [ton]	762.2	764.1	0.25%
Water consumption [ton]	11.85	11.70	1.27%

## 7. CONCLUSION

This paper presents the identification of PWARX models for the agglomeration process of a hydrometallurgical plant, and the design of hybrid controllers based on the identified models. The studied plant, which is composed of a crushing circuit, agglomeration drums and heap leaching, was modeled through a combination of static, dynamic, phenomenological and empirical models. Subsequent simulations of the identified models for the agglomeration process show a high degree of characterization of the studied processes. Similarly, the hybrid controller designed for the

agglomeration process exhibits a better performance when compared to an expert controller.

Most importantly, the framework of hybrid systems adopted in this paper offers a highly systematized methodology for the analysis, modeling and development of hybrid controllers for complex processes, as found in mining applications. When compared to an expert control scheme, where the structure of the controller is usually designed to suit the process specifically; the hybrid approach offers a much more flexible structure which can be adapted to any process, regardless of its structure.

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