Decentralized stabilizer design of wide-area interconnected power systems

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Abstract— **A decentralized feedback controller design is presented in this paper for wide-area interconnected power systems. The key feature is a simple methodology to tune the controller's parameters and to assure the synchronization of the power machines and the tracking of a constant reference via some error equations.**
The controller is tested in a two-area nonlinear interconnected power system and shows robustness to the effect of delays induced by the length of the transmission lines. Simulation results illustrate the viability of the proposed scheme.

Keywords: Electric power systems, interconnected systems, synchronization, decentralized control.

I. Introduction

Recently, the deregulation of power industry and the increasing demand of electricity have led to operate networks near to their transfer power limits. As a consequence, several modes of inter-area oscillations, which cannot be easily damped by traditional methods, have been detected (Chang y Xu, 2007).
Different Interconnected Power Systems (IPS) models and control techniques have been proposed to overcome the aforementioned problem. One of them, the use of remote signals from wide-area measurement systems (WAMS), has become an interesting alternative despite the high cost of the equipment (Ivanescu *et al.*, 2000; Snyder *et al.*, 2000). The major problem involved with this technology is the delay between the instant of measurement and that of the signals being available for the controller, which is in the range of 100-700 ms. depending on the communication links and other characteristics (Naduvathuparambil *et al.*, 2002). This has increased the attention to the overall effect of time delays in IPS and its crucial impact on the stability analysis (Jia *et al.*, 2008; Wu *et al.*, 2004).
Another delay, related to the length of the transmission lines, has been identified. However, it is usually ignored in practice due to its magnitude (less than 10 ms) (Wu y GT, 2003).
In (Kamwa *et al.*, 2001) and (Dou *et al.*, 2007) decentralized controllers for large IPS networks using the so-called hierarchical/decentralized architecture and a Takagi-Sugeno fuzzy model, respectively, have been proposed, without considering any WAMS delay.
The main contribution of this paper is a simple methodology to tune the parameters of a decentralized feedback controller and to assure the synchronization of the power machines and the tracking of a constant reference via some error equations. This solution is less costly and complex than standard WAMS-based ones.
The structure is organized as follows: Section II presents the nonlinear power system model with and without the inclusion of time delays, as well as preliminary definitions. The problem statement is the subject of Section III. The controller design and main results are shown in Section IV. Finally Section V concludes the paper.

II. Network model

A. Interconnected power systems model

Under some standard assumptions, the dynamics of N interconnected generators can be described by the classical model with flux decay dynamics. The network has been reduced to internal bus representation assuming the loads to be constant impedances. Furthermore, in practical power systems, line conductances G_{ij} can be neglected with respect to line susceptance B_{ij} ($G_{ij} \ll B_{ij}$). The dynamical model of the i-th machine without communication delays is represented by (Pai, 1989)

$$\dot{\delta}_i(t) = \omega_i(t) - \omega_s \tag{1}$$

$$\dot{\omega}_i(t) = \frac{\omega_s}{2H_i}[P_{m_i} - E'_{q_i}(t) I_{q_i}(t) - D_i(\omega_i(t) - \omega_s)]$$

$$\dot{E}'_{q_i}(t) = \frac{1}{T'_{d_i}}[E_{f_i}(t) - E'_{q_i}(t) - (X_{d_i} - X'_{d_i}) I_{d_i}(t)]$$

where

$$I_{q_i}(t)= \sum_{\substack{j=1\\ j\neq i}}^{N} E'_{q_j}(t)B_{ij}\sin(\delta_i(t)-\delta_j(t))$$

$$I_{d_i}(t)=- \sum_{\substack{j=1\\ j\neq i}}^{N} E'_{q_j}(t)B_{ij}\cos(\delta_i(t)-\delta_j(t))$$

$I_{q_i}(t)$ and $I_{d_i}(t)$ represent the currents in the d-q reference frame of the i-th generator,
$E'_{q_i}(t)$ is the transient EMF in the quadrature axis,
$E_{q_i}(t)$ is the EMF in the quadrature axis,
$E_{f_i}(t)$ is the equivalent EMF in the excitation coil,
P_{e_i} and Q_{e_i} are the active and reactive power, respectively,
$\delta_i(t)$ is the machine rotor angle with respect to a synchronously rotating frame, in radians, and
$\omega_i(t)$ represents the rotor speed.

Furthermore, the state of the i-th system (1) is represented by $X_i = [\delta_i(t), \omega_i(t), E'_{qi}(t)]^T \in I\!R^3$ for $i = 1, .., N$, while the system input $u_i(t) \in I\!R$ is $E_{fi}(t)$.

It is worthwhile pointing out that the accurate modelling of real generators may need more than three, say five or seven, differential equations. However, those extra equations only specify the dynamic behavior with respect to the very small time constant of the system, which has been proven to be negligible for designing a power system controller. The elementary model (1) includes only the slow dynamic behavior associated to the mechanical variables. The additional equations describing the faster electrical dynamics are ignored since they are reduced to purely algebraic equations (Pal y Chaudhuri, 2005).

Taking into account the communication delays, the following model is proposed for the state representation of the i-th machine with excitation control. It can be written as

$$\begin{aligned}\dot{\delta}_i(t)&=\omega_i(t)-\omega_s\\ \dot{\omega}_i(t)&=-a_i(\omega_i(t)-\omega_s)+b_i-c_iE'_{q_i}(t)\\ &\quad\sum_{\substack{j=1\\ j\neq i}}^{N}[E'_{q_j}(t-\tau_{ij})B_{ij}\sin(\delta_i(t)-\delta_j(t-\tau_{ij}))]\\ \dot{E}'_{q_i}(t)&=u_i(\cdot)-e_iE'_{q_i}(t)+d_i\\ &\quad\sum_{\substack{j=1\\ j\neq i}}^{N}[E'_{q_j}(t-\tau_{ij})B_{ij}\cos(\delta_i(t)-\delta_j(t-\tau_{ij}))]\end{aligned} \tag{2}$$

where $a_i = \dfrac{D_i}{2H_i}$, $b_i = \dfrac{\omega_s P_{m_i}}{2H_i}$, $c_i = \dfrac{\omega_s}{2H_i}$, $d_i = \dfrac{(X_{d_i} - X'_{d_i})}{T'_{d_i}}$, $e_i = \dfrac{1}{T'_{d_i}}$, are the system parameters, $u_i(t) = e_iE_{f_i}(t)$ is the control input and τ_{ij} are the delays inherent to the transmission lines.

Note that the model (2) in open loop does not involve the delay due to WAMS. The variables of the machine j are delayed due to the communication channel. However standard controllers require their value at present time. The common solution is to design a predictor to determine them, see for example (Chaudhuri *et al.*, 2004; Mohagheghi *et al.*, 2007), but it causes an extra effort in terms of time computation, equipment and depending on the number of variables to reconstruct.

The equilibrium points of systems (1) and (2) satisfy the following conditions

$$\begin{aligned}\omega_i^* &= \omega_s\\ b_i &= c_iE'^*_{q_i}\sum_{\substack{j=1\\ j\neq i}}^{N}(E'^*_{q_j}B_{ij}\sin(\delta_i^*-\delta_j^*))\\ \bar{u}_i &= e_iE'^*_{q_i}-d_i\sum_{\substack{j=1\\ j\neq i}}^{N}(E'^*_{q_j}B_{ij}\cos(\delta_i^*-\delta_j^*)).\end{aligned}$$

Given an operating point $(\delta_i^*, \omega_s, E'^*_{q_i})$, one can insert the constant excitation control $\bar{u}_i$ or vice-verse.

In power systems, two machines are said to be synchronized if their angles keep swinging together into pre-defined limits. The following definition is used throughout this paper.

Synchronization (Alberto y Bretas, 1999). The solutions $\delta_i(t)$ and $\delta_j(t)$ are considered in synchronism if, for each real number $L_0 > 0$, there exists a real number $L > 0$ such that for every initial condition $\delta_i(t_0)$ and $\delta_j(t_0)$ satisfying $\| \delta_i(t_0) - \delta_j(t_0) \| < L_0$, the solutions $\delta_i(t)$ and $\delta_j(t)$ satisfy the inequality $\| \delta_i(t) - \delta_j(t) \| < L$ for all $t > t_0$.

It is clear from previous definition that the synchronism property does not guarantee the stability. A system can be synchronized and yet be unstable. Now, we can define the problem statement, which is done in the following section.

III. Problem statement

Let consider a network which consists of three generators modeled each one by (2). For each system, the equivalent EMF in the excitation coil E_{f_i} is the available control input. The problem is to design a causal controller to stabilize the error signal $e(t) = \delta_1(t) - \delta_2(t)$.
Causality of the control law is meant for using only the available delayed data, in other words, the signal depends of the state at present and past values of the time only. This problem will be solved in the next section.

IV. Control Design

In this section, constructive conditions under which a linear decentralized state feedback that solves the stability problem stated in section III exists. Since only local states are used, there is no time delay involved and no state predictors are required.

A. Controller design procedure

In this section, a control design procedure that leads to a decentralized stabilizing control will be described. Generator No. 3 is an infinite bus, we have $E^{'}_{q_3} = constant = 1\angle 0°$. The steps are the following:

1. Linearization.
Linearizing around an equilibrium point ($\delta_i^* = \delta^*, \omega_i^*, E^{'*}_{q_i}, \bar{u}_i$) for $i = \{1, 2\}$.

$$\begin{aligned}\dot{x}_{i,1} &= x_{i,2}\\ \dot{x}_{i,2} &= p_i x_{i,1} - a_i x_{i,2} + q_i x_{i,3} + s_{ij} x_{j,1} \\ \dot{x}_{i,3} &= r_i x_{i,1} - e_i x_{i,3} + d_i B_{ij} x_{j,3} + u_i\end{aligned} \quad (3)$$

with $\begin{bmatrix} x_{i,1} & x_{i,2} & x_{i,3}\end{bmatrix}^T = \begin{bmatrix}\delta_i, \omega_i, E^{'}_{q_i}\end{bmatrix}^T$ and

$$\begin{aligned}p_i &= -c_i E^{'*}_{q_i}[(E^{'*}_{q_j} B_{ij}) + B_{iN}\cos(\delta^*)]\\ q_i &= -c_i B_{iN}\sin(\delta^*)\\ r_i &= -d_i B_{iN}\sin(\delta^*)\\ s_{ij} &= c_i E^{'*}_{q_i} E^{'*}_{q_j} B_{ij}\end{aligned}$$

for $i = \{1, 2\}$. The index j is defined as a function of i:

i	j
1	2
2	1

2. Transformation into canonical form.
The matrix A of (3) and their controllability matrix are of rank 6. Then there exists a transformation of the form (see (Luenberger, 1967) for details)

$$\begin{aligned}z_{i,1} &= \frac{1}{q_i} x_{i,1}\\ z_{i,2} &= \frac{1}{q_i} x_{i,2} \\ z_{i,3} &= \frac{p_i}{q_i} x_{i,1} - \frac{a_i}{q_i} x_{i,2} + x_{i,3} + \frac{s_{ji} B_{jN}}{q_j B_{iN}} x_{j,1}\end{aligned} \quad (4)$$

for $i = \{1, 2\}$, which allows to write the state equations in canonical form

$$\begin{aligned}&\dot{z}_{i,1} = z_{i,2}\\ &\dot{z}_{i,2} = z_{i,3} \\ &\dot{z}_{i,3} = \bar{p}_i z_{i,1} + \bar{q}_i z_{i,2} + \bar{r}_i z_{i,3} + [\bar{s}_i z_{j,1} + \bar{v}_i z_{j,2} + \bar{w}_i z_{j,3}]\\ &\quad + u_i\end{aligned} \quad (5)$$

with

$$\begin{aligned}\bar{p}_i &= -\frac{c_i d_i E^{'}_{q_i} E^{'}_{q_j} {B_{ij}}^2 B_{i3}}{B_{j3}} + c_i d_i \sin(\delta)^2 {B_{i3}}^2\\ &\quad - c_i\cos(\delta) e_i E^{'}_{q_i} B_{i3} - c_i e_i E^{'}_{q_i} E^{'}_{q_j} B_{ij}\\ \bar{q}_i &= -c_i \cos(\delta) E^{'}_{q_i} B_{i3} - c_i E^{'}_{q_i} E^{'}_{q_j} B_{ij} - a_i e_i\\ \bar{r}_i &= -e_i - a_i\\ \bar{s}_i &= \frac{c_j e_i E^{'}_{q_i} E^{'}_{q_j} B_{ij} B_{j3}}{B_{i3}} + c_j d_i E^{'}_{q_i} E^{'}_{q_j} {B_{ij}}^2\\ &\quad + c_j d_i \cos(\delta) E^{'}_{q_j} B_{ij} B_{j3}\\ \bar{v}_i &= \frac{c_j E^{'}_{q_i} E^{'}_{q_j} B_{ij} B_{j3}}{B_{i3}} + a_j d_i B_{ij}\\ \bar{w}_i &= d_i B_{ij}\end{aligned}$$

for $i = \{1, 2\}$.
3. Definition of error equations

$$\begin{aligned} e &= z_1 - z_4 = y_1 - y_2 \quad (6)\\ \varepsilon &= z_1 = y_1 \quad (7)\end{aligned}$$

to assure synchronization of the rotor angles and stabilization.
Differentiating (6) and (7) under the trajectories of (5) we get

$$\begin{aligned} e^{(3)}(t) &= (\bar{p}_1 - \bar{s}_2) y_1 + (\bar{q}_1 - \bar{v}_2)\dot{y}_1 + (\bar{r}_1 - \bar{\omega}_2)\ddot{y}_1\\ &\quad + (\bar{s}_1 - \bar{p}_2) y_2 + (\bar{v}_1 - \bar{q}_2)\dot{y}_2 + (\bar{\omega}_1 - \bar{r}_2)\ddot{y}_2\\ &\quad + u_1 - u_2 \quad (8)\\ \varepsilon^{(3)}(t) &= \bar{p}_1 y_1 + \bar{q}_1 \dot{y}_1 + \bar{r}_1 \ddot{y}_1 + \bar{s}_1 y_2 + \bar{v}_1 \dot{y}_2\\ &\quad + \bar{\omega}_1 \ddot{y}_2 + u_1 \quad (9)\end{aligned}$$

4. System stabilization.
A decentralized controller is designed. It means that only the local data of each generator will be used, without the WAMS. To find sufficient conditions under which such controller exists, define

$$\begin{aligned} P_1 &= \bar{r}_1 - \bar{\omega}_2 + k_{13}\\ Q_1 &= \bar{q}_1 - \bar{v}_2 + k_{12} q_1 + a_1 k_{13}\\ V_1 &= \bar{p}_1 - \bar{s}_2 - p_1 k_{13} + k_{11} q_1 - \frac{k_{23} s_1 B_{13}}{B_{23}}\\ P_2 &= \bar{p}_1 - p_1 k_{13} + k_{11} q_1 + \bar{s}_1 + \frac{k_{13} s_2 B_{23}}{B_{13}}\\ Q_2 &= \bar{q}_1 + k_{12} q_1 + a_1 k_{13} + \bar{v}_1\\ V_2 &= \bar{r}_1 + k_{13} + \bar{\omega}_1\end{aligned} \quad (10)$$

Proposition 1. *Consider System (2). If there exist parameters k_{11}, k_{12}, k_{13} such that the inequalities*

$$\begin{aligned} P_i &< 0,\\ Q_i &< 0,\\ V_i &< 0,\\ P_i Q_i &> -V_i, \text{ for } i = 1, 2.\end{aligned} \quad (11)$$

with P_i, Q_i, V_i defined by (10). Then the controller

$$\begin{aligned} u_i &= k_{i1}(x_{i,1}-x^*_{i,1})+k_{i2}(x_{i,2}-x^*_{i,2}) \\ &\quad +k_{i3}(x_{i,3}-x^*_{i,3});\ \textit{for } i=\{1,2\}, \end{aligned} \qquad (12)$$

stabilizes the system.

Proof.
Considering the linearized system (3), the equilibrium point is the origin. Thus, the control input (12) becomes:

$$u_i = k_{i1}x_{i,1}+k_{i2}x_{i,2}+k_{i3}x_{i,3} \qquad (13)$$

Applying the transformation $x(t)=T^{-1}z(t)$, inverse of (4), the inputs u_1 and u_2 are of the form

$$\begin{aligned} u_1 &= (-p_1k_{13}+k_{11}q_1)z_1+(k_{12}q_1+a_1k_{13})z_2 \\ &\quad +k_{13}z_3+\frac{k_{13}s_2B_{23}}{B_{13}}z_4 \qquad (14) \\ u_2 &= \frac{k_{23}s_1B_{13}}{B_{23}}z_1+(-p_2k_{23}+k_{21}q_2)z_4 \\ &\quad +(k_{22}q_2+a_2k_{23})z_5+k_{23}z_6. \end{aligned}$$

This is done to work with system (5). Now, the parameters k_{il} must be assigned to transform (8) and (9) into error equations as follows:

$$\begin{aligned} e^{(3)}(t) &= (\bar{p}_1-\bar{s}_2-p_1k_{13}+k_{11}q_1-\frac{k_{23}s_1B_{13}}{B_{23}})y_1 \\ &\quad +(\bar{q}_1-\bar{v}_2+k_{12}q_1+a_1k_{13})\dot{y}_1+(\bar{r}_1-\bar{\omega}_2+k_{13})\ddot{y}_1 \\ &\quad +(\bar{s}_1-\bar{p}_2+\frac{k_{13}s_2B_{23}}{B_{13}}+p_2k_{23}-k_{21}q_2)y_2 \qquad (15) \\ &\quad +(\bar{v}_1-\bar{q}_2-(k_{22}q_2+a_2k_{23}))\dot{y}_2+(\bar{\omega}_1-\bar{r}_2-k_{23})\ddot{y}_2 \end{aligned}$$

and

$$\begin{aligned} \varepsilon^{(3)}(t) &= (\bar{p}_1-p_1k_{13}+k_{11}q_1)y_1 \qquad (16) \\ &\quad +(\bar{q}_1+k_{12}q_1+a_1k_{13})\dot{y}_1+(\bar{r}_1+k_{13})\ddot{y}_1 \\ &\quad +(\bar{s}_1+\frac{k_{13}s_2B_{23}}{B_{13}})y_2+\bar{v}_1\dot{y}_2+\bar{\omega}_1\ddot{y}_2. \end{aligned}$$

From (15), assigning k_{21}, k_{22} and k_{23}, then

$$e^{(3)}(t) = P_1\ddot{e}+Q_1\dot{e}+V_1. \qquad (17)$$

from inequalities (11) it can be proved that (17) is asymptotically stable using the results from (Cahlon y Schmidt, 2006). Then, we have that

$$y_1 = y_2,$$

and (16) becomes

$$\varepsilon^{(3)}(t) = P_2\ddot{\varepsilon}+Q_2\dot{\varepsilon}+V_2\varepsilon. \qquad (18)$$

Now, using the error equations (6) and (7) one defines a change of coordinates such that

$$\frac{d}{dt}\begin{bmatrix} e(t) \\ \dot{e}(t) \\ \ddot{e}(t) \\ \varepsilon(t) \\ \dot{\varepsilon}(t) \\ \ddot{\varepsilon}(t) \end{bmatrix} = \begin{bmatrix} \dot{e}(t) \\ \ddot{e}(t) \\ l_1e(t)+l_2\dot{e}(t)+l_3\ddot{e}(t) \\ \dot{\varepsilon}(t) \\ \ddot{\varepsilon}(t) \\ l_7e(t)-\bar{v}_1\dot{e}(t)-\bar{\omega}_1\ddot{e}(t)+l_4\varepsilon(t)+l_5\dot{\varepsilon}(t)+l_6\ddot{\varepsilon}(t) \end{bmatrix} \qquad (19)$$

with

$$\begin{aligned} l_1 &= (\bar{p}_1-\bar{s}_2-p_1k_{13}+k_{11}q_1-\frac{k_{23}s_1B_{13}}{B_{23}}) \\ l_2 &= (\bar{q}_1-\bar{v}_2+k_{12}q_1+a_1k_{13}) \\ l_3 &= (\bar{r}_1-\bar{\omega}_2+k_{13}) \\ l_4 &= (\bar{p}_1-p_1k_{13}+k_{11}q_1+\bar{s}_1+\frac{k_{13}s_2B_{23}}{B_{13}}) \\ l_5 &= (\bar{q}_1+k_{12}q_1+a_1k_{13}+\bar{v}_1) \\ l_6 &= (\bar{r}_1+k_{13}+\bar{\omega}_1) \\ l_7 &= -(\bar{s}_1+\frac{k_{13}s_2B_{23}}{B_{13}}) \end{aligned}$$

Note that this change of coordinates allows to compute the system stability tuning just the parameters k_{1l} via pole placement. The stability of (19) and synchronization of the rotor angles depend on the stability of equations (17) and (18), which is given by inequalities (11) (Cahlon y Schmidt, 2006). ■

B. Simulation.

In this section we will illustrate the viability of the proposed procedure. In Table I the parameters used to describe the behavior of the machines are given. They are taken from (Xi, 2002), and used to compute the equilibrium point.

TABLE I
PARAMETERS, DESCRIPTION AND VALUES.

NOTATION	DESCRIPTION	VALUES Machine 1	VALUES Machine 2
X_{di}	Direct axis reactance.*	1.863	2.36
X'_{di}	Direct axis transient reactance.*	0.257	0.319
D_i	Damping factor.*	5	3
H_i	Inertia constant, in seconds.	4	5.1
T'_{di}	Direct axis transient short circuit time constant, in seconds.	6.9	7.96
ω_s	Synchronous machine speed, in rad/s.	120π	120π
B_{ij}	Element of the i-th row and j-th column of the nodal susceptance matrix, which is symmetric; at the internal nodes after eliminating all physical buses.*	$[B_{ij}] = \begin{bmatrix} -1.4187 & 0.3296 & 1.0891 \\ 0.3296 & -1.3040 & 0.9744 \\ 1.0891 & 0.9744 & -2.0635 \end{bmatrix}$	

* All parameters are in p.u..

The considered operating point is ($\delta_1 = 0.5236$, $\delta_2 = 0.5236$, $\omega_1 = 377$, $\omega_2 = 377$, $E'_{q_1} = 1.03$, $E'_{q_2} = 1.01$, $\bar{u}_1 = -0.1477$, $\bar{u}_2 = -0.1765$).

The linearized nonlinear system around this equilibrium point is

$$\begin{aligned}
\dot{x}_{1,1} &= x_{1,2}\\
\dot{x}_{1,2} &= -61.9395x_{1,1} - \frac{5}{8}x_{1,2} - 25.6620x_{1,3} + 16.1584x_{2,1}\\
\dot{x}_{1,3} &= -0.1268x_{1,1} - 0.1449x_{1,3} + 0.0767x_{2,3} + u_1\\
\dot{x}_{2,1} &= x_{2,2}\\
\dot{x}_{2,2} &= 12.6732x_{1,1} - 44.1746x_{2,1} - 0.2941x_{2,2} - 18.0073x_{2,3}\\
\dot{x}_{2,3} &= 0.0845x_{1,3} - 0.1249x_{2,1} - 0.1256x_{2,3} + u_2
\end{aligned}$$

where $\mathcal{X} = \begin{bmatrix}\delta_1 & \omega_1 & E'_{q_1} & \delta_2 & \omega_2 & E'_{q_2}\end{bmatrix}^T$. Applying transformation (4) the canonical system is

$$\begin{aligned}
\dot{z}_1 &= z_2\\
\dot{z}_2 &= z_3 \qquad (20)\\
\dot{z}_3 &= -7.1097z_1 - 62.0301z_2 - 0.7699z_3\\
&\quad +5.0322z_4 + 11.3611z_5 + 0.0767z_6 + u_1\\
\dot{z}_4 &= z_5\\
\dot{z}_5 &= z_6\\
\dot{z}_6 &= 7.5035z_1 + 18.1132z_2 + 0.0845z_3\\
&\quad -4.2583z_4 - 44.2116z_5 - 0.4198z_6 + u_2.
\end{aligned}$$

Let the inputs u_1 and u_2 be of the form (14). The error equation (8) becomes:

$$\begin{aligned}
e^{(3)}(t) &= (-14.6132 - 25.6410k_{11} + 61.8897k_{13}\\
&\quad +18.0462k_{23})e(t)\\
&\quad -(80.1433 + 25.6410k_{12} - 0.6256k_{13})\dot{e}(t)\\
&\quad +(-0.8544 + k_{13})\ddot{e}(t) \qquad (21)
\end{aligned}$$

and from (15) the coefficients of k_{2l} are

$$\begin{aligned}
k_{21} &= \frac{5.3227 + 25.641k_{11} - 50.5437k_{13} + 26.1556k_{23}}{18.018}\\
k_{22} &= \frac{24.5706 + 25.641k_{12} - 0.6256k_{13} + 0.2937k_{23}}{18.018}\\
k_{23} &= -0.3579 + k_{13}.
\end{aligned}$$

The stabilization of (21) gives to $y_1 = y_2$ and (16) becomes

$$\begin{aligned}
\varepsilon^{(3)}(t) &= (-2.0775 - 25.641k_{11} + 50.5437k_{13})\varepsilon(t)\\
&\quad +(-50.669 - 25.641k_{12} + 0.6256k_{13})\dot{\varepsilon}(t)\\
&\quad +(-0.6932 + k_{13})\ddot{\varepsilon}(t) \qquad (22)
\end{aligned}$$

After applying the change of coordinates based on the error equations the transformed system is

$$\frac{d}{dt}\begin{bmatrix} e(t)\\ \dot{e}(t)\\ \ddot{e}(t)\\ \varepsilon(t)\\ \dot{\varepsilon}(t)\\ \ddot{\varepsilon}(t)\end{bmatrix} = \begin{bmatrix} \dot{e}(t)\\ \ddot{e}(t)\\ p_1e(t) + p_2\dot{e}(t) + (-0.8544 + k_{13})\ddot{e}(t)\\ \dot{\varepsilon}(t)\\ \ddot{\varepsilon}(t)\\ p_3e(t) + 11.3611\dot{e}(t) + 0.0767\ddot{e}(t) + p_4\varepsilon(t)\\ +p_5\dot{\varepsilon}(t) + (k_{13} - 0.6932)\ddot{\varepsilon}(t)\end{bmatrix}$$

where

$$\begin{aligned}
p_1 &= 79.9359k_{13} - 25.641k_{11} - 21.0719\\
p_2 &= -80.1433 + 0.6256k_{13} - 25.641k_{12}\\
p_3 &= 5.0322 - 11.346k_{13}\\
p_4 &= 50.5437k_{13} - 25.641k_{11} - 2.0775\\
p_5 &= 0.6256k_{13} - 25.641k_{12} - 50.669
\end{aligned}$$

The stability of this transformed system can be guaranteed from (11), which yield the following inequalities:

$$\begin{aligned}
79.9359k_{13} - 25.641k_{11} &< 21.0719\\
-25.6410k_{12} + 0.6256k_{13} &< 80.1433\\
50.5437k_{13} - 25.641k_{11} &< 2.0775\\
0.6256k_{13} - 25.641k_{12} &< 50.669\\
k_{13} &< 0.6932\\
0.6256k_{13}^2 - 25.641k_{12}k_{13} - 0.7419k_{13} & \qquad (23)\\
+21.9077k_{12} - 25.641k_{11} &> -47.4025\\
0.6256k_{13}^2 - 25.641k_{12}k_{13} - 0.559k_{13} &\\
+17.7743k_{12} - 25.641k_{11} &> -33.0463.
\end{aligned}$$

The following control laws are proposed.

$$\begin{aligned}
u_1 &= -6x_1 - 1x_2 - 4x_3 \qquad (24)\\
u_2 &= -3.3484x_4 + 0.0084x_5 - 4.3579x_6
\end{aligned}$$

These values assures the stability of the system (19) and the solution of the inequalities (23). The simulation results are shown in Figure 1. The initial conditions are ($\delta_{11} = 0.8$, $\delta_2 = 0.6$, $\omega_{11} = 375$, $\omega_2 = 378$, $E'_{q_1} = 1.2$, $E'_{q_2} = 1.1$).

Figures 2 and 3 shown the effect of the transmission line delays, considering model (2) in closed loop with (24). Note that the performance decrease with higher magnitudes of time delays and the stability is lost for delays greater than 60 ms. However, this value is so big compared with the usual magnitudes of transmission line delays.

V. CONCLUSIONS

A simple procedure to design a decentralized state feedback controller has been proposed to tackle the stability control problem for two wide area interconnected power systems. It was shown that thanks to the full controllability of the linearized models there exist a transformation which allows to tune the controller's parameters easily. The asymptotic stabilization of the linearized models yields to the local stabilization of the nonlinear plants. This could be exploited

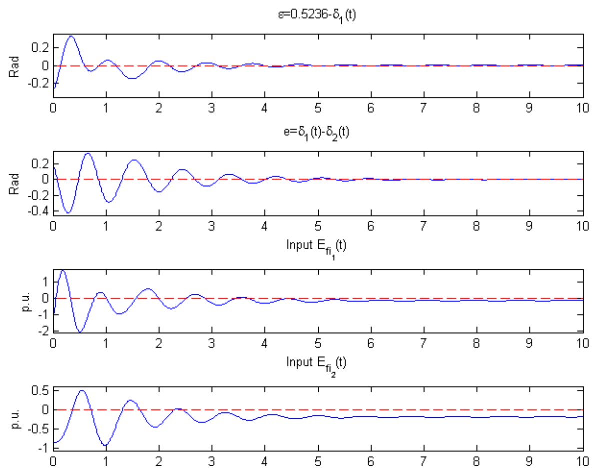

Figure 1. Error equations and control inputs.

to eliminate the need of WAMS in the control loop. Furthermore, we claim that this methodology is valid to design decentralized controllers for bigger networks, although the computation complexity is increased.

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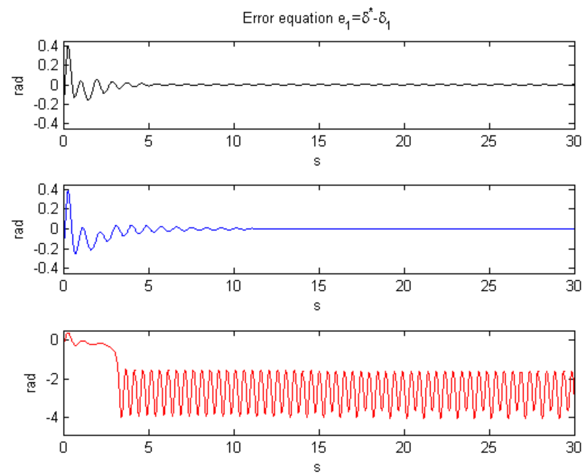

Figure 2. Synchronization error. The considered time delays are $\tau_{12} = 0, 50$ ms and 60 ms respectively.

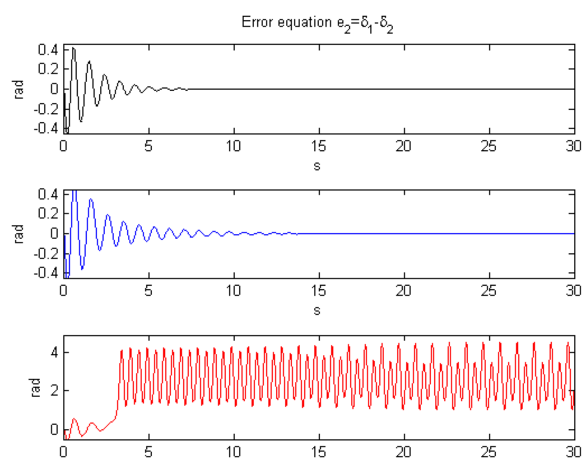

Figure 3. Tracking error. The considered time delays are $\tau_{12} = 0, 50$ ms and 60 ms respectively.