Observer based Trajectory-Tracking Control of an Omnidirectional Mobile Robot

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Abstract— **In this paper, the trajectory-tracking control problem without velocity measurement of an omnidirectional mobile robot (also known as a type (3,0)) is addressed and solved. It is shown that the conjunction of a passivity based controller and an Immersion and Invariance velocity observer produces an asymptotically stable closed–loop dynamics. Different from the classical approach that consider only the kinematic model, the proposed partial state feedback controller is designed taking into account the dynamic model. Numerical simulations are carried out to show the overall performance of the proposed scheme.**

Keywords: Mobile robots, Dynamic models, Observers, Tracking applications.

I. INTRODUCTION

In modern robotic systems, omnidirectional mobile robots have vast advantages compared to conventional robot designs in terms of mobility in congested environments. Such a capability gives to this class of mobile robots the potential to solve a number of challenges in industry and society. However, to fully exploit this mobility advantage it is necessary to implement an appropriate trajectory-tracking strategy.

The traditional control problems of trajectory-tracking and regulation have been extensively studied in the field of mobile robotics. In particular, the differential and the omnidirectional mobile robots, also known, respectively, as the (2,0) and the (3,0) robots (see (Bétourné and Campion, 1996), (Kalmár-nagy et al., 2004)), have attracted the interest of many control researchers.

It is a common practice in mobile robotics to address control problems taking into account only a kinematic representation. This approach assume that there exists a high level controller that ensures perfect velocity tracking. Unfortunately, the velocity tracker design is far from been obvious.

The regulation and trajectory-tracking problems of the omnidirectional mobile robot (3,0), has also received sustained attention. Following the approach that considers only the kinematic model, in (Liu et al., 2003) it is designed a nonlinear controller based on a Trajectory Linearization strategy and in (Velasco-Villa, Alvarez-Aguirre and Rivera-Zago, 2007), the remote control of the (3,0) mobile robot is developed based on a discrete-time strategy assuming a time-lag model of the robot. In (Velasco-Villa, del Muro-Cuellar and Alvarez-Aguirre, 2007) the trajectory-tracking problem is solved by means of an estimation strategy that predicts the future values of the system based on the exact nonlinear discrete-time model of the robot.

A reduced number of contributions have been focused on the dynamic model of the (3,0) robot. For example, in (Carter et al., 2001), it is described the mechanical design of the robot and based on its dynamic model it is proposed a PID control for each robot wheel. Authors in (Bétourné and Campion, 1996) consider an Euler-Lagrange model formulation and present an output feedback controller that solves the trajectory-tracking problem. In the same manner, in (Williams et al., 2002) the dynamic model of the mobile robot is considered to study the slipping effects between the wheels of the vehicle and the working surface. In (Vázquez and Velasco-Villa, 2008) the trajectory tracking problem is addressed and solved by considering a modification of the well known Computed-Torque strategy. Finally, in (Kalmár-nagy et al., 2004) the time-optimization problem of a desired trajectory is considered for a mobile robot subject to admissible input limits in order to obtain feedback laws that are based on the kinematic and dynamic models.

In this paper, we address and solve the trajectory-tracking problem of an omnidirectional mobile robot without velocity measurement. We show that the combination of a full information control strategy that renders the closed–loop dynamics globally asymptotically stable and a globally exponentially convergent velocity observer produces a partial state feedback controller that preserves asymptotic stability.

This paper is organized as follows: Section II presents the dynamic model of the considered mobile robot. Immediately, in Section III we present one of the main contributions of this paper, the velocity observer. Section IV, is devoted to the full information controller whereas in Section V we propose and analyse the partial state feedback control strategy. The evaluation through numerical simulations of the partial state feedback controller is performed in Section VI. Finally, Section VII presents some conclusions.

II. OMNIDIRECTIONAL MOBILE ROBOT

A top view of the configuration of a (3,0) mobile robot is depicted in Figure 1. The mobile reference frame $X_m - Y_m$ is located at the center of mass of the vehicle with the X_m axis aligned with respect to the wheel 3. Wheels 1 and 2 are

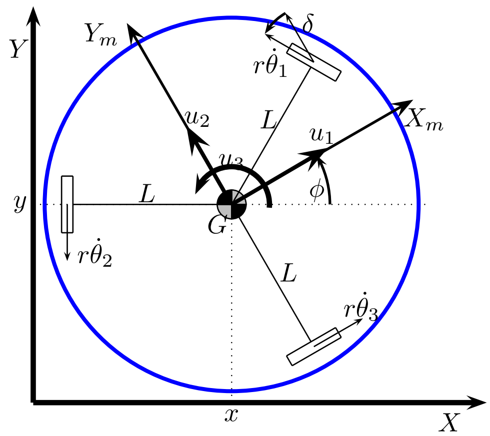

Figura 1. Omnidirectional Mobile Robot

placed symmetrically with an angle $\delta = 30°$ with respect to the Y_m axis. The fixed reference frame $X - Y$ provides the absolute localization of the vehicle on the workspace. The mobile robot is of the type (Canudas et al., 1996) $(\delta_m, \delta_s) = (3, 0)$, this is, it has three degrees of mobility and zero degrees of steerability allowing the displacements of the vehicle in all directions instantaneously.

A. Dynamic Model

A simple analysis of the velocity constrains (Campion et al., 1996) on Figure 1 produces,

$$J_1 R(\phi)\dot{q} - J_2\dot{\varphi} = 0 \tag{1}$$

with, $q = \begin{bmatrix} x & y & \phi \end{bmatrix}^\top$, and

$$J_1 = \begin{bmatrix} -\sin\delta & \cos\delta & L \\ -\sin\delta & -\cos\delta & L \\ 1 & 0 & L \end{bmatrix}, \; J_2 = \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix}$$

$$R(\phi) = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \; \varphi = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{bmatrix}$$

where φ_1, φ_2, φ_3 represent the angular displacements of wheels one, two and three, respectively; δ is the orientation of the i-wheel with respect to its longitudinal axis; L is the distance between the center of each wheel and the center of the vehicle and r is the radius of each wheel.

Following (Campion et al., 1996)-(Canudas et al., 1996), the kinetic energy of the robot is given by the wheel rotational energy plus the translational and rotational energy of the robot. Therefore, the Lagrangian of the system is obtained as,

$$\mathcal{L} = \frac{1}{2}\dot{q}^T R^T(\phi) M R(\phi)\dot{q} + \frac{1}{2}\sum_{i=1}^{3} \dot{\varphi_i}^T I_r \dot{\varphi_i}, \tag{2}$$

with $M = \mathrm{diag}\{M_p, M_p, I_p\}$ and $I_r = \mathrm{diag}\{I_\varphi, I_\varphi, I_\varphi\}$. M_p is the vehicle mass and I_p the moment of inertia about the Z axis of the vehicle, I_φ is the moment of inertia of each wheel about its rotation axis.

Considering that the kinematics restrictions (1) are satisfied for all t, from the Euler-Lagrange equations we obtain

$$D\ddot{q} + C(\dot{q})\dot{q} = B\tau, \tag{3}$$

where,

$$D = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_1 & 0 \\ 0 & 0 & d_3 \end{bmatrix}, \; C(\dot{q}) = a\begin{bmatrix} 0 & \dot{\phi} & 0 \\ -\dot{\phi} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$B = \frac{1}{r}\begin{bmatrix} -\sin(\delta+\phi) & -\sin(\delta-\phi) & \cos\phi \\ \cos(\delta+\phi) & -\cos(\delta-\phi) & \sin\phi \\ L & L & L \end{bmatrix},$$

with $d_1 = M_p + \frac{3I_r}{2r^2}$, $d_3 = I_p + \frac{3I_rL^2}{r^2}$ and $a = \frac{3I_r}{2r^2}$.

Note that the vector $C(\dot{q})\dot{q}$ does not possess a unique parameterization. For our developments, we consider the following parameterization,

$$\begin{aligned} C(\dot{q})\dot{q} &= a\begin{bmatrix} 0 & \dot{\phi} & 0 \\ -\dot{\phi} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\dot{q} \\ &= \frac{a}{2}\begin{bmatrix} 0 & \dot{\phi} & \dot{y} \\ -\dot{\phi} & 0 & -\dot{x} \\ -\dot{y} & \dot{x} & 0 \end{bmatrix}\dot{q} = C_a(\dot{q})\dot{q}. \end{aligned}$$

III. Velocity Observer

This Section contains one of the main contributions of this paper, a velocity observer. This observer is based on the Inmmersion and Invariance nonlinear design technique introduced in (Astolfi et al., 2008).

Consider the following general dynamic model of mechanical systems

$$\begin{aligned} \dot{x}_1 &= \Phi(x_1)x_2 \\ \dot{x}_2 &= B(x_1)u + \zeta(x,t), \end{aligned} \tag{4}$$

where x_1 is the angular or translational position, x_2 is the angular or translational velocity and u is the control force.

We assume that $\zeta(x,t)$ models all the rest of the mechanical system dynamics. Moreover, we assume that there exists α such that[1]

$$\zeta(x,t)^{(\alpha)} = 0. \tag{5}$$

Define now the estimation errors as:

$$\begin{aligned} z_1 &= x_2 - \hat{x}_2 - \beta_1(x_1) \\ z_2 &= \zeta - \rho_1 - \beta_2(x_1) \\ z_3 &= \dot{\zeta} - \rho_2 - \beta_3(x_1) \\ &\vdots \qquad \vdots \\ z_{\alpha+1} &= \zeta^{(\alpha-1)} - \rho_\alpha - \beta_{\alpha+1}(x_1). \end{aligned} \tag{6}$$

[1] In the following $x^{(\alpha)} = \frac{d^\alpha}{dt^\alpha}x$.

Straightforward computations show that,

$$\begin{aligned}\dot{z}_1 &= B(x_1)u + \zeta - \dot{\hat{x}}_2 - \tfrac{\partial \beta_1}{\partial x_1}\left[\Phi(x_1)x_2\right] \\ \dot{z}_2 &= \dot{\zeta} - \dot{\rho}_1 - \tfrac{\partial \beta_2}{\partial x_1}\left[\Phi(x_1)x_2\right] \\ \dot{z}_3 &= \ddot{\zeta} - \dot{\rho}_2 - \tfrac{\partial \beta_3}{\partial x_1}\left[\Phi(x_1)x_2\right] \\ &\vdots \\ \dot{z}_{\alpha+1} &= -\dot{\rho}_\alpha - \tfrac{\partial \beta_{\alpha+1}}{\partial x_1}\left[\Phi(x_1)x_2\right].\end{aligned}$$

In terms of the estimation errors the dynamics above reads as,

$$\begin{aligned}\dot{z}_1 &= B(x_1)u + z_2 + \rho_1 + \beta_2 - \dot{\hat{x}}_2 \\ &\quad -\tfrac{\partial \beta_1}{\partial x_1}\left[\Phi(x_1)\left(z_1 + \hat{x}_2 + \beta_1\right)\right] \\ \dot{z}_2 &= z_3 + \rho_2 + \beta_3 - \dot{\rho}_1 \\ &\quad -\tfrac{\partial \beta_2}{\partial x_1}\left[\Phi(x_1)\left(z_1 + \hat{x}_2 + \beta_1\right)\right] \\ \dot{z}_3 &= z_4 + \rho_3 + \beta_4 \\ &\quad -\dot{\rho}_2 - \tfrac{\partial \beta_3}{\partial x_1}\left[\Phi(x_1)\left(z_1 + \hat{x}_2 + \beta_1\right)\right] \\ &\vdots \\ \dot{z}_{\alpha+1} &= -\dot{\rho}_\alpha - \tfrac{\partial \beta_{\alpha+1}}{\partial x_1}\left[\Phi(x_1)\left(z_1 + \hat{x}_2 + \beta_1\right)\right].\end{aligned}$$

Let the following definition be made,

$$\begin{aligned}\dot{\hat{x}}_2 &= B(x_1)u + \rho_1 + \beta_2 - \tfrac{\partial \beta_1}{\partial x_1}\left[\Phi(x_1)\left(\hat{x}_2 + \beta_1\right)\right] \\ \dot{\rho}_1 &= \rho_2 + \beta_3 - \tfrac{\partial \beta_2}{\partial x_1}\left[\Phi(x_1)\left(\hat{x}_2 + \beta_1\right)\right] \\ \dot{\rho}_2 &= \rho_3 + \beta_4 - \tfrac{\partial \beta_3}{\partial x_1}\left[\Phi(x_1)\left(\hat{x}_2 + \beta_1\right)\right] \\ &\vdots \\ \dot{\rho}_\alpha &= -\tfrac{\partial \beta_{\alpha+1}}{\partial x_1}\left[\Phi(x_1)\left(\hat{x}_2 + \beta_1\right)\right].\end{aligned} \tag{7}$$

As a result of the previous definition the estimation error dynamics becomes,

$$\begin{aligned}\dot{z}_1 &= z_2 - \tfrac{\partial \beta_1}{\partial x_1}\left[\Phi(x_1)z_1\right] \\ \dot{z}_2 &= z_3 - \tfrac{\partial \beta_2}{\partial x_1}\left[\Phi(x_1)z_1\right] \\ \dot{z}_3 &= z_4 - \tfrac{\partial \beta_3}{\partial x_1}\left[\Phi(x_1)z_1\right] \\ &\vdots \\ \dot{z}_{\alpha+1} &= -\tfrac{\partial \beta_{\alpha+1}}{\partial x_1}\left[\Phi(x_1)z_1\right].\end{aligned} \tag{8}$$

Consider now,

$$\frac{\partial \beta_i}{\partial x_1} = C_i \Phi(x_1)^{-1},\ i = 1, \cdots \alpha + 1, \tag{9}$$

then, we have that,

$$\begin{aligned}\dot{z}_1 &= z_2 - C_1 z_1 \\ \dot{z}_2 &= z_3 - C_2 z_1 \\ \dot{z}_3 &= z_4 - C_3 z_1 \\ &\vdots \\ \dot{z}_{\alpha+1} &= -C_{\alpha+1} z_1\end{aligned} \tag{10}$$

which can be expressed as,

$$z_1^{(\alpha+1)} - C_1 z_1^{(\alpha)} - C_2 z_1^{(\alpha-1)} - \cdots - C_{\alpha+1} z_1 = 0. \tag{11}$$

We are now in position to present our velocity observer.

Proposition 1: Assume that there exist α and $\beta_i(x_1), i = 1, \cdots, \alpha+1$ such that (5) and (9) respectively hold. Then, there exists $C_i, i = 1, \cdots, \alpha+1$ such that (7) is a velocity observer for system (4) in the sense that,

$$\left[\hat{x}_2 + \beta_1(x_1)\right] \to x_2$$

exponentially.

Proof: Note that the observation error dynamics is described by the differential equation (11). This is a linear differential equation so that it is always possible to select $C_i, i = 1, \cdots, \alpha+1$ in such a way that z_1 exponentially converges to zero. ■

IV. Full state feedback controller for the (3,0) robot.

The full information controller is designed following ideas developed in the field of power electronics addressed as: Exact Tracking Error Dynamics Passive Output Feedback (ETEDPOF) (Sira-Ramírez, 2005), (Sira-Ramírez and Rodríguez-Cortés, 2008).

For, note that the dynamic model of the omnidirectional mobile robot (3) in closed loop with the controller

$$\tau = B^{-1}\left(-x_1 + u\right) \tag{12}$$

can be expressed in terms of the following model of physical systems,

$$\begin{aligned}\mathcal{A}\dot{x} &= \mathcal{J}(x,u)x - \mathcal{R}(x,u)x + \mathcal{B}(x)u + \mathcal{E}(t) \\ y &= \mathcal{B}^\top x,\end{aligned} \tag{13}$$

with $x = \left[\begin{array}{cc} x_1^\top & x_2^\top \end{array}\right]^\top$,

$$q = x_1 = \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \end{bmatrix}, \quad \dot{q} = x_2 = \begin{bmatrix} x_{21} \\ x_{22} \\ x_{23} \end{bmatrix}. \tag{14}$$

and

$$\mathcal{A} = \begin{bmatrix} I & 0 \\ 0 & D \end{bmatrix}, \quad \mathcal{J}(x,u) = \begin{bmatrix} 0 & I \\ -I & -C_a(x_2) \end{bmatrix},$$
$$\mathcal{R}(x,u) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathcal{B}(x) = \begin{bmatrix} 0 \\ I \end{bmatrix},$$

where I is the 3×3 identity matrix. As stated in (Sira-Ramírez, 2005) system (13) can be expressed in terms of error coordinates dynamics as follows,

$$\begin{aligned}\mathcal{A}\dot{e} &= \left[\mathcal{J}^*(x,u,t) - \mathcal{R}^*(x,u,t)\right]e + \mathcal{B}^*(x,t)e_u \\ y_e &= \mathcal{B}^*(x,t)^\top e\end{aligned} \tag{15}$$

where $e = x - x^*$, $e_u = u - u^*$ with x^* a feasible smooth bounded reference trajectory for which there exist a smooth open loop control input u^*, such that all trajectories starting at $x(0) = x^*(0)$, the tracking error e is identically zero for all $t \geq 0$. And, in particular for the closed–loop dynamics (3)-(12), one has,

$$\mathcal{J}^*(x,u,t) = \begin{bmatrix} 0 & I \\ -I & J_{22}^* \end{bmatrix},\ \mathcal{R}^*(x,u,t) = \begin{bmatrix} 0 & 0 \\ 0 & R_{22}^* \end{bmatrix},\quad \mathcal{B}^* = \begin{bmatrix} 0 & I \end{bmatrix},^\top \tag{16}$$

with

$$J_{22}^* = \begin{bmatrix} 0 & -ax_{23} - ax_{23}^* & -ax_{22} \\ ax_{23} + ax_{23}^* & 0 & ax_{21} \\ ax_{22} & -ax_{21} & 0 \end{bmatrix}$$

and

$$R_{22}^* = \begin{bmatrix} 0 & 0 & ax_{22}^* \\ 0 & 0 & -ax_{21}^* \\ ax_{22}^* & -ax_{21}^* & 0 \end{bmatrix}.$$

As a result of this, we have:

Proposition 2: Consider the dynamic model of the omnidirectional mobile robot (3) in closed loop with the controller

$$\tau = B^{-1}\left[-x_1 + u^* - K_1 e_1 - K_2 e_2\right] \tag{17}$$

where K_1 and K_2 are symmetric positive definite matrices. Assume that there exist smooth bounded reference trajectories x_1^*, $x_2(t)^*$ and a smooth open loop control input u^*. Then, for any $\gamma_3 > 0$, the closed–loop system (3)-(17) renders the equilibrium point $e_1 = 0$, $e_2 = 0$ globally asymptotically stable.

Proof: To show the convergence of the tracking error notice first that $(e_1, e_2) = (0, 0)$ is an equilibrium point of the closed–loop system (3)-(17). Consider, now a candidate Lyapunov function of the form,

$$V(e_1, e_2) = \frac{1}{2} e_2^T D e_2 + \frac{1}{2} e_1^T e_1 + \epsilon e_1^T D e_2 + \frac{1}{2}\epsilon e_1^T K_2 e_1. \tag{18}$$

It is not difficult to see that this function is positive definite for sufficiently small ϵ. Taking the time derivative of equation (18) along the closed–loop system (3)-(17) we obtain,

$$\dot{V} = -\epsilon e_1^T\left[I + K_1\right] e_1 - e_2^T\left[R_{22}^* + K_2 - \epsilon D\right] e_2 + \epsilon e_1^T\left[(J_{22}^* - R_{22}^*) - K_1\right] e_2. \tag{19}$$

Note now that R_{22}^* can be written as $R_{22}^* = aF(x_{2d})$ with,

$$F(x_{2d}) = \begin{bmatrix} 0 & 0 & x_{22d} \\ 0 & 0 & -x_{21d} \\ x_{22d} & -x_{21d} & 0 \end{bmatrix},$$

and also, $J_{22}^* - R_{22}^*$ can be rewritten as $J_{22}^* - R_{22}^* = C_a(e_2) + aG(x_{2d})$ with

$$G(x_{2d}) = \begin{bmatrix} 0 & -x_{23d} & -x_{22d} \\ x_{23d} & 0 & x_{21d} \\ 0 & 0 & 0 \end{bmatrix}.$$

. From the fact that,

$$\left\|F(x_{2d})\right\| \leq \left\|x_{2d}\right\| \text{ and } \left\|G(x_{2d})\right\| \leq \left\|x_{2d}\right\|,$$

lengthy but simple computations show that,

$$\begin{aligned}\dot{V} \leq -\Big\{&\left[\lambda_m(K_1) + \epsilon\right]\left\|e_1\right\|^2 + \big[\lambda_m(K_2) + \lambda_m(R_{22}^*) \\ &- \epsilon\lambda_M(D) - a\left\|x_{2d}\right\| - \epsilon a\left\|e_1\right\|\big]\left\|e_2\right\|^2 \\ &- \left[\epsilon\lambda_M(R_{22}^*) + \epsilon a\left\|x_{2d}\right\| + \lambda_M(k_1)\right]\left\|e_1\right\|\left\|e_2\right\|\Big\}.\end{aligned}$$

Notice now that the above relation can be rewritten as:

$$\dot{V} \leq -\left\|e\right\|^\top P\| \left\|e\right\|$$

with $e = \begin{bmatrix} e_1^\top & e_2^\top \end{bmatrix}^\top$, and

$$P = \begin{bmatrix} \left[\lambda_m(K_1) + \epsilon\right] & p_{12} \\ p_{12} & p_{22} \end{bmatrix},$$

where

$$\begin{aligned} p_{12} &= -\tfrac{1}{2}\left[\epsilon\lambda_M(R_{22}^*) + \epsilon a\left\|x_{2d}\right\| + \lambda_M(K_1)\right] \\ p_{22} &= \lambda_m(K_2) + \lambda_m(R_{22}^*) - \epsilon\lambda_M(D) - a\left\|x_{2d}\right\| \\ &\quad -\epsilon a\left\|e_1\right\|.\end{aligned}$$

Therefore, the closed-loop system will be stable if the conditions,

$$\begin{aligned} (i) &\quad \lambda_m(K_1) + \epsilon > 0 \\ (ii) &\quad \det\{P\} > 0 \end{aligned}$$

are satisfied. Condition (i) is trivially satisfied while condition (ii) can be written as a second order function of ϵ, that is,

$$-\gamma_1\epsilon^2 + \gamma_2\epsilon + \gamma_3 > 0 \tag{20}$$

where,

$$\begin{aligned} \gamma_1 &= \lambda_M(D) + a\left\|e_1\right\| + \frac{1}{4}\left[\lambda_M(R_{22}^*) + a\left\|x_{2d}\right\|\right]^2 \\ \gamma_2 &= \lambda_m(K_2) + \lambda_m(R_{22}) - a\left\|x_{2d}\right\| \\ &\quad - \lambda_m(K_1)\left[\lambda_M(D) + a\left\|e_1\right\|\right] \\ &\quad - \lambda_M(K_1)\left[\lambda_M(R_{22}^*) + a\left\|x_{2d}\right\|\right] \\ \gamma_3 &= \lambda_m(K_1)\left[\lambda_m(K_2) + \lambda_m(R_{22}) - a\left\|x_{2d}\right\|\right] \\ &\quad - \frac{1}{4}\lambda_M^2(K_1).\end{aligned}$$

It is clear now, from equation (20), that the system will be asymptotically stable for a sufficiently small ϵ. Notice that when $\epsilon \to 0$ the required stability condition is reduced to $\gamma_3 > 0$ that can be easily obtained by an adequate selection of the control gains together with a bounded desired velocity. This completes the proof. ■

V. PARTIAL STATE FEEDBACK CONTROLLER

In this section we present a partial state feedback controller for the same vehicle. This control strategy is obtained by combining the full information controller (17) with the velocity observer presented in Proposition 2.

Proposition 3: Let $\alpha = 3$. Consider the omnidirectional mobile robot dynamics (3) in closed–loop with the dynamic controller

$$\tau = B^{-1}\left[-x_1 + u^* - K_1 e_1 - K_2\left(\hat{x}_2 + C_1 x_1 - x_2^*\right)\right] \tag{21}$$

where

$$\begin{aligned}\dot{\hat{x}}_2 &= B\tau + \rho_1 + C_2 x_1 - C_1 \left[(\hat{x}_2 + C_1 x_1)\right] \\ \dot{\rho}_1 &= \rho_2 + C_3 x_1 - C_2 \left[(\hat{x}_2 + C_1 x_1)\right] \\ \dot{\rho}_2 &= -C_3 \left[(\hat{x}_2 + C_1 x_1)\right]\end{aligned} \quad (22)$$

with

$$C_i = \text{diag}\left\{c_{i1}, c_{i2}, c_{i3}\right\}, \ i = 1, 2, 3$$

Then, there exists positive definite matrices C_i, $i = 1, 2, 3$ such that all trajectories of the closed–loop system are bounded and are such that

$$\lim_{t\to\infty} x = x^*$$

Proof: Straight forward computations show that the mobile robot dynamics (3) can be expressed in the form of system (4) with $\Phi(x) = I$. Thus, it follows that (22) is a velocity observer for (3) in the sense of Proposition 1 with

$$\beta_i = C_i x_1, \ i = 1, 2, 3$$

therefore, the velocity observation error satisfies the following differential equation,

$$z_1^{(3)} + C_1 \ddot{z}_1 + C_2 \dot{z}_1 + C_3 z_1 = 0 \quad (23)$$

with

$$z_1 = x_2 - \hat{x}_2 - C_1 x_1. \quad (24)$$

Note now that, taking into account (24), the partial state feedback (21) can be written as follows

$$\tau = B^{-1}\left[-x_1 + u^* - K_1 e_1 - K_2 e_2\right] + B^{-1} K_2 z_1 \quad (25)$$

that is, it is obtained the full information controller (17) perturbed by a vanishing additive term. Consider now the Lyapunov function (18). Straight forward computations give

$$\dot{V} \leq -\left\|e\right\|^\top P\| \left\|e\right\| + \kappa \|e\| \|z_1\|$$

with κ a positive definite constant. Thus, by Theorem 4.7 of (Sepulchre et al., 1997) we conclude that the closed–loop dynamics (3)-(25) is globally asymptotically stable. Note that the term that interconnects the closed-loop dynamics (3)-(25) with the velocity observer dynamics (23) is bounded by a linear term in $z_1.e$. ∎

VI. Numerical simulations

We carried out numerical simulations to assess the performance of the controller given in Proposition 3. The values of the parameters correspond to a laboratory prototype built in our institution and they are $M_p = 9.58$Kg, $I_r = 0.52$Kgm^2, $L = 0.1877$m, $r = 0.03812$m and $\delta = 30^o$. The initial conditions of the mobile robot are $x_1(0) = \left[\begin{array}{ccc} 0, & 0, & 0.45 \end{array}\right]^\top$ and $x_2(0) = \left[\begin{array}{ccc} 0, & 0, & 0 \end{array}\right]^\top$. The controller parameters are summarized in Table I while the ones corresponding to the observer are $c_{i1} = 504$, $c_{i2} = 191$ and $c_{i3} = 24$ for $i = 1, 2, 3$.

It is desired to follow a circular trajectory or radius 0.5m centered at the origin with initial conditions $x_{1d}(0) = \left[\begin{array}{ccc} 0.5, & 0, & \frac{\pi}{2} \end{array}\right]^\top$.

Parameter	Value	Parameter	Value
k_{11}	200	k_{21}	200
k_{12}	200	k_{22}	200
k_{13}	100	k_{23}	100
r_1, r_2	200	r_3	30

TABLE I
FEEDBACK CONTROL LAW PARAMETERS

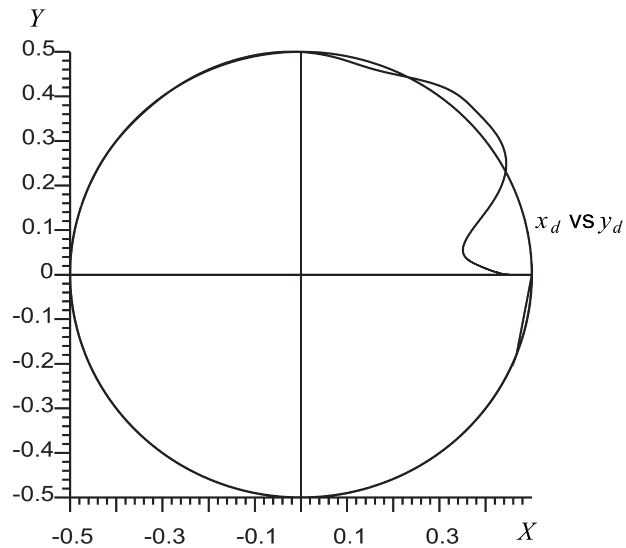

Figura 2. Evolution on the $X - Y$ plane.

On Figure 2 it is shown the evolution on the plane of the mobile robot when it is considered the proposed observer based control strategy. The evolution of the position errors are shown on Figure 3. The observer error convergence is depicted on Figure 4. The closed-loop torque input signals are shown on Figure 5.

VII. Conclusions

The trajectory-tracking problem for the omnidirectional mobile robot considering its dynamic model and without velocity measurement has been addressed and solved by means of a partial state time varying feedback based on a methodology that exploits the passivity properties of the exact tracking error dynamics and an globally exponentially convergent velocity observer. The asymptotic stability of the closed loop system interconnected with the velocity observer is formally proved. Numerical simulations are proposed to illustrate the properties of the plant-controller-observer dynamics.

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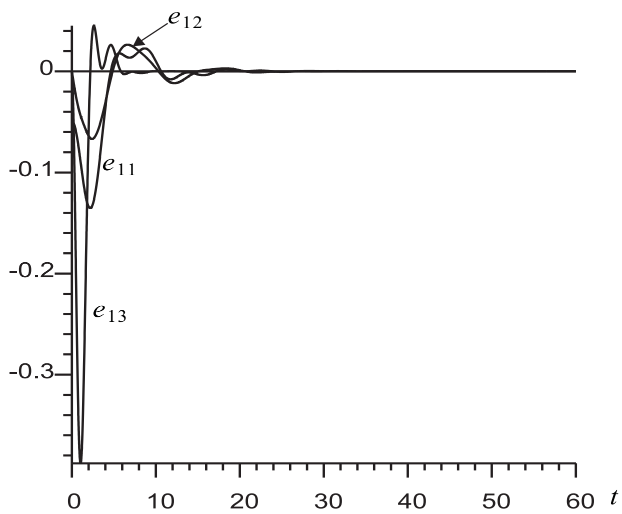

Figura 3. Time position errors evolution.

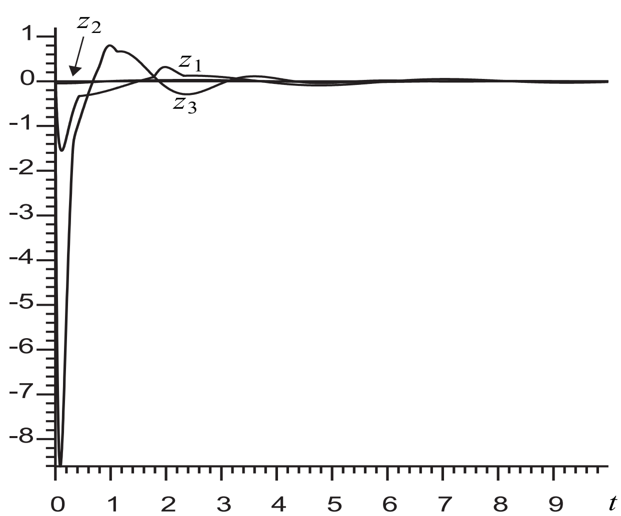

Figura 4. Observer errors convergence.

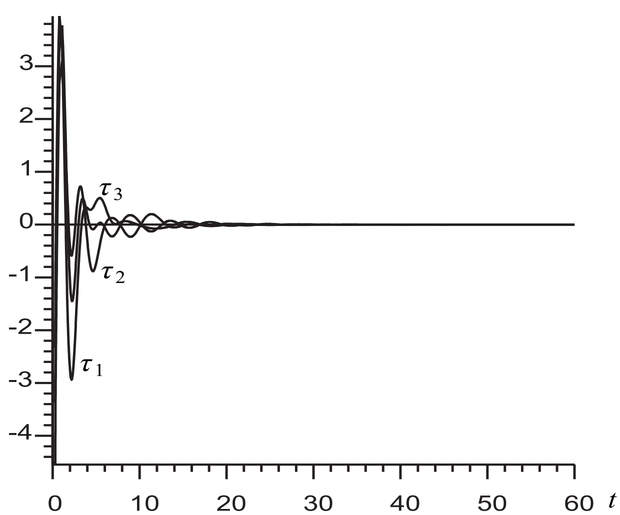

Figura 5. Closed-loop applied torques.

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