Lateral Dynamic Control of a Single Rotor Airship

G. Murguía-Rendon, H. Rodríguez-Cortés and M. Velasco-Villa
Sección de Mecatrónica, CINVESTAV-IPN
Av. IPN 2508, Col. San Pedro Zacatenco, México D.F.
{cmurguia,hrodriguez,mvelasco}@cinvestav.mx

Abstract— **This paper addresses and solves the trajectory tracking problem for the planar dynamics of a thrust vectored airship. The proposed controller is based on a combination of two nonlinear control design techniques, backstepping and Exact Tracking Error Dynamics Passive Output Fedback (ETDPOF). Numerical simulations show the performance of the proposed controller and some concluding remarks complete the paper.**

Key words: Nonlinear Control, trajectory tracking, airship dynamics.

I. INTRODUCTION

Among the topologies of unmanned aerial vehicles capable of hovering airships represent an interesting option for low speed, low altitude, long term monitoring and surveillance missions. Airships have the benefits, with respect to rotary wing aerial vehicles, from the absecence of rotors, which generally imply high structural design costs and the low power requirements to hover thanks to aerostatic lift. In fact, the attribute of low power consumption to hover makes airships noisless, ecological and very useful for long term enviromental applications (Maggiore, 2005) such as oceanographic, agricultural and climate studies.

The undoubtly regain of interest in potential applications of airships has attracted the attention of several research institutes and private companies. Innovative research projects devoted to airship operation have already provided interesting results in some places. The Aurora project from Campinas Information Technology has proposed some efficient blimp navigation algorithms (Carvalho, 2001), the Lotte project from the University of Stuttgart and the Aztec project from the University of Virginia have introduced new power plants based on solar energy (Kungl, 2001), the Elettra Twin Flyiers airship designed and patented by Nautilus S.p.A. and the Polytechnic of Turin has introduced new concepts on airship topology.

The ability of an airship to autonomously achieve a mission requires a successful flight control system in conjuction with an efficient algorithm to fusion different sensor's measurements to determine the airship spatial position and attitude. In developing the flight control system difficulties arise because the airship dynamic model is described by a set of nonlinear differential equations with the particularity that the traslational dynamics is coupled to the rotational dynamics through acceleration terms. This makes the airship flight control design a difficult task attracting the interest of many control researchers who had proposed control algorithms based on linear and nonlinear control design techniques. In particular, classical linear design tools have been used in (Carvalho, 2001), (Yamada, 2007), whereas the application of advanced nonlinear methods, such as dynamical decoupling (Solaque, 2008), backstepping (Repoulias, 2008), dynamic inversion (Moutinho, 2005) and neural networks (Park, 2003) has also been investigated.

In this paper the trajectory tracking problem for the lateral dynamics of an airship driven by a single rotor is addressed and solved. It is shown that a combination of two nonlinear control design techniques yields a locally asymtotically stable closed–loop system. The rest of the paper is organized as follows. In Section II we present the airship planar dynamics. Section III is committed to the design of the nonlinear controller. Finally, in Section IV we illustrate the performance of the proposed nonlinear controller with numerical simulations and in Section V we present some concluding remarks.

II. PLANAR AIRSHIP DYNAMIC MODEL

In order to describe the airship dynamic model we consider two coordinate systems. The right-hand inertial coordinate system (Earth axis) denoted as $x^e y^e z^e$ and the non inertial coordinate system (body axes) denoted as $x^b y^b z^b$, whose origin is at the airship volumetric center. See Figure 1. The airship dynamics expressed in terms of the body axis coordinates is described by (Thor, 1994)

$$\begin{array}{rcl} m\left[\dot V_{CV} + \Omega\times V_{CV} + \dot\Omega\times r_{cg} + \Omega\times(\Omega\times r_{cg})\right] & = & F_e^b \\ I\dot\Omega + \Omega\times I\Omega + m\left[r_{cg}\times\left(\dot V_{CV} + \Omega\times V_{CV}\right)\right] & = & M_e^b \end{array} \quad (1)$$

where $V_{CV} = \begin{bmatrix} u & v & W \end{bmatrix}^\top$ is the volumetric center velocity, $\Omega = \begin{bmatrix} p & q & r \end{bmatrix}^\top$ is the volumetric center angular velocity, $r_{cg} = \begin{bmatrix} x_{cg} & y_{cg} & z_{cg} \end{bmatrix}^\top$ is the center of gravity position with respect to the volumetric center and

$$\begin{array}{rcl} F_e^b & = & F_{AM}^b + F_R^b + F_P^b + F_A^b \\ M_e^b & = & M_{AI}^b + M_R^b + M_P^b + M_A^b \end{array}$$

the external applied forces and moments. Considering that the kinetic energy induced in the air by the airship motion

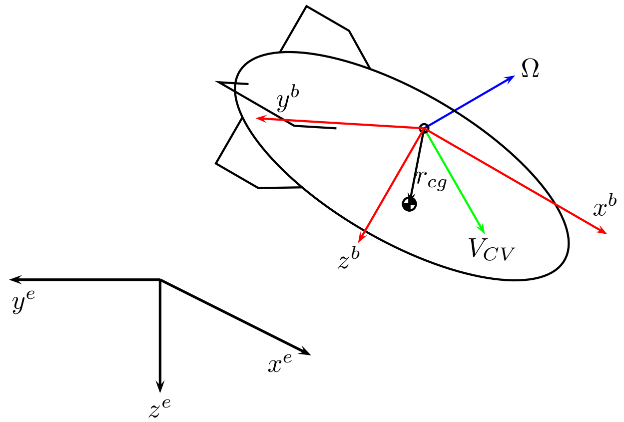

Figure 1. Earth axes and body axes.

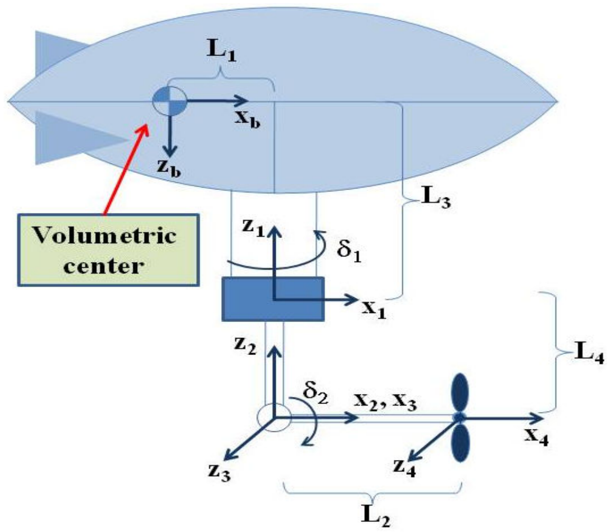

Figure 2. Propulsion system.

is given as(Thor, 1994)

$$T_A = \frac{1}{2}\left(X_{\dot{u}}u^2 + Y_{\dot{v}}v^2 + Z_{\dot{w}}w^2 + K_{\dot{p}}p^2 + M_{\dot{q}}q^2 + N_{\dot{r}}r^2\right)$$

then, the added mass and inertia forces are

$$\begin{aligned} F_{AM}^b &= M_A\dot{V}_{CV}^b + C_{12}\Omega \\ M_{IM}^b &= I_A\dot{\Omega} + C_{12}V_{CV}^b + C_{22}\Omega \end{aligned}$$

where

$$M_A = \mathrm{diag}\{X_{\dot{u}}, Y_{\dot{v}}, Z_{\dot{w}}\},\ I_A = \mathrm{diag}\{K_{\dot{p}}, M_{\dot{q}}, N_{\dot{r}}\}$$

$$C_{12} = \begin{bmatrix} 0 & Z_{\dot{w}}w & -Y_{\dot{v}}v \\ -Z_{\dot{w}}w & 0 & X_{\dot{u}}u \\ Y_{\dot{v}}v & -X_{\dot{u}}u & 0 \end{bmatrix}$$

$$C_{22} = \begin{bmatrix} 0 & N_{\dot{r}}r & -M_{\dot{q}}q \\ -N_{\dot{r}}r & 0 & K_{\dot{p}}p \\ M_{\dot{q}}q & -K_{\dot{p}}p & 0 \end{bmatrix}$$

The restoring forces are given by

$$F_R^b = (W - B)r_3,\ M_R^b = (W - B)r_{cg} \times r_3$$

with r_3 the third column of the rotation matrix that describes the orientation of the body frame relative to the inertial frame, that is,

$$R = \begin{bmatrix} c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\ c_\psi s_\theta s_\phi - s_\psi c_\phi & s_\psi s_\theta s_\phi + c_\psi c_\phi & c_\theta s_\phi \\ c_\psi s_\theta c_\phi + s_\psi s_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi & c_\theta c_\phi \end{bmatrix}$$

The airship is driven by a vectored thrust whose configuration is described in Figure 2

We apply the Denavit Hartenberg procedure, whose parameters are

α	a	d	θ
π	ℓ_1	$-\ell_3$	$-\delta_1$
0	0	$-\ell_4$	0
$\frac{\pi}{2}$	0	0	δ_2
0	ℓ_2	0	0

Table 1. Denavit Hartenberg parameters

then, we have that the transformation matrix from the coordinate system $x^4y^4z^4$ to $x^by^bz^b$ is given by

$$T_4^b = \begin{bmatrix} c_{\delta_1}c_{\delta_2} & -c_{\delta_1}s_{\delta_2} & -s_{\delta_1} & \ell_1 + \ell_2 c_{\delta_1}c_{\delta_2} \\ c_{\delta_2}s_{\delta_1} & -s_{\delta_1}s_{\delta_2} & c_{\delta_1} & \ell_2 c_{\delta_2}s_{\delta_1} \\ -s_{\delta_2} & -c_{\delta_2} & 0 & \ell_3 + \ell_4 - \ell_2 s_{\delta_2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In terms of the $x^4y^4z^4$ coordinates the propeller thrust is

$$T_P = \begin{bmatrix} T & 0 & 0 \end{bmatrix}^\top$$

moreover, the propeller position with respect to the body axis is given by

$$r_P = \begin{bmatrix} \ell_1 + \ell_2 c_{\delta_1}c_{\delta_2} & -\ell_2 c_{\delta_2}s_{\delta_1} & \ell_3 + \ell_4 - \ell_2 s_{\delta_2} \end{bmatrix}^\top$$

finally,

$$F_P^b = \begin{bmatrix} Tc_{\delta_1}c_{\delta_2} \\ Tc_{\delta_2}s_{\delta_1} \\ -Ts_{\delta_2} \end{bmatrix},\ M_P^b = \begin{bmatrix} -(\ell_3 + \ell_4)Tc_{\delta_2}s_{\delta_1} \\ (\ell_3 + \ell_4)Tc_{\delta_1}c_{\delta_2} + \ell_1 Ts_{\delta_2} \\ \ell_1 Tc_{\delta_2}s_{\delta_1} \end{bmatrix}$$

The airship lateral dynamics is described by the following equations

$$\begin{aligned} \dot{X}_L &= \begin{bmatrix} R_\psi & 0 \\ 0 & 1 \end{bmatrix} V_L \\ \mathcal{M}_L\dot{V}_L &= \Big[\mathcal{J}_L(V_L) - \mathcal{D}_L\Big]V_L + \mathcal{B}_L u_L + F_L \end{aligned} \qquad (2)$$

where $X_L = \begin{bmatrix} x & y & \psi \end{bmatrix}^\top$, $V_L = \begin{bmatrix} u & v & r \end{bmatrix}^\top$, $u_L = \begin{bmatrix} Tc_{\delta_1}c_{\delta_2} & Tc_{\delta_1}s_{\delta_2} \end{bmatrix}$ and

$$\mathcal{M}_L = \begin{bmatrix} m_x & 0 & 0 \\ 0 & m_y & mx_{cg} \\ 0 & mx_{cg} & I_z \end{bmatrix}, \ \mathcal{D}_L = \begin{bmatrix} X_u & 0 & 0 \\ 0 & Y_u & 0 \\ 0 & 0 & Z_u \end{bmatrix}$$

$$\mathcal{J}_L(V_L) = \begin{bmatrix} 0 & mr & -Y_{\dot{v}}v + mx_{cg}r \\ -mr & 0 & X_{\dot{u}}u \\ Y_{\dot{v}}v - mx_{cg}r & -X_{\dot{u}}u & 0 \end{bmatrix}$$

$$\mathcal{B}_L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \ell_1 \end{bmatrix}^\top, \ R_\psi = \begin{bmatrix} c_\psi & -s_\psi \\ s_\psi & c_\psi \end{bmatrix}$$

with $m_x = m - X_{\dot{u}}$, $m_y = m - Y_{\dot{v}}$, $I_z = I_{zz} - N_{\dot{r}}$ and F_L the external forces and moments.

III. Nonlinear controller

Consider the following general model of physical systems,

$$\begin{aligned} \mathcal{A}\dot{x} &= \mathcal{J}(x,u) - \mathcal{R}(x,u)x + \mathcal{B}(x)u + \mathcal{E}(t) \\ y &= \mathcal{B}^\top x \end{aligned} \tag{3}$$

where x is an n dimensional vector, $\mathcal{A}$ is a constant symmetric, positive define matrix, $\mathcal{J}(x,u)$ is a skew symmetric matrix, $\mathcal{R}(x,u)$ is a symmetric positive definite matrix and $\mathcal{E}(t)$ is an n-dimensional smooth vector function of t or sometimes, a constant vector, y is an m dimensional vector. Moreover we assume that,

$$\begin{aligned} \mathcal{J}(x,u) &= \mathcal{J}_0 + \textstyle\sum_{j=1}^m \mathcal{J}_j^u u_j + \sum_{k=1}^n \mathcal{J}_k^x x_k \\ \mathcal{R}(x,u) &= \mathcal{R}_0 + \textstyle\sum_{j=1}^m \mathcal{R}_j^u u_j + \sum_{k=1}^n \mathcal{R}_k^x x_k \\ \mathcal{B}(x) &= \mathcal{B}_0 + \textstyle\sum_{k=1}^n \mathcal{B}_k u_k \end{aligned} \tag{4}$$

Consider now that

A1 Given a feasible smooth bounded reference trajectory $x^*(t) \in \mathbb{R}^n$ there exist a smooth open loop control input $u^*(t) \in \mathbb{R}^m$, such that for all trajectories starting at $x(t_0) = x^*(t_0)$, the tracking error $e(t) = x(t) - x^*(t)$ is identically zero for all $t \geq t_0$.

A2 For any constant positive definite symmetric matrix K the following relation is uniformly satisfied,

$$\mathcal{R}^*(x,u,t) + \mathcal{B}^*(x,t)K\mathcal{B}^*(x,t)^\top > 0$$

Theorem 1: Consider the system (3)-(4) in closed loop with the controller,

$$u = u^*(t) - K\mathcal{B}^*(x,t)e. \tag{5}$$

Then, under assumption A1 and A2, the tracking error $e(t)$ is globally asymptotically stabilized to zero.

Proof: Let us define $e_u = u - u^*(t)$ and the following,

$$\begin{aligned} \mathcal{M}^*(t) &= [(\mathcal{J}_1^x - \mathcal{R}_1^x)x^* \cdots (\mathcal{J}_n^x - \mathcal{R}_n^x)x^*] \\ \mathcal{L}^*(t) &= [B_1 x^* \cdots B_n x^*] \\ \mathcal{Q}^*(t) &= [(\mathcal{J}_1^u - \mathcal{R}_1^u)x^* \cdots (\mathcal{J}_m^u - \mathcal{R}_m^u)x^*] \end{aligned} \tag{6}$$

where $x^* = x^*(t)$. Straightforward computations show that the error dynamics reads as,

$$\begin{aligned} \mathcal{A}\dot{e} &= \mathcal{J}^*(x,u,t) - \mathcal{R}^*(x,u,t)e + \mathcal{B}^*(x,t)e_u \\ y_e &= \mathcal{B}^*(x,t)^\top e \end{aligned} \tag{7}$$

where

$$\begin{aligned} \mathcal{J}^*(x,u,t) &= \mathcal{J}(x,u) + \tfrac{1}{2}\left[\mathcal{P}^*(t) - \mathcal{P}^*(t)^\top\right] \\ \mathcal{R}^*(x,u,t) &= \mathcal{R}(x,u) - \tfrac{1}{2}\left[\mathcal{P}^*(t) + \mathcal{P}^*(t)^\top\right] \\ \mathcal{B}^*(x,t) &= \mathcal{B}(x) + \mathcal{Q}^*(t) \end{aligned} \tag{8}$$

with

$$\mathcal{P}(t) = \mathcal{M}^*(t) + \mathcal{L}^*(t).$$

We refer to (7) as the exact open loop error dynamics. Take now the following Lyapunov function candidate,

$$V = \tfrac{1}{2} e^\top \mathcal{A} e \tag{9}$$

whose time derivative is given by.

$$\dot{V} = -e^\top \mathcal{R}^*(x,u,t)e + e^\top \mathcal{B}^*(x,t)e_u$$

Introducing (5) into the above equation, we have,

$$\dot{V} = -e^\top \left[\mathcal{R}^*(x,u,t) + \mathcal{B}^*(x,t)K\mathcal{B}^*(x,t)^\top\right] e,$$

By A2 the proof is completed. ■

Now, we are in position to present the main contribution of this paper, that is to say a nonlinear controller yielding local asymptotic stability of the trajectory tracking error for the thrust vectored airship.

Proposition 1: Assume $F_L = 0$ and let x_d and y_d be the desired airship trajectories with bounded time derivatives. Consider the lateral airship dynamics (2) in closed–loop with the dynamic state feedback control

$$u_L = u_{Ld} - K\mathcal{B}_L^\top e_{V_L} \tag{10}$$

where $K = \text{diag}\{k_1, k_2\}$,

$$e_{V_L} = V_L - V_{Ld} = \begin{bmatrix} e_u \\ e_v \\ e_r \end{bmatrix} = \begin{bmatrix} u - u_d \\ v - v_d \\ r - r_d \end{bmatrix}$$

$$\begin{bmatrix} u_d \\ v_d \end{bmatrix} = -K_X \tanh(n) + R_\psi^\top \dot{X}_d$$

$K_X = \text{diag}\{k_x, k_y\}$and

$$n = R_\psi^\top \begin{bmatrix} e_x \\ e_y \end{bmatrix}, \ X_d = \begin{bmatrix} x_d \\ y_d \end{bmatrix}$$

Moreover, $e_x = x - x_d$, $e_y = y - y_d$, r_d the solution of the differential equation

$$\mathcal{B}^\perp \left\{\mathcal{M}_L \dot{V}_{Ld} - \Big[\mathcal{J}_L(V_{Ld}) - \mathcal{D}_L\Big] V_{Ld}\right\} = 0 \tag{11}$$

and

$$u_{Ld} = \left[\mathcal{B}^\top\mathcal{B}\right]^{-1} \mathcal{B}^\top \left\{\mathcal{M}_L \dot{V}_{Ld} + \Big[\mathcal{J}_L(V_{Ld}) - \mathcal{D}_L\Big] V_{Ld}\right\}$$

Assume that in the subspace $\mathcal{S} \subset \mathbb{R}^7$ there exist a bounded solution for (11). Then, there exist k_1, k_2, k_x and k_y positive constants such that

$$\lim_{t\to\infty} e_x = 0, \quad \lim_{t\to\infty} e_y = 0 \tag{12}$$

holds in $\mathcal{S}$.

Proof: The selection of u_d and v_d is based on the fact that if the system follows these velocity references automatically it follows the desired path over the $(x-y)$ plane. From the first equation of (2) we can see that,

$$\dot{X} = R_\psi V \tag{13}$$

where $X = \begin{bmatrix} x & y \end{bmatrix}^\top$ and $V = \begin{bmatrix} u & v \end{bmatrix}^\top$. In terms of n equation (13) reads as

$$\dot{n} = V - R_\psi^\top \dot{X}_d + J(r)n \tag{14}$$

with

$$J(r) = \begin{bmatrix} 0 & r \\ -r & 0 \end{bmatrix}$$

In (14) we consider V as a virtual control and we design it in such a way that the system tracks a desired path. A possible virtual control choice is given by

$$V = -K_X \tanh(n) + R_\psi^\top \dot{X}_d \tag{15}$$

Consider now the kinematic equation (14) in closed loop with the virtual control (15),

$$\dot{n} = -K_X \tanh(n) + J(r)n \tag{16}$$

With the choice of the Lyapunov function $V_n = 1/2 n^\top n$ whose time derivative along (16) is

$$\dot{V}_n = -n^\top K_X \tanh(n)$$

we conclude that the velocity defined in (15) solves the trajectory tracking problem, at the kinematic level, as a result this velocity is selected as the desired velocity for the airship dynamic model.

Note that the closed–loop dynamics (2)–(10) expressed in terms of the n, ψ, r_d and e_{V_L} coordinates reads as,

$$\begin{aligned}
\dot{n} &= -K_X \tanh(n) + J(e_r + r_d)n + \bar{e}_{V_L} \\
\dot{\psi} &= e_r + r_d \\
\mathcal{M}_L \dot{e}_{V_L} &= \mathcal{J}_L^* e_{V_L} - \left[\mathcal{D}_L^* + \mathcal{B}_L K \mathcal{B}_L^\top\right] e_{V_L} \\
a_0 \dot{r}_d &= a_1(n,\psi,t) r_d + a_2(e_{V_L}, n, t)
\end{aligned}$$

where $\bar{e}_{V_L} = \begin{bmatrix} e_u & e_v \end{bmatrix}^\top$, and following the exact tracking error dynamics passive output feedback technique,

$$\begin{aligned}
\mathcal{J}_L^* &= \mathcal{J}_L(e_{V_L}, t) + \frac{1}{2}\left[\mathcal{P}(n,\psi,r_d,t) - \mathcal{P}(n,\psi,r_d,t)^\top\right] \\
\mathcal{D}_L^* &= \mathcal{D}_L - \frac{1}{2}\left[\mathcal{P}(n,\psi,r_d,t) + \mathcal{P}(n,\psi,r_d,t)^\top\right]
\end{aligned}$$

with

$$\mathcal{P}(n,\psi,r_d,t) = \begin{bmatrix} J_1^u V_{Ld} & J_2^v V_{Ld} & J_3^r V_{Ld} \end{bmatrix}$$

$$J_1^u = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & X_{\dot{u}} \\ 0 & -X_{\dot{u}} & 0 \end{bmatrix}, \; J_2^v = \begin{bmatrix} 0 & 0 & -Y_{\dot{v}} \\ 0 & 0 & 0 \\ Y_{\dot{v}} & 0 & 0 \end{bmatrix}$$

$$J_3^r = \begin{bmatrix} 0 & m & mx_{cg} \\ -m & 0 & 0 \\ -mx_{cg} & 0 & 0 \end{bmatrix}$$

moreover,

$$\begin{aligned}
a_0 &= I_z - mx_{cg}\ell_1 \\
a_1 &= (m_x \ell_1 - mx_{cg})\, u_d - N_r \\
&\quad + (m_y \ell_1 - mx_{cg}) \left[\frac{\partial v_d}{\partial \psi} - \frac{\partial v_d}{\partial n_2} n_1\right] \\
a_2 &= (m_y \ell_1 - mx_{cg}) \Bigg[\frac{\partial v_d}{\partial n_2}(e_v - k_y \tanh(n_2) - e_r n_1) \\
&\quad + \frac{\partial v_d}{\partial t} + \frac{\partial v_d}{\partial \psi} e_r\Bigg] \\
&\quad + \left[\ell_1 Y_v - (X_{\dot{u}} + Y_{\dot{v}}) u_d\right] v_d
\end{aligned}$$

$g = \begin{bmatrix} 0 & 1 \end{bmatrix}^\top$.

By defining the Lyapunov function

$$V_{e_{V_L}} = \frac{1}{2} e_{V_L}^\top \mathcal{M}_L e_{V_L}$$

and computing its time derivative we obtain

$$\dot{V}_{e_{V_L}} = -e_{V_L}^\top \left[\mathcal{D}_L^* + \mathcal{B}_L K \mathcal{B}_L^\top\right] e_{V_L}$$

where

$$\mathcal{D}_L^* + \mathcal{B}_L K \mathcal{B}_L^\top = \mathcal{D}_{TL} = \begin{bmatrix} A_1 & B_1 \\ B_1^\top & C_1 \end{bmatrix}$$

with

$$A_1 = \begin{bmatrix} k_1 + X_u & \frac{1}{2}(Y_{\dot{v}} - X_{\dot{u}}) r_d \\ \frac{1}{2}(Y_{\dot{v}} - X_{\dot{u}}) r_d & k_1 + Y_v \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -\frac{1}{2}(m_x v_d - x_{cg} r_d) \\ \frac{1}{2} m_y u_d + k_2 \ell_1 \end{bmatrix}$$

$$C_1 = N_r + \ell_1^2 k_2 + mx_{cg} u_d$$

In order to make $\dot{V}_{e_{V_L}}$ negative its should be possible to select k_1 and k_2 in such a way that $\mathcal{D}_{TL}$ is positive definite. Note that the first minor of $\mathcal{D}_{TL}$ is trivially positive, the second minor will be positive selecting k_1 and k_2 large enough to satisfy

$$(k_1 + X_u)(k_2 + Y_v) - \frac{1}{4}(Y_{\dot{v}} - X_{\dot{u}})\,\delta_r^2 > 0$$

with $r_d \le \delta_r$. The third minor of $\mathcal{D}_{TL}$ is given by

$$\det(\mathcal{D}_{TL}) = \det(A_1)\left[C_1 - B_1^\top A_1^{-1} B_1\right]$$

thus, $\det(\mathcal{D}_{TL})$ will be positive if and only if $C_1 - B_1^\top A_1^{-1} B_1 > 0$. Notice that for a desired trajectory with bounded time derivatives we do have

$$\begin{aligned}
|u_d| &\le k_x + |\dot{x}_d| + |\dot{y}_d| = k_x + \kappa_T \\
|v_d| &\le k_y + |\dot{x}_d| + |\dot{y}_d| = k_y + \kappa_T
\end{aligned}$$

then, there exist k_2 such that $C_1 > 0$. Straightforward algebraic manipulations show that

$$B_1^\top A_1^{-1} B_1 = \frac{b_0 r_d^2 + b_1 r_d + b_2}{-(Y_{\dot{v}} - X_{\dot{u}})^2 r_d^2 + 4(X_u + k_1)(Y_v + k_2)}$$

with

$$\begin{aligned} b_0 &= m x_{cg} \left[x_{cg}(Y_v + k_2) + (m_y u_d + 2\ell_1 k_2)(Y_{\dot{v}} - X_{\dot{u}}) \right] \\ b_1 &= m_x v_d \left[2 m x_{cg}(Y_v + k_2) \right. \\ &\quad \left. -(m_y u_d + 2\ell_1 k_2)(X_{\dot{u}} - Y_{\dot{v}}) \right] \\ b_2 &= (k_1 + X_u)(m_y^2 u_d^2 + 4 m_y u_d \ell_1 + 4\ell_1^2 k_2^2) \\ &\quad + m_x^2 v_d^2 (k_2 + Y_v) \end{aligned}$$

Due to the fact that r_d, u_d and v_d are bounded we have

$$|B_1^\top A_1^{-1} B_1| \leq \frac{\kappa_1 k_2 + \kappa_2 + (k_1 + X_u) 4\ell_1^2 k_2^2}{4(X_u + k_1)(Y_v + k_2) - \kappa_3}$$

where κ_i, $i = 1, 2, 3$ positive constants. Note that the upper bounds on u_d and v_d can be arbitrarily small, consequently we have $|B_1^\top A_1^{-1} B_1| \leq \kappa_5 + \kappa_6 k_2$ for large enough values of k_1 and k_2, with κ_5 and κ_6 arbitrarily small positive constants. Finally, the third minor of $\mathcal{D}_{TL}$ will be positive definite provided

$$N_r + \ell_1^2 k_2 + m x_{cg} u_d - \kappa_5 - \kappa_6 k_2 > 0$$

as u_d, κ_5 and κ_6 are arbitrarily small for large values of k_1 and k_2 and small values of k_x, k_y, $|\dot{x}_d|$ and $|\dot{y}_d|$ the inequality is satisfied. As a consequence e_{V_L} is bounded and converges to zero.

Consider now the Lyapunov function $V_n = \frac{1}{2} n^\top n$ whose time derivative is given by

$$\dot{V}_n = -n^\top K_X \tanh(n) + n^\top \bar{e}_{V_L}$$

as $\bar{e}_{V_L}$ is a vanishing disturbance (12) is concluded.

In all previous developments the corner stone is the assumption on the boundedness on r_d. To show that $\mathcal{S}$ is not an empty set, we observe that

$$a_0 \dot{r}_d = a_1 r_d + \kappa_4 e_v + \kappa_5 e_r + \kappa_6 n_1 \qquad (17)$$

where κ_i, $i = 4, 5, 6$ positive constants and we have used the fact that

$$\left| \frac{\partial v_d}{\partial \psi} \right| \leq |\dot{x}_d| + |\dot{y}_d|, \quad \left| \frac{\partial v_d}{\partial t} \right| \leq |\ddot{x}_d| + |\ddot{y}_d|, \quad \left| \frac{\partial v_d}{\partial n_2} \right| \leq k_y$$

Note that the solutions of equation (17) are bounded around $e_v = 0$, $e_r = 0$ and $n_1 = 0$ provided $a_0 > 0$, $a_1 < 0$. The proposed controller is not able to modify the sign of a_0 and the only way to satisfy $a_1 < 0$ is to choose k_x, $|\dot{x}_d|$ and $|\dot{y}_d|$ sufficiently small as N_r is a positive constant. ■

Remark 1: Note that a drawback of the controller in Proposition 1 depends on the airship parameters, mainly N_r. In order to address this problem the controller can be modified as

$$u_{L_d} = -K_X \tanh(n) + R_\psi^\top \dot{X}_d + G_r r_d$$

where $G_r = \begin{bmatrix} 0 & -k_r \end{bmatrix}^\top$ with k_r a positive constant. In this case, the control objective (12) can be achieved with $N_r = 0$. Unfortunately, the proof of closed–loop stability gets even more involved.

IV. Numerical simulations

We performed a few numerical simulations to illustrate the results of Proposition 1 and Remark . The airship parameter values are $m = 2.36$, $X_{\dot{u}} = -0.2879$, $Y_{\dot{v}} = -1.89508$, $N_{\dot{r}} = -0.0089$, $I_{zz} = 2.13$, $x_{cg} = 0.01$, $X_u = 1$, $Y_v = 1$, $N_r = 1$ and $\ell_1 = 0.3$. The desired trajectory is defined as $x_d = 10 \sin\left(\frac{\pi}{100} t\right)$, $y_d = 10 \cos\left(\frac{\pi}{100} t\right)$. Finally, the controller parameters are $k_x = 0.25$, $k_y = 0.25$, $k_r = -0.45$, $k_1 = 1$ and $k_2 = 1$. In order to test the robustness of the proposed controller with respect to unmodeled dynamics we consider

$$F_L = \frac{1}{2} \rho \sqrt{u^2 + v^2} \begin{bmatrix} C_D V_o^{2/3} & C_Y V_o^{2/3} & C_N V_o \end{bmatrix}^\top$$

with $C_D = 0.6$, $C_Y = 0.2$, $C_N = 0.2$ the aerodynamic coefficients, $V_o = 3.39$ the airship volume and $\rho = 1.18$ the air density.

Figure 8 and figure 9 show the values of our controls through the time, but this information does not really help us to understand the system behavior because the actual physical controls are the force T and the angle δ_2 mentioned in 2 (the angle δ_1 is not taken into account because we are analyzing only the lateral dynamics).

Figure 6 and 7 shows the time history of the angle control and the force control with and without aerodynamic forces.

Figure 5 shows the time history of the airship planar position with and without aerodynamic forces. As can be observed closed–loop stability is preserved in the presence of unmodeled dynamics.

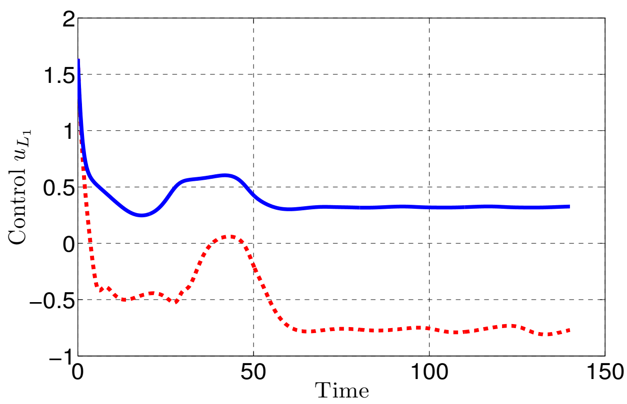

Figure 3. Control u_{L1}. Aerodynamic forces equal zero (continuos line), non zero aerodynamic forces (dashed line).

V. Conclusions

The problem of trajectory tracking for the planar dynamics of a thrust vectored airship has been addressed and solved by means of a nonlinear controller. Numerical simulations have been proposed to illustrate the closed–loop dynamics properties.

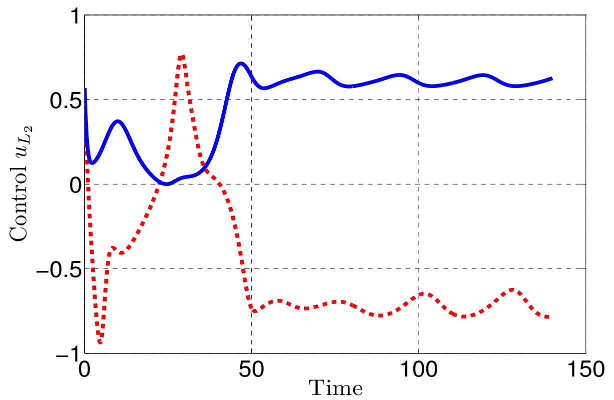

Figure 4. Control u_{L2}. Aerodynamic forces equal zero (continuos line), non zero aerodynamic forces (dashed line).

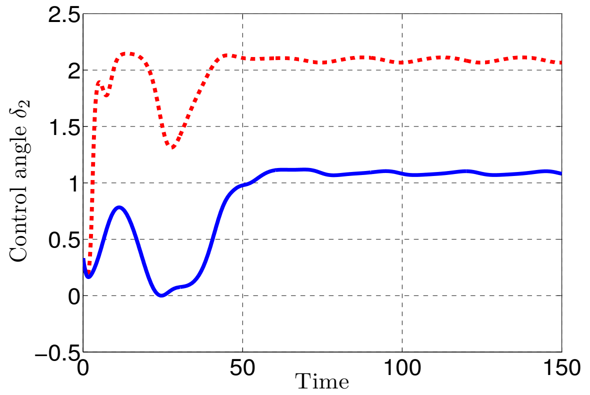

Figure 5. Control angle δ_2. Aerodynamic forces equal zero (continuos line), non zero aerodynamic forces (dashed line).

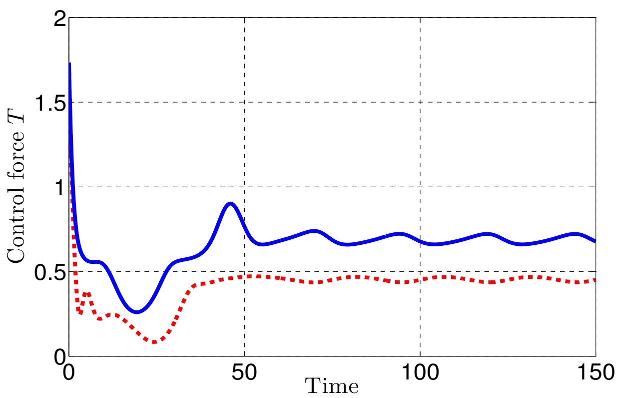

Figure 6. Control force T. Aerodynamic forces equal zero (continuos line), non zero aerodynamic forces (dashed line).

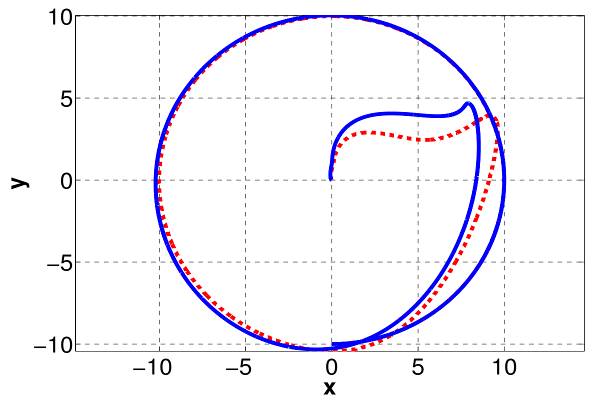

Figure 7. Airship trajectory. Aerodynamic forces equal zero (continuos line), non zero aerodynamic forces (dashed line).

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