Output Robust Feedback Control with Exact Unmatched Uncertainties Compensation Based on HOSM Observation

(Invited Paper)

Alejandra Ferreira and Francisco Bejarano
Department of Control and Robotics,
Division of Electrical Engineering, Engineering Faculty,
National Autonomous University of Mexico (UNAM),
04510, Mexico, D.F.
Email: ferreira@astroscu.unam.mx, javbejarano@yahoo.com.mx

***Abstract*—This manuscript tackles the regulation problem of linear time invariant systems with unmatched perturbations. A hierarchical sliding mode observer is used allowing theoretically exact state and perturbation estimation. A compensation control approach based on the identified perturbation values is proposed ensuring exact regulation of the unmatched states. A simulation example shows the feasibility of this approach.**

I. INTRODUCTION

Motivation. Control under heavy uncertainties is one of the main problems of modern control theory. One of the most prospering control strategies insensitive w.r.t. uncertainties is the Sliding-mode control (SMC) (see, e.g., [1]). This robust technique is well known for its ability to withstand external disturbances and model uncertainties satisfying the matching condition. This condition is presented when the perturbation or parameters variations are implicit at the input channels, for example in the case of completely actuated systems.

The SMC design methodology involves two distinct stages: the design of a switching function which provides desirable system performance in the sliding mode and the design of the control law which will ensure that the system states are driven to the sliding manifold and thus the desired performance is attained and maintained in spite of the matched uncertainties. Nevertheless, there are some disadvantages: the necessity to measure the whole state and the lack of robustness against unmatched uncertainties of the resulting controller.

A possible solution to overcome the full state requirement is to use an observer to estimate the state, while on the other hand, to address the issue of robustness against the unmatched perturbation, the main solution has been the combination of sliding mode technique with other robust strategies. In order to reduce the effects of the unmatched uncertainties, a method that combines H_∞ and integral sliding mode control is proposed in [2]. The main idea is to choose such a projection matrix, ensuring not only that unmatched perturbations are not amplified, but even more, that its effects are minimized. In [4] the linear time-varying system with unmatched disturbances is replaced by a finite set of dynamic models such that each one describes a particular uncertain case then, applying a min-max SMC they develop an optimal robust sliding-surface design. A new control scheme, based on block control and quasi-continuous HOSM techniques, is proposed in [5] for control of nonlinear systems with unmatched perturbations, this method assures exact finite time tracking using only output information.

Aim of the paper. A new methodology to compensate the unmatched uncertainties, while simultaneously stabilizing the underactuated dynamics is suggested here. We propose an output sliding mode type approach based on the estimation of states and the identification of perturbations.

Methodology. In this paper a robust output control law is designed to reject the unmatched uncertainties and stabilize the underactuated dynamics using a high order sliding mode observer to reconstruct the states and perturbations in finite time.

Contribution. The proposed control law stabilizes the underactuated dynamics compensating the perturbations. At the same time, it is guaranteed that the trajectories of the remaining states are bounded. In order to achieve this:

- A sliding manifold is designed such that the system's motion along the manifold meets the specified performance: the regulation of the non-actuated states and the rejection of unmatched uncertainties.
- A discontinuous control law is designed such that the system's state is driven toward the manifold and stay|s there for all future time, regardless of disturbances and uncertainties.

Paper Structure. In Section II the problem formulation and control challenge are presented. The hierarchical high order sliding mode observer is introduced in Section III as well as the perturbations identification algorithm. In Section IV an output sliding mode controller rejecting the unmatched uncertainty is presented. A simulation example illustrates the performance of the robust exact unmatched uncertainties compensation controller in Section V.

II. PROBLEM STATEMENT

Let us consider a linear time invariant system with unknown inputs

$$\dot{x}(t) = Ax(t) + Bu(t) + Dw(t), \quad (1)$$
$$y(t) = Cx(t) \quad (2)$$

where $x(t) \in \Re^n$, $u(t) \in \Re^m$, and $y(t) \in \Re^p$ $(1 \le p < n)$ are the state vector, the control and the output of the system, respectively. The unknown inputs are represented by the vector $w(t) \in \Re^q$, and $\texttt{rank}C = p$ and $\texttt{rank}B = m$. The (A, B) pair is assumed to be controllable.

Thus, throughout the paper the following conditions are assumed to be fulfilled.

A1. For $u = 0$, the system is strongly observable, (or (A, C, D) has no invariant zeros).
A2. $w(t)$ is absolutely continuous, there is a constant w^+ such that $\|w(t)\| \le w^+$ and $\|\dot{w}(t)\| \le w^+$ for all $t \ge 0$.

Here $\|\cdot\|$ is understood as the vector Euclidean norm.

Control goal.

The main contribution of this paper is to design a controller to regulate the non-actuated states in the presence of unknown unmatched perturbations.

First, let us consider that system (1) can be decomposed in two subsystems

$$\dot{x}_1(t) = A_{11}x_1(t) + A_{12}x_2(t) + D_1 w(t) \quad (3)$$
$$\dot{x}_2(t) = A_{21}x_1(t) + A_{22}x_2(t) + D_2 w(t) + B_2 u(t) \quad (4)$$

The objective of this work is to design a controller to regulate the subsystem (3) which is non-actuated and affected by perturbations.

Before designing the control law it is necessary to estimate the state and identify the perturbations, next a brief description of the Hierarchical Super-Twisting (HST) observer is given.

III. RECALLING THE HSMO

Now, to realize the observation of the state, the HST observer is applied. Such observer provides the exact value of the state vector in a finite time. Basically, the observer works in two stages: first, a linear observer is used to maintain the estimation error between a linear observer and the original state bounded; then, by means of a differentiation scheme, such a linear estimation error is found. Thus, the hierarchical state observer is equal to a linear observer plus the linear estimation error. Below we give a general description of the observer, for more details see [8].

Stage 1: Design the linear observer $\dot{\tilde{x}}(t) = A\tilde{x}(t) + Bu(t) + L(y(t) - \tilde{y}(t))$, where $\tilde{y}(t) = C\tilde{x}(t)$. The L gain must be designed so that the matrix $\tilde{A} := (A - LC)$ is Hurwitz. Let $e(t) := x(t) - \tilde{x}(t)$. Thus, $e(t)$ converges to a ball of known radius in a finite time T_1, such that

$$\|e(t)\| \le e^+, \text{ for all } t > T_1 \quad (5)$$

Stage 2: This part of the state reconstruction is based on an algorithm that allows the decoupling of the unknown inputs from the successive derivatives of the output of the linear estimation error system $y_e = y - \tilde{y}$. The super-twisting algorithm (second order sliding mode) is used as a differentiator. The following notation must be introduced, for any matrix $F \in \Re^{r\times q}$ having rank $F = h$, $F^\perp \in \Re^{r-h\times r}$ represents one of the matrices fulfilling $F^\perp F = 0$ and rank $F^\perp = r - h$. The algorithm for the reconstruction of $e(t)$ is as follows:

a) Design each term of the *output injection* $v^{(j)}$ *at the j-th level as a super-twisting controller* [15], i.e.,

$$v_i^{(j)} = z_i^{(j)} + \lambda_j \left|s_i^{(j)}\right|^{1/2} \operatorname{sign} s_i^{(j)}, \quad \dot{z}_i^{(j)} = \alpha_j \operatorname{sign} s_i^{(j)} \quad (6)$$

where λ_j and α_j for $j \in \{1, \ldots, k-1\}$ are constants satisfying

$$\left.\begin{array}{c} \alpha_j > \beta_j \ge M_j\left(\left\|\tilde{A}\right\| e^+ + \|B\| w^+\right) \\ \lambda_j > \frac{(\alpha_j+\beta_j)(1+\theta)}{(1-\theta)}\sqrt{\frac{2}{\alpha_j-\beta_j}},\ 0<\theta<1 \end{array}\right\}$$
$$\left.\begin{array}{c} \alpha_{k-1} > \beta_{k-1} \ge \left\|\tilde{A}\right\| e^+ + \|B\| w^+ \\ \lambda_{k-1} > \frac{(\alpha_{k-1}+\beta_{k-1})(1+\theta)}{(1-\theta)}\sqrt{\frac{2}{\alpha_{k-1}-\beta_{k-1}}},\ 0<\theta<1 \end{array}\right\} \quad (7)$$

where e^+ satisfies (5) and

$$M_j = \begin{bmatrix} (M_{j-1}B)^\perp M_{j-1}\tilde{A} \\ C \end{bmatrix}, \quad M_1 := C, \text{ for } j = 1, \ldots, k \quad (8)$$

where the constant k is the smallest integer such that rank $M_k = n$.

b) The variables $s^{(j)}$ and $z^{(j)}$ satisfy the equations

$$s^{(k-1)}\left(y_e, z^{(k-2)}\right) = \left\{\begin{array}{c} \begin{pmatrix} (M_1B)^\perp y_e(t) \\ \int\limits_{\tau=0}^{t} y_e(\tau)\,d\tau \end{pmatrix} - \int\limits_{\tau=0}^{t} v^{(1)}(\tau)\,d\tau, \\ j=1 \\ \begin{pmatrix} (M_jB)^\perp z^{(j-1)} \\ \int\limits_{\tau=0}^{t} y_e(\tau)\,d\tau \end{pmatrix} - \int\limits_{\tau=0}^{t} v^{(j)}(\tau)\,d\tau, \\ \text{for } j \in \{2, \ldots, k-2\} \\ M_k^+\begin{pmatrix} (M_{k-1}B)^\perp z^{(k-2)} \\ \int\limits_{\tau=0}^{t} y_e(\tau)\,d\tau \end{pmatrix} - \int\limits_{\tau=0}^{t} v^{(k-1)}(\tau)\,d\tau. \end{array}\right. \quad (9)$$

where $M_k^+ := \left(M_k^T M_k\right)^{-1} M_k^T$.

It must be noticed that the general idea of the previous algorithm is to reconstruct $M_j e(t)$ (see, e.g., [18]) by means of the extended vector

$$\begin{bmatrix} \frac{d}{dt}(M_{j-1}B)^\perp M_{j-1}e(t) \\ y_e(t) \end{bmatrix} = M_j e(t)$$

Thus, each vector $M_j e(t)$ can be reconstructed from the second order sliding dynamics $s^{(j-1)} = \dot{s}^{(j-1)} = 0$. M_k^+ is included in the variable $s^{(k-1)}$ since $M_k^+ M_k = I$; this allows for the obtention of the $e(t)$ state representation directly from $z^{(k-1)}$ [8].

Finally, the second order sliding sets are achieved $(k-1)$ times and the following equation is obtained

$$z^{(k-1)}(t) = e(t), \forall t \geq t_{k-1} \tag{10}$$

where t_{k-1} is the reaching time for the second order sliding set $s^{(k-1)} = \dot{s}^{(k-1)} = 0$. Since $z^{(k-1)}$ and $\tilde{x}$ are available on-line, the equation characterizing the trajectories of the observer $\hat{x}$ is defined as

$$\hat{x}(t) = \tilde{x}(t) + z^{(k-1)}(t), \; t \geq 0, \tag{11}$$

where $\hat{x}$ represents the estimated value of x. In this way, a comparison between (10) and (11) yields the identity

$$\hat{x}(t) \equiv x(t), \qquad \forall t \geq t_{k-1}. \tag{12}$$

IV. Uncertainties Identification

Now, having $x(t)$ available, the uncertainty vector $w(t)$ can be identified. This can be done by finding the derivative of the projection of state to the space of $w(t)$.

Firstly, we design the variable $\bar{x}$ which satisfies the equation

$$\dot{\bar{x}}(t) = A\hat{x}(t) + Bu(t) + B\bar{u}(t) \tag{13}$$

Define the variable $\sigma(t)$ in the form $\sigma(t) = B^+(\hat{x}(t) - \bar{x}(t))$ where $B^+ := \left(B^T B\right)^{-1} B^T$. Since $\hat{x}(t) \equiv x(t)$ for $t \geq t_{k-1}$, (see (12)) and from (1) and (13), the time derivative of $\sigma(t)$ for all $t > t_{k-1}$ turns out to be

$$\dot{\sigma}(t) = w(t) - \bar{u}(t) \tag{14}$$

Now, let us design $\bar{u}$ using STA, that is, $\bar{u} = \Gamma|\sigma|^{1/2}\operatorname{sign}(\sigma) + \bar{u}_1$, $\dot{\bar{u}}_1 = \Lambda \operatorname{sign}(\sigma)$. Note that the criteria to choose Γ and Λ should satisfy the inequalities ([15]):

$$\begin{aligned} \Lambda &> w^+ \\ \Gamma &> \sqrt{\frac{2}{\Lambda - w^+}} \frac{(\Lambda + w^+)(1+p)}{(1-p)}, \; 0 < p < 1. \end{aligned}$$

Thus, after a finite time $t_\sigma > t_{k-1}$, we will have $\sigma(t) = 0$, $\dot{\sigma}(t) = 0$. Therefore, theoretically, $\hat{w} := \bar{u}_1 = w$.

V. Robust control design

Now, let us transform the system into a suitable canonical form. In this form, the system is decomposed into two connected subsystems.

A. LTI system in canonical form

Consider an invertible matrix of elementary row operations $T \in \Re^{m \times n}$

$$T = \begin{bmatrix} B^{\perp} \\ B^+ \end{bmatrix}$$

such that

$$TB = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

where $I \in \Re^{m \times m}$. By using the coordinate transformation $x \leftrightarrow Tx$. The states are partitioned such that $x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$. Applying the transformation to system (1) yields

$$\begin{aligned} \dot{x}_1(t) &= A_{11}x_1(t) + A_{12}x_2(t) + D_1 w(t) && (15) \\ \dot{x}_2(t) &= A_{21}x_1(t) + A_{22}x_2(t) + D_2 w(t) + u(t) && (16) \end{aligned}$$

where $x_1 \in \Re^{n-m}$, $x_2 \in \Re^m$, $D_1 \in \Re^{(n-m)\times q}$, $D_2 \in \Re^{m \times q}$.

Design a controller to regulate the non-actuated coordinate x_1 in spite of the presence of unknown bounded unmatched perturbations.

The proposed control law is

$$u(t) = -\rho(x)\frac{s(t)}{\|s(t)\|} \tag{17}$$

where the proposed switching function is designed as

$$s(t) = Kx_1(t) + x_2(t) + G\hat{w} \tag{18}$$

The matrix $K \in \Re^{m \times (n-m)}$ could be designed to prescribe the required performance of the reduced-order system.The term $G\hat{w}$ is added to compensate the unmatched uncertainties.

First, it is necessary guarantee that the proposed control law (17) induces a sliding motion despite the presence of the uncertainties.

B. Existence of the Ideal Sliding Mode

From (18), the time derivative of $s(t)$ is given by

$$\dot{s}(t) = \Phi x + (KD_1 + D_2)w + G\dot{\hat{w}} + u(t) \tag{19}$$

where matrix $\Phi \in \Re^{m \times n}$ is defined as $\Phi := \begin{bmatrix} KA_{11} + A_{21} & KA_{12} + A_{22} \end{bmatrix}$.

Choosing a Lyapunov candidate $V(s) = \frac{s^T s}{2}$ and taking its derivative along the time yields:

$$\begin{aligned} &\dot{V}(s) \\ &= s^T\left(\Phi x + (KD_1 + D_2)w + G\dot{\hat{w}} - \rho(x)\frac{s}{\|s\|}\right) \\ &\leq -\|s\|(\rho(x) - \|\Phi\|\|x\| - \phi) \end{aligned} \tag{20}$$

the scalar gain $\rho(x)$ satisfies the condition

$$\rho(x) - \|\Phi\|\|x\| - \phi \geq \zeta > 0$$

where ζ is a constant and $\phi := \|(KD_1 + D_2)\| w^+ + \|G\| w^+$.

$$\rho(x) > \|\Phi\|\|x\| + \phi + \zeta \tag{21}$$

Combining inequalities (20) and (21), it follows that the derivative of the Lyapunov function satisfies $\dot{V}(s) \leq -\zeta V^{\frac{1}{2}}$. Gain $\rho(x)$ will induce the sliding motion.

C. Description of the sliding mode motion

The equation representing the motion when confined to the sliding surface is obtained when $s(t) = 0$.When the system reaches the sliding surface $s = 0$, we have

$$x_2 = -Kx_1 - G\hat{w} \quad (22)$$
$$\dot{x}_1 = (A_{11} - A_{12}K)\, x_1 - A_{12}G\hat{w} + D_1 w \quad (23)$$

As (A, B) pair is controllable, it is well known that (A_{11}, A_{12}) pair will be controllable [11] so that, it is possible to design a matrix K in order to matrix $A_s \triangleq (A_{11} - A_{12}K)$ has stable eigenvalues. The G gain matrix should be selected in order to compensate the unmatched uncertainties. In order to compensate w from x_1, matrix D_1 must be matched with respect to A_{12}; therefore, it will be assumed that:

A3. $\mathtt{Im}D_1 \in \text{span}\,(A_{12})$

Then there is a matrix $G \in \Re^{m\times p}$ such that

$$A_{12}G = D_1 \quad (24)$$

[1] Then the equation (23) yields

$$\dot{x}_1\,(t) = (A_{11} - A_{12}K)\, x_1\,(t) + D_1\,(w - \hat{w}) \quad (25)$$

so, in the ideal case,

$$\dot{x}_1\,(t) = A_s x_1\,(t) \quad (26)$$

Since the eigenvalues of A_s have negative real part, equation (26) is exponentially stable. So, the unmatched uncertainties are compensated and coordinate x_1 is stabilized. The trajectories of the state x_1 will converge to a bounded region, i.e. there exist some constants $a_1,\ a_2 > 0$ such that

$$\|x_1(t)\| \le a_1\,\|x_1(0)\| \exp^{-a_2 t} \quad \forall t > t_\sigma$$

Furthermore, x_2 is bounded as well indeed during sliding motion. Taking the norm of equation (22) we have

$$\|x_2\,(t)\| \le \|K\|\,\|x_1\,(t)\| + \|G\|\,w^+ \quad \forall t > t_\sigma \quad (27)$$

From the above equation, it is clear that the trajectories of x_2 are bounded.

VI. SIMULATION EXAMPLE

Here, we present an academic example in order to show the feasibility of the proposed methodology. Consider the system

$$\dot{x} = \begin{bmatrix} 0 & 0 & 1 & 0.25 \\ 0 & 1 & 0.25 & 0 \\ 0 & -4 & 0.25 & 0 \\ 1 & 0.25 & 0 & -0.5 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ -1 \\ 0 \\ -1 \end{bmatrix} w \quad (28)$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x \quad (29)$$

where the unknown input $w = 0.5\sin(3t) + 0.3$.

Observer design. It can be verified that system (28)-(29)does not have invariant zeros. The Luemberger-type observer is designed such that matrix $A - LC$ has a set of eigenvalues given by $\{-1, -2, -3, -4\}$. For triplet (A, C, D) , $k = 2$, i.e. we need to derive two times in order to reconstruct x and w. The M_k matrices are:

$$M_1 = C;\ M_2 = \begin{bmatrix} -3.5 & 0 & 0 & 0.25 \\ 0 & 0 & -2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The STA gains are $\alpha = 5$, $\lambda = 2$.The identification process has parameters $\Gamma = 3.11$, $\Lambda = 1.45$. The sampling step is $\delta = 10(\mu s)$. First, transform the system (28) to the canonical form (15)-(16) with:

$$T = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

It is clear from the above equation that there are matched and unmatched perturbations. Selecting the sliding mode controller gain as suggested in (21) we have $\rho\,(x) = 20\,\|x\,(t)\| + 2$. For this example matrix K was selected using the quadratic minimization approach [1], such that the reduced order system has a pair of complex eigenvalues $-1.34 \pm i1.7015$.

The simulation was carried comparing a conventional sliding mode controller design using a $s = x_1 + Kx_2$ surface against the methodology proposed in this manuscript. Fig. (1) shows the states of the regularized system, column (A) shows the results when no compensation is carried: the perturbation effects are present in all the states. The column (B) shows the states when the compensation of unmatched uncertainties is done through the sliding surface, here the stabilization of state x_1 (solid-line plot) is achieved, while the trajectories of state x_2 (dotted-line) remains bounded.

VII. CONCLUSIONS

A output regulation for linear systems with unmatched uncertainties was presented. Based on the exact reconstruction of the unknown inputs (unmatched uncertainties), we propose

[1] In particular, when $\text{rank}(A_{12}) = n - m$, matrix $G = A_{12}^{+} D_1$ where A_{12}^{+} is understood as the right inverse of A_{12}, that is $A_{12}^{+} = A_{12}^{T}\left(A_{12}A_{12}^{T}\right)^{-1}$.

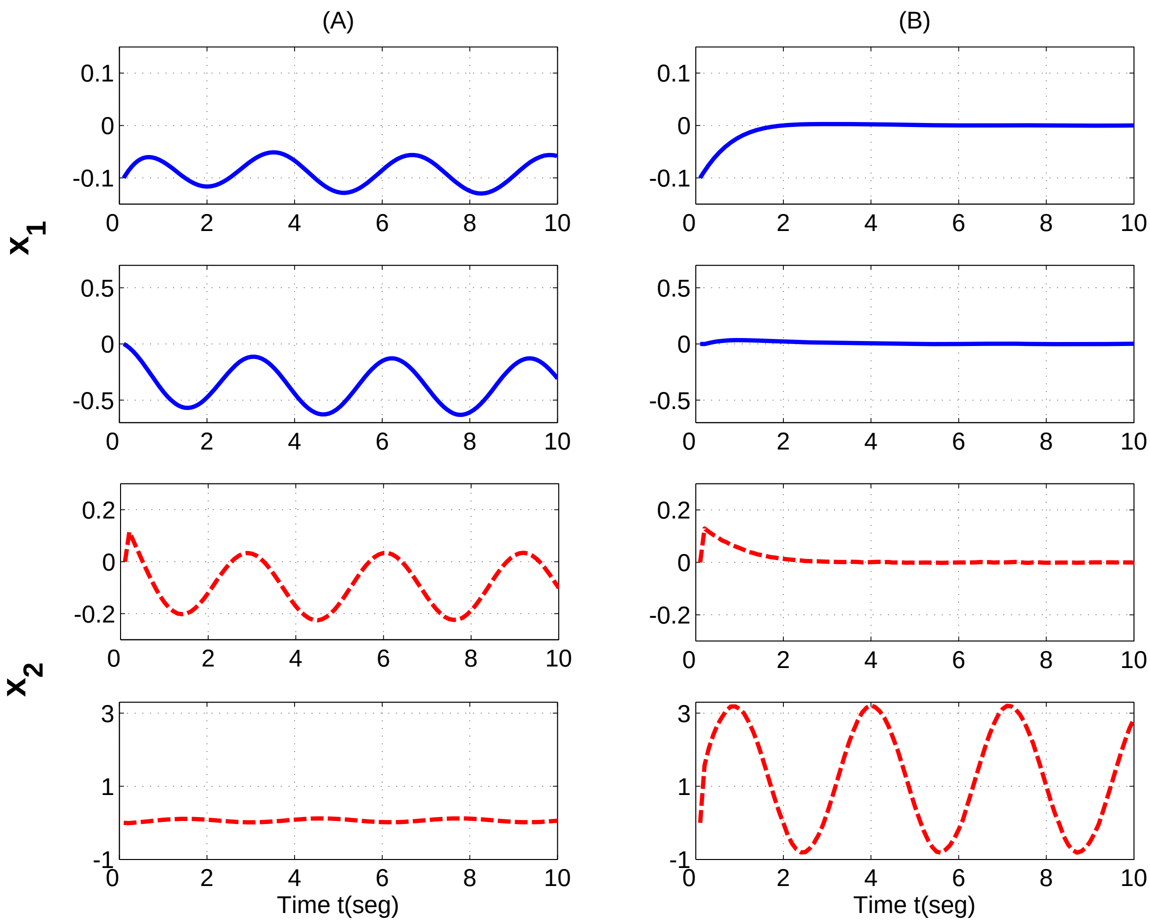

Fig. 1. Column (A) shows the canonical form system without compensation and in column (B) the compensated system. The underactuated states are plotted in solid-line, while the completely actuated states are plotted in (dashed-line)

designing a sliding mode control which allows compensating the uncertainties from the non actuated system dynamics and maintains the trajectories of the remained states bounded.

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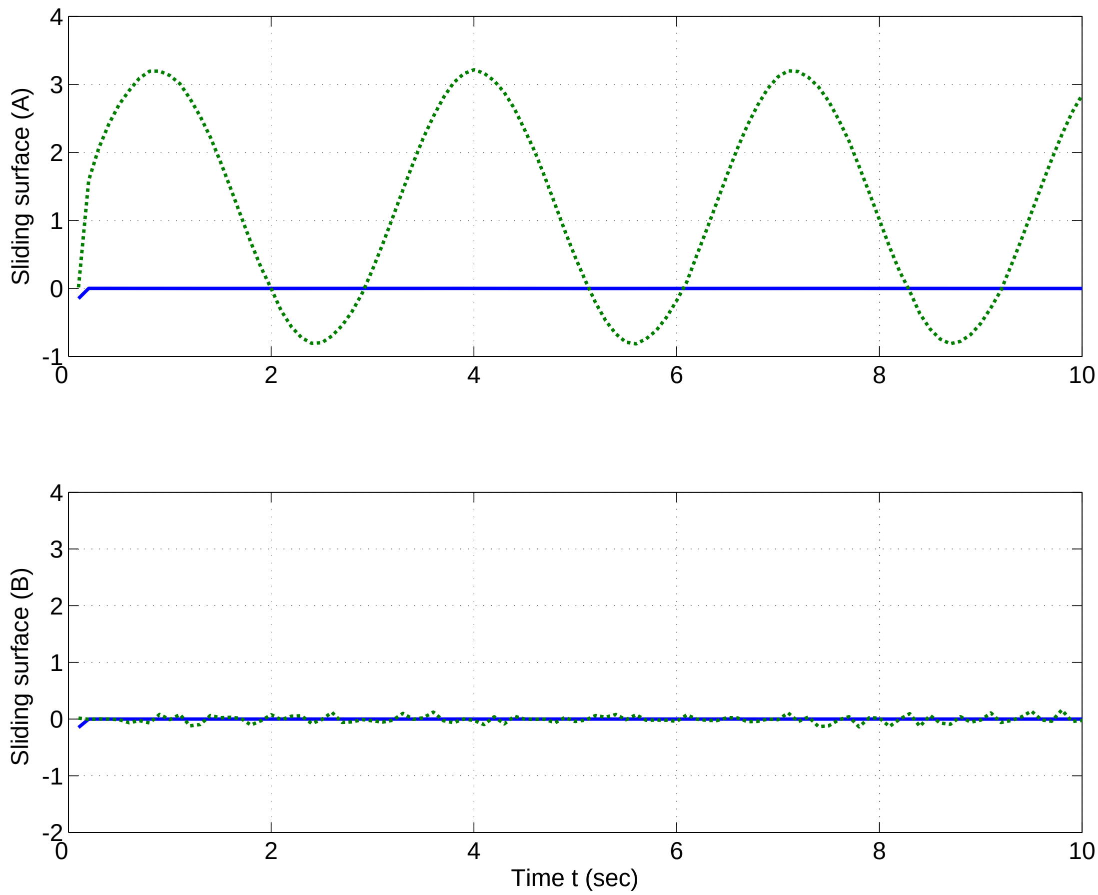

Fig. 2. Column (A) shows the system without compensation and in column (B) the compensated system

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