Haptic Steering by Wire based on High Gain GPI Observer - Controller techniques

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Abstract—A steering by wired haptic system based on disturbance rejection control techniques is presented. High gain Generalized Proportional Integral (GPI) observers are considered for the estimation of tire and steering wheel dynamic disturbances. These disturbances are on line canceled to ensure efficient tracking between the commanded steering wheel angle and the tire orientation angle. The estimated disturbances at the steering rack are fed back to the steering wheel to provide a haptic interface with the driver. The system can be seen as a master-slave haptic system, where the steering wheel acts as a master system, and the steering rack and tire behave as slave system. The proposed approach requires very few sensors and only the input gain of the systems, which is a major advantage compared to other approaches.

Keywords: High gain GPI observer, steering, haptic feedback, PD control.

I. INTRODUCTION

In conventional steering systems, the driver commanded direction is transmitted by a column or steering shaft, including universal joints and gearboxes, to the front tires. A major advanced on such system was introduced in 1950, with the hydraulic power steering system. Based on hydraulic pressure, the steering system complements the effort required by the driver to steer the vehicle. More recently, the so called Electric Power Assisted Systems (EPAS) have been introduced, (Kim and Song, 2002), (Zaremba et al., 1998), (Amberkar et al., 2000), (Peter and Gerhard, 1999). Among the EPAS, in the steering by wired (SBW) the goal is to completely do away with as many mechanical components (steering shaft, column, gear reduction mechanism, etc.) as possible (Cetin et al., 2010).

SBW systems allow better structures for crash energy absorption (Bertacchini et al., 2006), and benefits related to passengers comfort and driver feeling (Baviskar et al, 2009), (Cetin et al., 2010). In SBW systems the forces, torques and driving conditions transmitted by the steering wheel to the driver are important for a proper and safe vehicle driving. The SBW must provide the driver the opportunity to “feel” what the road driving conditions are. As a consequence, the SBW must behave as a haptic device (Baviskar et al., 2009), (Bertacchini et al., 2006).

There exist several dynamic models for SBW systems in the literature, (Baviskar et al., 2009), (Bertacchini et al., 2006). Uncertain and disturbance terms such as friction, damping, inertia, realigning forces, among others, affect the dynamics and behavior of the SBW. Their influence is rather important to guarantee a good performance of the SBW and to provide a reliable feedback of the road and driving conditions to the driver. The previously mentioned phenomena can be regarded as perturbations since they are difficult to be known with certainty due to the changing conditions on the road and the driving, e.g., speed, roadway texture, tire wear, tire air pressure, rain, etc.. For the purpose of determining the effects of such uncertain disturbances, the use of observers has been proposed, (Cetin et al., 2010), (Kim et al., 2008). The implementation of such observers often requires complex dynamic models and the used of specialized sensors, such as GPS and INS, measurement of lateral acceleration (Yhi and Gerdes, 2005), current and torque motor measurements (Nguyen and Ryu, 2009).

In this work high gain GPI observers (Sira-Ramirez, 2003) are considered for an Active Disturbance Rejection Control approach to the trajectory tracking problems on the perturbed interconnected subsystems (steering wheel, steering rack subsystems). Different to other approaches, only input gains and angle position feedback are required, thus, it can be implemented with common low cost encoder sensors. The high gain GPI observers estimate the effects of non modeled dynamics phenomena and additive perturbations on the steering rack and tire, such as, continuous and discontinuous friction, aligning forces, inertia effects, damping. For background on Active Disturbance Rejection Control (ADRC) the reader is referred to (Han, 2009) and (Zheng et al., 2009). Experimental results on a real platform shows good agreement with the theoretical results that allows concluding convergence of the tracking and estimation errors to a small vicinity around zero.

II. SBW DYNAMIC MODEL

For modeling, control and implementation the SBW system might be viewed as a master-slave system (Im et al., 2007). The steering wheel acts as a master subsystem, and the steering rack is the slave subsystem. Both subsystems are interconnected through a PD controller with active
disturbance rejection provided by a high gain GPI observer.

The dynamics of the master subsystem is affected by physical phenomena in its three fundamental components: steering wheel, gearbox and DC motor (Figure 1). The DC motor is in charge of reflecting to the driver the forces that are estimated from the dynamics phenomena presented on the steering rack and tire (slave subsystem). Dynamic effects on the steering wheel include discontinuous Coulomb friction, as well as common phenomena such as damping and stiffness (Yhi and Gerdes, 2005) and (Im et al., 2007).

Figure 1 shows an equivalent mechanical-electrical diagram of the master subsystem. The relevant parameters are, $V_v$ the input voltage, $L_v$ armature inductance, $R_v$ armature resistance, $i_v$ armature current, $i_f$ current through the winding field, $e_v$ electromotive force, $k_2$ electromotive force constant gain, $\phi_v$ output angle of the motor, $J_1$ inertia moment at the reduction input, $B_1$ damping coefficient at the reduction input, $F_1$ Coulomb friction coefficient at the reduction input, $k_1$ motor torque constant gain, $\tau_v$ motor torque, $F_{Tv}$ Coulomb friction coefficient at gearbox, $B_{Tv}$ damping coefficient at gearbox, $J_2$ inertia moment at the reduction output, $B_2$ damping coefficient at the reduction output, $F_2$ Coulomb friction coefficient at the reduction output, $\tau_r$ torque at the reduction output, $C_v$ wheel Coulomb friction coefficient, $B_v$ wheel damping coefficient, $J_v$ wheel inertia moment, $N_v = \frac{N_v_1}{N_v_2}$ gearbox reduction ratio, $N_v_1$ the number of teeth on the input gear, $N_v_2$ the number of teeth on the output gear, $\theta_v$ wheel angular position.

![Figure 1. Mechanical electrical diagram of the master subsystem.](image)

The relation between $\tau_v$ and $V_v$, is given by

$$\tau_v = \frac{N_v k_1}{R_v} V_v - \frac{N_v^2 k_1 k_2}{R_v} \dot{\theta}_v$$ (1)

While the relation between $\tau_v$ and $\theta_v$ is given by

$$\tau_v = J_{Tv} \ddot{\theta}_v + B_{Tv} \dot{\theta}_v + F_{Tv} \text{sign}(\dot{\theta}_v)$$ (2)

Where the equivalent coefficients of the reduction gearbox are given by: $J_{Tv} = N_v^2 J_1 + J_2$, $B_{Tv} = N_v^2 B_1 + B_2$ and $F_{Tv} = N_v^2 F_1 + F_2$, with $N_v = \frac{N_v_1}{N_v_2}$.

Finally, taking into account (1) and (2) and the dynamic effects of the steering wheel, the dynamic model of the master subsystem is given by equation (3), with $J_{ve} = J_v + J_{ve}$ the equivalent inertia of the steering wheel system.

$$J_{ve} \ddot{\theta}_v + (B_{Tv} + B_r) \dot{\theta}_v + (F_{Tv} + C_r) \text{sign}(\dot{\theta}_v) = \frac{N_v k_1}{R_v} V_v - \frac{N_v^2 k_1 k_2}{R_v} \dot{\theta}_v$$ (3)

A schematic diagram of the steering rack and tire (slave subsystem) is presented in Figure 2, with its main components (tire, gearbox, and DC motor). Notice the similarities with the steering wheel system depicted at Figure 1.

![Figure 2. Mechanical electrical diagram of the slave subsystem.](image)

The dynamic model of the slave subsystem is given by equation (4). This model greatly simplifies the tire dynamics by neglecting tire-road interaction and aligning forces due to traction, which are always present on real driving conditions (Kim et al., 2008), (Li et al., 2006). Such tire-road interaction would be further addressed when traction is added to the experimental platform. The parameters in model (4) have a direct equivalence to those of the master subsystem, this is done by replacing the subindex $v$ by $r$.

$$J_r \ddot{\theta}_r + (B_{Tr} + B_r) \dot{\theta}_r + (F_{Tr} + C_r) \text{sign}(\dot{\theta}_r) = \frac{N_r k_1}{R_r} V_r - \frac{N_r^2 k_1 k_2}{R_r} \dot{\theta}_r$$ (4)

Notice that the dynamic models here developed (3) and (4) are not used for the design of the PD controller and the high gain GPI observers. The only parameter that is required for the high gain GPI observer is the input gain. Nevertheless, the dynamic models are presented to show the dynamic effects that affect the SBW system.

III. PD + HIGH GAIN GPI OBSERVER

Based on the dynamic model of the steering wheel or master subsystem (3), the nonlinear terms, including perturbations and unmodeled dynamics, are substituted by a time variant disturbance term $\xi_v(t)$. Lumping all uncertainties and unknown dynamics on the time varying term $\xi_v(t)$
might be considered as a rough approximation, nevertheless under certain conditions on boundedness and smoothness such approximation results valid. (Sira-Ramírez, 2003). In state space \( x_1 = \theta_v \) and \( x_2 = \dot{\theta}_v \), model (3) is represented as a linear perturbed system depicted by

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{N_v k_1}{J_{ve} R_v} V_v(t) + \xi_v(t)
\end{align*}
\]  

(5)

where \( \xi_v(t) \) is given by

\[
\xi_v(t) = -\frac{1}{J_{ve}} \left\{ (B T_v + B_v) \dot{\theta}_v + (F T_v + C_v) \text{sign}(\dot{\theta}_v) + \frac{N_v^2 k_1 k_2}{R_v} \dot{\theta}_v \right\}
\]

(6)

Note that the equality established by (6) is enforced by the proposed model (3), if other dynamic effects or disturbances are to be considered, these phenomena would be included in the term \( \xi_v(t) \) as well, such that, the approximated linear model (5) remains valid in local basis. The relation (6) is presented for the sake of completeness, since the term \( \xi_v(t) \) is estimated by a high gain GPI observer.

A high gain GPI observer, given by (7), is proposed for the linear perturbed system. (5) The number of integrators required at the high gain GPI observer depends on the order and complexity of the non linearity represented by the term \( \xi_v(t) \). In this case the number of integrators was adjusted by trial and error based on the value of the estimation error \( \tilde{e}_v = x_1 - \dot{x}_1 \) of a set of experiments. The term \( \dot{\varphi}_1 \) corresponds to an estimate of the disturbance term \( \xi_v(t) \).

\[
\begin{align*}
\dot{x}_1 &= \dot{x}_2 + \lambda_7 v (x_1 - \dot{x}_1) \\
\dot{x}_2 &= \frac{N_v k_1}{J_{ve} R_v} V_v(t) + \dot{\varphi}_1 + \lambda_{6v} (x_1 - \dot{x}_1) \\
\dot{\varphi}_1 &= \dot{\varphi}_2 + \lambda_5 v (x_1 - \dot{x}_1) \\
\dot{\varphi}_2 &= \dot{\varphi}_3 + \lambda_4 v (x_1 - \dot{x}_1) \\
\dot{\varphi}_3 &= \dot{\varphi}_4 + \lambda_3 v (x_1 - \dot{x}_1) \\
\dot{\varphi}_4 &= \dot{\varphi}_5 + \lambda_2 v (x_1 - \dot{x}_1) \\
\dot{\varphi}_5 &= \dot{\varphi}_6 + \lambda_1 v (x_1 - \dot{x}_1) \\
\dot{\varphi}_6 &= \lambda_{6v} (x_1 - \dot{x}_1)
\end{align*}
\]

(7)

The estimation error of the high gain GPI observer \( \tilde{e}_v \) satisfies the perturbed dynamics given by (8), which corresponds to a non homogeneous linear dynamic system.

\[
\tilde{e}_v^{(8)} + \lambda_7 v \tilde{e}_v^{(7)} + \ldots + \lambda_{2v} \tilde{e}_v + \lambda_{1v} \dot{\tilde{e}}_v + \lambda_{0v} \tilde{e}_v = \xi_v^{(6)}(t)
\]

(8)

Then, to achieve convergence of the error dynamics to a small vicinity around zero, the gains \( \lambda_{iv} \), \( i = 0, 1, 2, \ldots, 7 \) are chosen such that the roots of the associated characteristic polynomial are located far enough into the left half of the complex plane, that usually implies high gain values. Among all possibilities to selecting the gains \( \lambda_{iv} \), \( i = 0, 1, 2, \ldots, 7 \), a simple selection of the gains is done by assigning the associated characteristic polynomial to a suitable polynomial with known adequate roots, e.g. as follows

\[
s^8 + \lambda_7 v s^7 + \lambda_{6v} s^6 + \ldots + \lambda_{2v} s^2 + \lambda_{1v} s + \lambda_{0v} = \left( s^2 + 2 \xi_{obs,v} \omega_{n(\text{obs},v)} s + \omega_{n(\text{obs},v)}^2 \right)^4
\]

(9)

Since the observer (7) explicitly estimates the disturbance term \( \xi_v(t) \), then it is possible to proposed a PD control with disturbance rejection (10). \( \dot{\theta}_v(t) \) and \( \dot{\theta}_v(t) \) are the desired angular velocity and acceleration on the steering wheel, which might be obtained from the corresponding time derivatives of the angular position imposed by the driver \( \dot{\theta}_v(t) \), although this might raise problems due to noise.

\[
V_v(t) = -\frac{J_{ve} R_v}{k_1 N_v} \left[ \dot{\varphi}_1 + \omega_{n(\text{e},v)} \left( \theta_v - \dot{\theta}_v(t) \right) + 2 \xi_{e,v} \omega_{n(\text{e},v)} \left( \dot{x}_2 - \dot{\varphi}_1 \right) - \dot{\varphi}_1 + K_h \dot{\varphi}_1 \right]
\]

(10)

Notice that in (10), \( \dot{x}_2 \rightarrow \dot{\varphi}_1(t) \) corresponds to the steering wheel angular velocity estimation, and \( \dot{\varphi}_1 \rightarrow \dot{\xi}_v(t) \) is the estimated disturbance term acting on the steering wheel subsystem, so that both estimates are simultaneously obtained by the high gain GPI observer (7). Meanwhile, the term \( \dot{\sigma}_1 \) corresponds to the estimate of the disturbance signal on the steering rack subsystem and it is also obtained by a high gain GPI observer designed for such particular system (13).

The feedback reflection of the estimated disturbance term \( \dot{\sigma}_1 \) from the steering rack subsystem to the steering wheel subsystem yields the haptic loop in the form of a feedback signal proportional to the lumped perturbation torques present at the rack system. This recreates, to the driver, the forces and dynamic effects and torques affecting the steering rack and tire mechanism. The gain \( K_h \) weights the amount of haptic feedback to the driver, and can be related to driving sensation and feeling of the road conditions. This feedback is highly related to safety and comfort (Baviskar et al., 2009), (Cetin et al., 2010), and its value is related to driving style, similar as the feeling of a hard or soft steering system. The feedback of \( \dot{\sigma}_1 \) changes the closed loop (8), but its effects might be included in the disturbance term \( \xi_v(t) \) as an external influence.

From Figures 1 and 2, it is straightforward to find out the similarities between the steering wheel (master) system and the rack and tire (slave) system. Due to the similarities between both systems, also a high gain observer and PD controller with disturbance rejection similar to (7) and (10) are proposed for the slave system. Thus, based on the dynamic model of the steering rack, or slave subsystem (4), the nonlinear terms, including uncertain and unmodeled dynamics, are substituted by a time variant disturbance term \( \xi_v(t) \). Then, the state space dynamic model (4) is represented as in equation (11), with states \( y_1 = \theta_r \) and
\[ y_2 = \dot{\theta}_r. \]

\[ \dot{y}_1 = y_2 \quad (11) \]
\[ \dot{y}_2 = \frac{N_r k_1}{J_{re} R_r} V_r(t) + \xi_r(t) \]

with
\[ \xi_r(t) = -\frac{1}{J_{re}} \left\{ \left( B_T r + B_r \right) \dot{\theta}_r + \left( F_T + C_r \right) \text{sign}(\dot{\theta}_r) + \frac{N_r^2 k_2 k_3}{R_r} \dot{\theta}_r \right\} \quad (12) \]

The equality established by (12) is enforced by the proposed model (4), if other dynamic effects or disturbances are to be considered, these phenomena would be included in the term \( \xi_r(t) \) as well, such that, the approximated linear model (11) remains valid in local basis. The term \( \xi_r(t) \) would be estimated by a high gain GPI observer (13), its estimate is denoted by the variable \( \hat{\sigma}_1 \).

\[ \dot{\hat{y}}_1 = \hat{y}_2 + \lambda_{r7} (y_1 - \hat{y}_1) \]
\[ \dot{\hat{y}}_2 = \frac{N_r k_1}{J_{re} R_r} V_r(t) + \hat{\sigma}_1 + \lambda_{r6} (y_1 - \hat{y}_1) \]
\[ \dot{\hat{\sigma}}_1 = \hat{\sigma}_2 + \lambda_{r5} (y_1 - \hat{y}_1) \]
\[ \dot{\hat{\sigma}}_2 = \hat{\sigma}_3 + \lambda_{r4} (y_1 - \hat{y}_1) \]
\[ \dot{\hat{\sigma}}_3 = \hat{\sigma}_4 + \lambda_{r3} (y_1 - \hat{y}_1) \]
\[ \dot{\hat{\sigma}}_4 = \hat{\sigma}_5 + \lambda_{r2} (y_1 - \hat{y}_1) \]
\[ \dot{\hat{\sigma}}_5 = \hat{\sigma}_6 + \lambda_{r1} (y_1 - \hat{y}_1) \]
\[ \dot{\hat{\sigma}}_6 = \lambda_{r0} (y_1 - \hat{y}_1) \quad (13) \]

The observer estimation error \( \hat{e}_r = y_1 - \hat{y}_1 \) satisfies the dynamics given by
\[ \hat{e}_r^{(8)} + \lambda_{r7} \hat{e}_r^{(7)} + \ldots + \lambda_{r2} \hat{e}_r + \lambda_{r1} \hat{e}_r + \lambda_{r0} \hat{e}_r = \xi_r^{(6)}(t) \quad (14) \]

The convergence properties of the estimation error in the steering rack \( \hat{e}_r \) to a small vicinity of the origin can be established in a similar manner as for the steering wheel subsystem. This is assigning the associated characteristic polynomial to a suitable polynomial with known adequate roots, as in (9).

Based on the estimated angular velocity of the steering rack subsystem \( \hat{y}_2 \) along with the estimated perturbation \( \hat{\sigma}_1 \), an active disturbance rejection control is synthesized in the form of a PD controller with a perturbation term, (15). Notice that the reference trajectory for the PD steering rack controller corresponds to the steering wheel angle \( \theta_v(t) \) and the angular velocity and acceleration estimates \( \hat{x}_2 \) and \( \hat{x}_2 \), which are obtained by the high gain GPI observer (7). Therefore, the closed loop system can be seen as a master - slave system with a haptic loop. The haptic loop is due to feedback reflecting the estimate of the steering rack perturbation term \( \hat{\sigma}_1 \) to the steering wheel PD control, given by
\[ V_r(t) = \frac{J_{re} R_r}{k_1 N_r} \left[ \hat{\sigma}_1 + \omega_n^2(c,r)(\theta_r - \theta_v(t)) + 2\xi_{c,r} \hat{\omega}_n(c,r)(\hat{y}_2 - \hat{x}_2) - \hat{x}_2 \right] \quad (15) \]

Notice that the full control system given by the controllers (10) and (15) together with the high gain GPI observers (7) and (13) only require measurement of the steering wheel angle \( \theta_v \) and the tire orientation angle \( \theta_r \). It is also important to point out that the proposed approach is based on the input gain of the systems, thus minimum knowledge of the dynamic models is required. Figure 3 shows a schematic representation of the proposed SBW system and the interconnections between the master and slave subsystems.

IV. EXPERIMENTAL RESULTS

A low cost experimental platform has been built with a steering wheel as a master subsystem, and half of the steering rack of a beetle VW vehicle, including suspension system and tire. This type of steering system is still available in economic commercial cars. The steering column was modified as shown in Figure 4 including 2 DC motors and encoders. The communication and control are programmed in a PC with Windows® as operating system. The SBW platform is still under development and it lacks traction for the tire. As future work traction will be provided to the experimental platform.

The whole control approach, PD and high gain GPI observer (7, 10) for the steering wheel, and (13, 15) for the steering rack are programmed in Simulink® available in MATLAB®. The angular position on the steering wheel and on the rack are measured by incremental encoders OMRON, model E6B2-CWZ1X, 2000PPR, 0.5M. The PC is provided with a data acquisition card Sensoray Model 626, the sampling period is 0.0005 seconds. Two operational amplifiers, model STK4050 II, are used to conditioning the voltages that are sent to the DC motors NISCA MOTOR Model NC5475B. Each motor is connected to a gearbox, such that for the steering wheel (master subsystem) a reduction rate of \( N_v = 16 \) is obtained, and for the steering rack (slave subsystem) the reduction ratio is \( N_r = 48 \).

The experimental results are first shown for the steering wheel (master subsystem) and later on for the steering rack (slave subsystem), all gains were easily tuned by trial and error taking into account the performance of the tracking errors \( e_v = \theta_v - \theta_v^* \), \( e_r = \theta_r - \theta_v \), and the estimation errors \( \hat{e}_v \) and \( \hat{e}_r \). For the master subsystem the control gains involve in equation (10) are \( \xi_{c,v} = 1.01 \), \( \omega_{n(c,v)} = 5.92 \), \( K_h = 1 \), and for its GPI observer (7)
and 9) $\xi_{obs,v} = 2.2$, $\omega_{n(obs,v)} = 15.2$. Meanwhile, for steering rack (slave subsystem) the control gains, see (15), are $\xi_{c,r} = 2.53$, $\omega_{n(c,r)} = 5.92$ and the GPI observer (13) the gains are $\xi_{obs,r} = 10$, $\omega_{n(obs,r)} = 25$.

Figure 5 shows the tracking error $e_v = \theta_v - \theta^*_v$, while tracking a random reference $\theta^*_v(t)$ set by a human operator (driver), turning the steering wheel.

It can be observed at Figure 5, that there is an acceptable tracking error, this allows one to conclude rejection of the disturbance signal at the steering wheel subsystem, which is estimated by the GPI observer (7). The voltage signal $V_v$ at the steering wheel DC motor is shown in Figure 6.

As for the steering rack (slave subsystem), Figure 7 shows tracking between the rack angular position $\theta_r$ an the steering wheel angular position $\theta_v$, while the angular rack tracking error $e_r = \theta_r - \theta_v$ is shown in Figure 8. For the value of the tracking error $e_r$, which is around zero, it can be concluded that the high gain GPI observer (13) properly estimates the disturbance signal at the steering rack and tire, and therefore the PD plus disturbance cancellation (15) is able to effectively reject the disturbance signal effects.

The convergence between the steering rack angle (tire angle orientation) $\theta_r$ and its desired angular position given by the steering wheel angle $\theta_v$ allows concluding good tracking between both systems, this is further supported by the tracking error $e_r = \theta_r - \theta_v$ convergence to a small vicinity around zero, see Figure 8. On the other hand, feedback of the disturbance phenomena $\hat{\sigma}_1(t)$, shown in Figure 9, from the steering rack to the steering wheel generates a haptic loop such that the operator "feels" the dynamic perturbations that affect the steering rack and tire.

The DC motor voltage on the steering rack subsystem $V_r(t)$ required to properly track the rack angular reference is shown in Figure 10.

V. CONCLUSIONS AND PERSPECTIVES

This article has presented a practical application of Active Disturbance Rejection Control based on high-gain GPI observers. The high gain GPI technique requires a minimum amount of information from the dynamic model; only the system input gain is required, as for sensors only encoders are employed, which is an advantage compared to other techniques of SBW systems that required specialized sensors. The tracking error between the desired angle settled
by the human operator, the steering wheel angle and the tire orientation angle can be made arbitrarily small by a proper gain selection. The technique is capable of estimating the perturbation inputs and the uncertainty terms that affect the dynamics. In particular, the SBW system represents a twofold application of the GPI observer, on one hand it helps in rejection or compensating the perturbation in each subsystem, on the other hand the estimated disturbances on the steering rack are fed back to the steering wheel to yield a haptic system. It is worth to notice that the high gain GPI observer together with the PD control rely only on measurements of an encoder, thus highly diminishing the use of sensors. To validate this last conclusion, as future work the installation of force, acceleration and other sensors is being considered. This will allow to establish clear comparisons between the estimated perturbation and measurements on the experimental platform. Also, the influence of traction on the wheel will be further investigated.

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