An Adaptive Scheme for Bilateral Teleoperators with Time-Delays

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***Abstract*— An adaptive controller for teleoperators with time-delays, which ensures synchronization of positions and velocities of the master and slave manipulators, and does not rely on the use of the ubiquitous scattering transformation is proposed in (Chopra *et al.* Automatica. 44(8):2142-2148, Aug. 2008). In the present paper it is shown that such controller will tend to drive to zero the positions of the joints where gravity forces are non–zero. Hence, such scheme is, in general, applicable only to systems without gravity. It is also proved that this limitation can be obviated replacing the positions and velocities—that are used in the coordinating torques and the adaptation laws—by their *errors*. Simulation results illustrate the performance of both schemes.**

Keywords—Bilateral teleoperation; Adaptive control; Passivity-based control; Time delay.

I. Introduction

It is widely known, amongst the teleoperation literature, that Anderson and Spong have created the basis of *modern* teleoperators control, providing the first scheme rendering a stable teleoperation despite any constant time-delay. Their approach was to render passive the communications using the analogy of a lossless transmission line with the scattering theory. Under the reasonable assumption that the human operator and the contact environment define passive (force to velocity) maps, stability of the overall system is then ensured (Anderson y Spong, 1989; Niemeyer y Slotine, 1991). Nevertheless, the classic scattering transformation may give raise to position drift. In order to overcome this obstacle, PD plus damping injection controllers have been reported in (Lee y Spong, 2006; Nuño *et al.*, 2008; Nuño *et al.*, 2009).

(Chopra y Spong, 2006) proposed to formulate the teleoperation problem in terms of synchronization, which also avoids the scattering transformation. An adaptive version of this scheme that aims at synchronizing the local and remote positions and velocities is proposed in (Chopra *et al.*, 2008). In the present paper it is proved that such scheme is, in general, only applicable to systems without gravity. Moreover, an adaptive controller that overcomes this obstacle is presented. The main, simple but essential, difference between the proposed controller and the one in (Chopra *et al.*, 2008) is the use of the position and velocity *errors* in the coordinating torques and the robot dynamics parametrization—and, consequently, in the adaptation laws.

Notation. $\mathbb{R} := (-\infty,\infty)$, $\mathbb{R}_{>0} := (0,\infty)$, $\mathbb{R}_{\geq 0} := [0,\infty)$. $\lambda_m\{\mathbf{A}\}$ and $\lambda_M\{\mathbf{A}\}$ represent the minimum and maximum eigenvalue of matrix $\mathbf{A}$, respectively. $|\mathbf{x}|$ stands for the Euclidean norm of vector $\mathbf{x}$. For any function $\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, the $\mathcal{L}_\infty$-norm is defined as $\|\mathbf{f}\|_\infty = \sup_{t\in[0,\infty)} |\mathbf{f}(t)|$, and the $\mathcal{L}_2$-norm as $\|\mathbf{f}\|_2 = \left(\int_0^\infty |\mathbf{f}(t)|^2 dt\right)^{\frac{1}{2}}$. The subscript i takes the values l and r for local and remote robot manipulators, respectively.

II. Dynamic Model of the Teleoperator

The local and remote manipulators are modeled as a pair of *n*-Degrees of Freedom (DOF) serial links. Their corresponding nonlinear dynamics, together with the human operator and environment torques, are given by

$$\begin{aligned}\mathbf{M}_l(\mathbf{q}_l)\ddot{\mathbf{q}}_l + \mathbf{C}_l(\mathbf{q}_l,\dot{\mathbf{q}}_l)\dot{\mathbf{q}}_l + \mathbf{g}_l(\mathbf{q}_l) &= \boldsymbol{\tau}_h - \boldsymbol{\tau}_l \\ \mathbf{M}_r(\mathbf{q}_r)\ddot{\mathbf{q}}_r + \mathbf{C}_r(\mathbf{q}_r,\dot{\mathbf{q}}_r)\dot{\mathbf{q}}_r + \mathbf{g}_r(\mathbf{q}_r) &= \boldsymbol{\tau}_r - \boldsymbol{\tau}_e,\end{aligned} \tag{1}$$

where: $\ddot{\mathbf{q}}_i, \dot{\mathbf{q}}_i, \mathbf{q}_i \in \mathbb{R}^n$ are the joint acceleration, velocity and position; $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{n\times n}$ are the inertia matrices; $\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i) \in \mathbb{R}^{n\times n}$ capture the Coriolis and centrifugal effects; $\mathbf{g}_i(\mathbf{q}_i) \in \mathbb{R}^n$ are the gravitational forces; $\boldsymbol{\tau}_i \in \mathbb{R}^n$ are the control signals; and $\boldsymbol{\tau}_h \in \mathbb{R}^n$, $\boldsymbol{\tau}_e \in \mathbb{R}^n$ are the forces exerted by the human and the environment. It is assumed that the manipulators are composed by actuated revolute joints, and that friction can be neglected.

These dynamical models have some important well–known properties (Kelly *et al.*, 2005; Spong *et al.*, 2005).

- P1. The inertia matrix is lower and upper bounded, i.e., $0 < \lambda_m\{\mathbf{M}_i\}\mathbf{I} \leq \mathbf{M}_i(\mathbf{q}_i) \leq \lambda_M\{\mathbf{M}_i\}\mathbf{I} < \infty$.
- P2. The Coriolis and inertia matrices are related as $\dot{\mathbf{M}}_i(\mathbf{q}_i) = \mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i) + \mathbf{C}_i^\top(\mathbf{q}_i,\dot{\mathbf{q}}_i)$.
- P3. The Coriolis forces are bounded as follows $\forall\, \mathbf{q}_i, \mathbf{x}, \mathbf{y} \in \mathbb{R}^n\ \exists k_{c_i} \in \mathbb{R}_{>0}$ such that $|\mathbf{C}_i(\mathbf{q}_i,\mathbf{x})\mathbf{y}| \leq k_{c_i}|\mathbf{x}||\mathbf{y}|$. Consequently, $|\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i| \leq k_{c_i}|\dot{\mathbf{q}}_i|^2$.
- P4. The Lagrangian dynamics are linearly parameterizable, such that

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \mathbf{g}_i(\mathbf{q}_i) = \mathbf{Y}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i,\ddot{\mathbf{q}}_i)\boldsymbol{\theta}_i$$

where $\mathbf{Y}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i) \in \mathbb{R}^{n\times p}$ are matrices of known functions and $\boldsymbol{\theta}_i \in \mathbb{R}^p$ are constant vectors of the manipulator physical parameters (link masses, moments of inertia, etc.).

III. Previous Results

In this section we briefly review the results reported in (Chopra *et al.*, 2008) and point out the constraint it imposes on the gravity.

A. A Synchronization Result

Let $\mathbf{e}_i \in \mathbb{R}^n$ denote the position error vectors, defined, for a constant time-delay T, by

$$\mathbf{e}_l = \mathbf{q}_r(t-T) - \mathbf{q}_l; \quad \mathbf{e}_r = \mathbf{q}_l(t-T) - \mathbf{q}_r. \tag{2}$$

The control objective of (Chopra *et al.*, 2008) is to drive the coordination errors, $\mathbf{e}_i, \dot{\mathbf{e}}_i$, to zero, independently of the constant time-delay T, and without using the scattering transformation. In this case, it is said that the manipulators synchronize. In order to achieve the synchronization objective the following adaptive controller is proposed

$$\begin{aligned} \boldsymbol{\tau}_l &= \hat{\mathbf{M}}_l(\mathbf{q}_l)\lambda\dot{\mathbf{q}}_l + \hat{\mathbf{C}}_l(\mathbf{q}_l,\dot{\mathbf{q}}_l)\lambda\mathbf{q}_l - \hat{\mathbf{g}}_l(\mathbf{q}_l) - \bar{\boldsymbol{\tau}}_l \quad (3)\\ \boldsymbol{\tau}_r &= \bar{\boldsymbol{\tau}}_r - \hat{\mathbf{M}}_r(\mathbf{q}_r)\lambda\dot{\mathbf{q}}_r - \hat{\mathbf{C}}_r(\mathbf{q}_r,\dot{\mathbf{q}}_r)\lambda\mathbf{q}_r + \hat{\mathbf{g}}_r(\mathbf{q}_r) \end{aligned}$$

where $\hat{\mathbf{M}}_i, \hat{\mathbf{C}}_i, \hat{\mathbf{g}}_i$ are the estimates of the inertia and Coriolis matrices and the gravity forces, respectively, $\lambda \in \mathbb{R}_{>0}$,[1] and $\bar{\boldsymbol{\tau}}_i$ are the coordinating torques given by

$$\bar{\boldsymbol{\tau}}_l = K[\mathbf{s}_r(t-T) - \mathbf{s}_l]; \quad \bar{\boldsymbol{\tau}}_r = K[\mathbf{s}_l(t-T) - \mathbf{s}_r] \tag{4}$$

where $K \in \mathbb{R}_{>0}$. The signals $\mathbf{s}_i$ are defined as

$$\mathbf{s}_i = \dot{\mathbf{q}}_i + \lambda\mathbf{q}_i. \tag{5}$$

The control signals (3) can be also written as

$$\boldsymbol{\tau}_l = \mathbf{Y}_l(\mathbf{q}_l,\dot{\mathbf{q}}_l)\hat{\boldsymbol{\theta}}_l - \bar{\boldsymbol{\tau}}_l; \quad \boldsymbol{\tau}_r = \bar{\boldsymbol{\tau}}_r - \mathbf{Y}_r(\mathbf{q}_r,\dot{\mathbf{q}}_r)\hat{\boldsymbol{\theta}}_r,$$

where $\mathbf{Y}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)$ are matrices of known functions, and $\hat{\boldsymbol{\theta}}_i$ are the physical estimated parameters.

Replacing (3) in (1) and adding the term

$$\mathbf{Y}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\boldsymbol{\theta}_i = \mathbf{M}_i(\mathbf{q}_i)\lambda\dot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\lambda\mathbf{q}_i - \mathbf{g}_i(\mathbf{q}_i), \tag{6}$$

at each side, yields

$$\begin{aligned} \mathbf{M}_l(\mathbf{q}_l)\dot{\mathbf{s}}_l + \mathbf{C}_l(\mathbf{q}_l,\dot{\mathbf{q}}_l)\mathbf{s}_l &= \mathbf{Y}_l(\mathbf{q}_l,\dot{\mathbf{q}}_l)\tilde{\boldsymbol{\theta}}_l + \bar{\boldsymbol{\tau}}_l + \boldsymbol{\tau}_h \quad (7)\\ \mathbf{M}_r(\mathbf{q}_r)\dot{\mathbf{s}}_r + \mathbf{C}_r(\mathbf{q}_r,\dot{\mathbf{q}}_r)\mathbf{s}_r &= \mathbf{Y}_r(\mathbf{q}_r,\dot{\mathbf{q}}_r)\tilde{\boldsymbol{\theta}}_r + \bar{\boldsymbol{\tau}}_r - \boldsymbol{\tau}_e \end{aligned}$$

where $\tilde{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_i$ are the estimation errors. The parameter update laws are given by

$$\dot{\hat{\boldsymbol{\theta}}}_i = \boldsymbol{\Gamma}_i\mathbf{Y}_i^\top(\mathbf{q}_i,\dot{\mathbf{q}}_i)\mathbf{s}_i \tag{8}$$

where $\boldsymbol{\Gamma}_i$ are constant positive definite matrices.

Proposition 1: (Chopra *et al.*, 2008). Consider the bilateral teleoperator (1) in free motion ($\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$) controlled by (3) using the parameter update law (8) and coordinating torques (4), (5). Then, for any constant time-delay T, all signals in the system are bounded and $|\mathbf{e}_i| \to |\dot{\mathbf{e}}_i| \to 0$ as $t \to \infty$.

B. A Practical Limitation

In this subsection it is shown that the controller of (Chopra *et al.*, 2008) will tend to drive to zero the positions of the joints where gravity forces are non–zero.

Consistently to the convergence claim of Proposition 1, we will study the constant position equilibria of the closed–loop system (1), (3), (4), (5), (8), whose state vector is $(\mathbf{q}_l, \mathbf{q}_r, \dot{\mathbf{q}}_l, \dot{\mathbf{q}}_r, \hat{\theta}_l, \hat{\theta}_r)$.[2] Evaluating (5), (8) at the equilibrium we get

$$\mathbf{q}^\top\mathbf{Y}(\mathbf{q},\mathbf{0}) = \mathbf{0}. \tag{9}$$

Notice that the equilibrium constraint (9) is imposed even if the manipulators are not in free motion. It will be shown that this constraint implies a restriction on the gravity forces. Indeed, from (6) we get

$$\mathbf{Y}(\mathbf{q},\mathbf{0})\boldsymbol{\theta} = -\mathbf{g}(\mathbf{q}), \tag{10}$$

which establishes a relationship between $\mathbf{Y}(\mathbf{q},\mathbf{0})$ and $\mathbf{g}(\mathbf{q})$. Before considering the general case let us analyze the implications of the equilibrium constraint (9) in some simple examples. Consider the five–bar linkage system studied in (Spong *et al.*, 2005), whose gravity forces are

$$\mathbf{g}(\mathbf{q}) = \begin{bmatrix} a_1\cos(q_1) \\ a_2\cos(q_2) \end{bmatrix},$$

with $a_i \in \mathbb{R}_{>0}$. Selecting a minimal parametrization we get

$$\mathbf{Y}(\mathbf{q},\mathbf{0}) = -\begin{bmatrix} 0 & 0 & \cos(q_1) & 0 \\ 0 & 0 & 0 & \cos(q_2) \end{bmatrix},$$

and the equilibrium constraint (9) is

$$q_1\cos(q_1) = q_2\cos(q_2) = 0.$$

Hence, the controller tends to drive to zero the position of the joints where gravity forces are nonzero. A similar situation happens for of a 2-DOF manipulator with rotational joints, whose gravity forces are

$$\mathbf{g}(\mathbf{q}) = \begin{bmatrix} a_1\cos(q_1) + a_2\cos(q_1+q_2) \\ a_2\cos(q_1+q_2) \end{bmatrix},$$

and

$$\mathbf{Y}(\mathbf{q},\mathbf{0}) = -\begin{bmatrix} 0 & \cdots 0 & \cos(q_1) & \cos(q_1+q_2) \\ 0 & \cdots 0 & 0 & \cos(q_1+q_2) \end{bmatrix}.$$

The equilibrium constraint (9) becomes $q_1\cos(q_1) = 0$, $q_2\cos(q_1+q_2) = 0$, implying, again, that the position of the second joint will go to zero if the corresponding gravity force is not zero. It can be easily shown that similar restrictions apply for the 3-DOF and the Puma manipulators.

[1] In (Chopra *et al.*, 2008) λ is a positive definite matrix. As will become clear below, taking it to be a scalar, does not change our main argument. See remark (i) in Subsection 3.2.

[2] To avoid cluttering, and with some obvious abuse of notation, we will not distinguish the constant equilibria from the variable itself, that is, we omit the standard upperbar notation. Also, when clear from the context, the subindex $(\cdot)_i$, that identifies the local and remote manipulator, is omitted.

The question that arises naturally is whether there exists manipulators for which (9) *does not* impose a constraint. That is, is there a manipulator with gravity forces that satisfy (9) *for all* $\mathbf{q}$? In the proposition below it is proven that the answer to this question is negative.

Proposition 2: Consider the bilateral teleoperator (1) in closed–loop with (3), (4), (5), (8). The set of achievable (constant) equilibrium positions is strictly contained in

$$\{(\mathbf{q}_l, \mathbf{q}_r) \in \mathbb{R}^{2n} \mid \mathbf{q}_i^\top \mathbf{Y}_i(\mathbf{q}_i, \mathbf{0}) = \mathbf{0},\ i = l, r\}.$$

Moreover, for all manipulators of the form (1), composed by kinematic open chains of revolute or prismatic joints there is no (non-zero) gravity force vector that satisfies (9), (10) *for all* $\mathbf{q}$, therefore, this set is a *strict* subset of $\mathbb{R}^{2n}$.

Proof: First, notice that (9) and (10) imply

$$\mathbf{q}^\top \mathbf{g}(\mathbf{q}) = 0. \tag{11}$$

Now, since gravity forces are the gradient of the potential energy function $U(\mathbf{q})$, that is $\mathbf{g}(\mathbf{q}) = \frac{\partial}{\partial \mathbf{q}} U(\mathbf{q})$, (11) becomes the partial differential equation (PDE)

$$\mathbf{q}^\top \frac{\partial}{\partial \mathbf{q}} U(\mathbf{q}) = 0. \tag{12}$$

It will be proved that the only potential energy function $U(\mathbf{q})$ that satisfies this PDE is $U(\mathbf{q})$ =constant, that is $\mathbf{g}(\mathbf{q}) = \mathbf{0}$. Toward this end, recall that for manipulators with prismatic and revolute joints, $U(\mathbf{q})$ is a polynomial function in the arguments q_i, $\sin(q_i)$ and $\cos(q_i)$, that is, a function of the form

$$P(\mathbf{q}) = \sum_{j=1}^{n} a_j \prod_{i=1}^{n} q_i^{b_i} \prod_{i=1}^{n} \sin^{c_i}(q_i) \prod_{i=1}^{n} \cos^{d_i}(q_i), \tag{13}$$

where a_j are real numbers and b_j, c_j, d_j are nonnegative integers (Kelly *et al.*, 2005; Spong *et al.*, 2005). It will be now proved that the only solution of (12) of the form (13) is the constant solution.

It is possible to prove that all solutions of the partial differential equation (12) are homogenous, that is, they satisfy $U(\mathbf{q}) = U(r\mathbf{q})$ for any $r \in \mathbb{R}_{>0}$.[3] Indeed, evaluating the time derivative of $U(\mathbf{q})$ we get $\dot{U} = \frac{\partial}{\partial \mathbf{q}} U^\top(\mathbf{q})\dot{\mathbf{q}}$, which, in view of (12), is equal to zero along the flow of the system $\dot{\mathbf{q}} = \mathbf{q}$, *i.e.* along $\mathbf{q}(t) = e^t \mathbf{q}(0)$. Hence,

$$U(\mathbf{q}(t)) = U(e^t \mathbf{q}(0)),\ \forall t \geq 0,$$

establishing the claim of homogeneity.

Now, notice that the function $P(\mathbf{q})$ has a limit at zero, that is $P(0) = \lim_{|\mathbf{q}| \to 0} P(\mathbf{q})$. Second, because of homogeneity, if $P(\mathbf{q})$ is a solution of (12), then $P(\mathbf{q}) = P(r\mathbf{q})$. Hence, the limits satisfy $\lim_{|\mathbf{q}| \to 0} P(\mathbf{q}) = \lim_{r \to 0} P(r\mathbf{q})$. But $\lim_{r \to 0} P(r\mathbf{q}) = \lim_{r \to 0} P(\mathbf{q}) = P(\mathbf{q})$. Hence, $P(\mathbf{q}) = P(\mathbf{0})$. The proof is concluded noting that the only polynomial function that is constant for all $\mathbf{q}$ is the constant function, and consequently $\mathbf{g}(\mathbf{q}) = \mathbf{0}$. ■

The following remarks are in order.

(i) In the derivations above it has been assumed that the tuning parameter λ is a scalar. If it is a matrix, (9) becomes $\mathbf{q}^\top \boldsymbol{\lambda} \mathbf{Y}(\mathbf{q}, \mathbf{0}) = \mathbf{0}$, leaving the scenario discussed above, essentially, unmodified.

(ii) The experiments in (Chopra *et al.*, 2008) yield the desired behavior because the manipulators are composed by two revolute joints whose DOFs lie on the horizontal plane. Thus, in such case $\mathbf{g}_i(\mathbf{q}_i) = \mathbf{0}$.

IV. A New Adaptive Controller

Let us define $\boldsymbol{\epsilon}_i$ as

$$\boldsymbol{\epsilon}_i = \dot{\mathbf{q}}_i - \boldsymbol{\lambda} \mathbf{e}_i, \tag{14}$$

where $\mathbf{e}_i$ has been previously defined in (2) and $\boldsymbol{\lambda}$ is a diagonal positive definite matrix.

The proposed controllers are given by

$$\begin{aligned} \boldsymbol{\tau}_l &= \mathbf{Y}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l, \mathbf{e}_l, \dot{\mathbf{e}}_l)\hat{\boldsymbol{\theta}}_l + \bar{\boldsymbol{\tau}}_l \\ \boldsymbol{\tau}_r &= -\mathbf{Y}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r, \mathbf{e}_r, \dot{\mathbf{e}}_r)\hat{\boldsymbol{\theta}}_r - \bar{\boldsymbol{\tau}}_r \end{aligned} \tag{15}$$

where,

$$\mathbf{Y}_i \hat{\boldsymbol{\theta}}_i = -\hat{\mathbf{M}}_i(\mathbf{q}_i)\boldsymbol{\lambda}\dot{\mathbf{e}}_i - \hat{\mathbf{C}}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\boldsymbol{\lambda}\mathbf{e}_i - \hat{\mathbf{g}}_i(\mathbf{q}_i). \tag{16}$$

Substituting the controllers (15) on the teleoperator dynamics (1) and using (14), yields

$$\begin{aligned} \mathbf{M}_l(\mathbf{q}_l)\dot{\boldsymbol{\epsilon}}_l + \mathbf{C}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l)\boldsymbol{\epsilon}_l &= \mathbf{Y}_l \tilde{\boldsymbol{\theta}}_l - \bar{\boldsymbol{\tau}}_l + \boldsymbol{\tau}_h \\ \mathbf{M}_r(\mathbf{q}_r)\dot{\boldsymbol{\epsilon}}_r + \mathbf{C}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\boldsymbol{\epsilon}_r &= \mathbf{Y}_r \tilde{\boldsymbol{\theta}}_r - \bar{\boldsymbol{\tau}}_r - \boldsymbol{\tau}_e. \end{aligned} \tag{17}$$

The dynamics of the estimations of the uncertain parameters are given by

$$\dot{\hat{\boldsymbol{\theta}}}_i = \boldsymbol{\Gamma}_i \mathbf{Y}_i^\top \boldsymbol{\epsilon}_i. \tag{18}$$

where $\boldsymbol{\Gamma}_i$ are positive definite matrices. The torques $\bar{\boldsymbol{\tau}}_i$ are

$$\bar{\boldsymbol{\tau}}_i = \mathbf{K}_i \boldsymbol{\epsilon}_i - \mathbf{B}_i \dot{\mathbf{e}}_i \tag{19}$$

where $\mathbf{K}_i$ are positive definite matrices and $\mathbf{B}_i$ is diagonal positive definite.

A. Asymptotic Regulation in Free Motion

Proposition 3: Consider the bilateral teleoperator (1) in free motion ($\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$) controlled by (15) using the parameter update law (18) and coordinating torques (19) together with (14). Then, for any constant time-delay T, all signals in the system are bounded. Moreover, position errors and velocities asymptotically converge to zero, *i.e.*, $|\mathbf{e}_i| \to |\dot{\mathbf{q}}_i| \to 0 \quad t \to \infty$.

Proof: Let us propose a Lyapunov-Krasovskiĭ candidate function V as the following

$$V = \frac{1}{2} \sum_{i \in \{l,r\}} \left[\boldsymbol{\epsilon}_i^\top \mathbf{M}_i \boldsymbol{\epsilon}_i + \tilde{\boldsymbol{\theta}}_i^\top \boldsymbol{\Gamma}_i^{-1} \tilde{\boldsymbol{\theta}}_i + \mathbf{e}_i^\top \boldsymbol{\lambda} \mathbf{B} \mathbf{e}_i + \int_{t-T}^{t} \dot{\mathbf{q}}_i^\top \mathbf{B} \dot{\mathbf{q}}_i d\sigma \right]$$

[3] This conclusion also follows noting that all solutions of (12) are of the form

$$U(\mathbf{q}) = f\left(\frac{q_2}{q_1}, \frac{q_3}{q_1}, \ldots, \frac{q_n}{q_1}\right),$$

where $f : \mathbb{R}^{n-1} \to \mathbb{R}$ is an arbitrary C^1 map (Ibragimov, 1999).

This function is positive definite and radially unbounded in $\boldsymbol{\epsilon}_i, \tilde{\boldsymbol{\theta}}_i, \mathbf{e}_i$. Its time-derivative along (17) and (19), using P2, is given by

$$\dot{V} = \sum_{i\in\{l,r\}} \Big[-\boldsymbol{\epsilon}_i^\top \mathbf{K}_i \boldsymbol{\epsilon}_i + \dot{\mathbf{q}}_i^\top \mathbf{B}\dot{\mathbf{e}}_i + \frac{1}{2}\dot{\mathbf{q}}_i^\top \mathbf{B}\dot{\mathbf{q}}_i - \frac{1}{2}\dot{\mathbf{q}}_i^\top(t-T)\mathbf{B}\dot{\mathbf{q}}_i(t-T)\Big].$$

Notice that, for $i = l$, $\dot{\mathbf{q}}_l^\top \mathbf{B}\dot{\mathbf{e}}_l = \dot{\mathbf{q}}_l^\top \mathbf{B}(\dot{\mathbf{q}}_r(t-T) - \dot{\mathbf{q}}_l)$. Hence, when $i = r$ and gathering the crossed terms $-\frac{1}{2}[\dot{\mathbf{q}}_l^\top \mathbf{B}\dot{\mathbf{q}}_l - 2\dot{\mathbf{q}}_l^\top \mathbf{B}\dot{\mathbf{q}}_r(t-T) + \dot{\mathbf{q}}_r^\top(t-T)\mathbf{B}\dot{\mathbf{q}}_r(t-T)]$, yields

$$\dot{V} = -\sum_{i\in\{l,r\}} \Big[\boldsymbol{\epsilon}_i^\top \mathbf{K}_i \boldsymbol{\epsilon}_i + \frac{1}{2}\dot{\mathbf{e}}_i^\top \mathbf{B}\dot{\mathbf{e}}_i \Big].$$

Due to $V \geq 0$ and $\dot{V} \leq 0$, $\boldsymbol{\epsilon}_i, \dot{\mathbf{e}}_i \in \mathcal{L}_2$ and $\boldsymbol{\epsilon}_i, \tilde{\boldsymbol{\theta}}_i, \mathbf{e}_i \in \mathcal{L}_\infty$. From (14) it can be shown that $\dot{\mathbf{q}}_i \in \mathcal{L}_\infty$, implying that $\dot{\mathbf{e}}_i \in \mathcal{L}_\infty$. All these bounded signals together with P1 and P3 guarantee that $\mathbf{Y}_i \in \mathcal{L}_\infty$. Now, from (17), using P1 and P3 together with boundedness of $\bar{\boldsymbol{\tau}}_i, \mathbf{Y}_i, \tilde{\boldsymbol{\theta}}_i, \boldsymbol{\epsilon}_i, \dot{\mathbf{q}}_i$, it can be conclude that $\dot{\boldsymbol{\epsilon}}_i \in \mathcal{L}_\infty$. Hence, $\boldsymbol{\epsilon}_i \in \mathcal{L}_\infty \cap \mathcal{L}_2$, $\dot{\boldsymbol{\epsilon}}_i \in \mathcal{L}_\infty$ support that $|\boldsymbol{\epsilon}_i| \to 0$.

$\dot{\boldsymbol{\epsilon}}_i, \dot{\mathbf{e}}_i \in \mathcal{L}_\infty$ imply that $\ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$, hence $\ddot{\mathbf{e}}_i \in \mathcal{L}_\infty$. This last, and the fact that $\dot{\mathbf{e}}_i \in \mathcal{L}_\infty \cap \mathcal{L}_2$ prove that $|\dot{\mathbf{e}}_i| \to 0$.

Now, for $i = l$, $\mathbf{e}_l, \dot{\mathbf{e}}_l, \ddot{\mathbf{e}}_l \in \mathcal{L}_\infty$ and $|\dot{\mathbf{e}}_l| \to 0$ imply that $\lim_{t\to\infty}\int_0^t \dot{\mathbf{e}}_l d\sigma = \mathbf{e}_l - \mathbf{e}_l(0) = k_l < \infty$. On the other hand,

$$\lim_{t\to\infty}[|\boldsymbol{\epsilon}_l| = |\dot{\mathbf{q}}_l - \boldsymbol{\lambda}\mathbf{e}_l|] = \lim_{t\to\infty}|\dot{\mathbf{q}}_l - \boldsymbol{\lambda}(k_l + \mathbf{e}_l(0))| = 0$$

imply that when $t \to \infty$, $\dot{\mathbf{q}}_l \to \boldsymbol{\lambda}(k_l + \mathbf{e}_l(0))$ that is constant, and because $\ddot{\mathbf{q}}_l \in \mathcal{L}_\infty$ the only constant is $|\dot{\mathbf{q}}_l| \to 0$. Hence, $|\mathbf{e}_l| \to 0$. The same analysis applies for $i = r$. This completes the proof. ■

V. SIMULATIONS

In the simulations performed, the local and remote manipulators are modeled as a pair of 2 DOF serial links with revolute joints. Their corresponding nonlinear dynamics are modeled by (1). The inertia matrices $\mathbf{M}_i(\mathbf{q}_i)$ are given by

$$\mathbf{M}_i(\mathbf{q}_i) = \begin{bmatrix} \alpha_i + 2\beta_i \mathrm{c}_{2_i} & \delta_i + \mathrm{c}_{2_i} \\ \delta_i + \beta_i \mathrm{c}_{2_i} & \delta_i \end{bmatrix}.$$

c_{2_i} is the short notation for $\cos(q_{2_i})$. q_{k_i} is the articular position of link k of manipulator i, with $k \in \{1, 2\}$. The Coriolis and centrifugal effects are modeled with the matrices

$$\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) = \begin{bmatrix} -2\beta_i \mathrm{s}_{2_i}\dot{q}_{2_i} & -\beta_i \mathrm{s}_{2_i}\dot{q}_{2_i} \\ \beta_i \mathrm{s}_{2_i}\dot{q}_{1_i} & 0 \end{bmatrix}.$$

s_{2_i} is the short notation for $\sin(q_{2_i})$. $\dot{q}_{1_i}$ and $\dot{q}_{2_i}$ are the respective revolute velocities of the two links. The gravity forces $\mathbf{g}_i(\mathbf{q}_i)$ for each manipulator are represented by

$$\mathbf{g}_i(\mathbf{q}_i) = \begin{bmatrix} \frac{1}{l_{2_i}} g\delta_i \mathrm{c}_{12_i} + \frac{1}{l_{1_i}}(\alpha_i - \delta_i)\mathrm{c}_{1_i} \\ \frac{1}{l_{2_i}} g\delta_i \mathrm{c}_{12_i} \end{bmatrix}.$$

c_{12_i} stands for $\cos(q_{1_i} + q_{2_i})$. $\alpha_i = l_{2_i}^2 m_{2_i} + l_{1_i}^2(m_{1_i} + m_{2_i})$, $\beta_i = l_{1_i} l_{2_i} m_{2_i}$ and $\delta_i = l_{2_i}^2 m_{2_i}$. l_{k_i} and m_{k_i} are the respective lengths and masses of each link.

A. Controller of (Chopra et al. 2008)

As an illustrative example, let us take $\boldsymbol{\lambda} = \mathbf{I}$. Thus, matrices $\mathbf{Y}_i$, for the controllers (3), can be written as

$$\begin{bmatrix} \dot{q}_{1_i} & Y_{21_i} & \dot{q}_{2_i} & -g\mathrm{c}_{12_i} & -g\mathrm{c}_{1_i} \\ 0 & \mathrm{c}_{2_i}\dot{q}_{1_i} + \mathrm{s}_{2_i}\dot{q}_{1_i}q_{1_i} & \dot{q}_{1_i} + \dot{q}_{2_i} & -g\mathrm{c}_{12_i} & 0 \end{bmatrix}$$

where $Y_{21_i} = 2\mathrm{c}_{2_i}\dot{q}_{1_i} + \mathrm{c}_{2_i}\dot{q}_{2_i} - \mathrm{s}_{2_i}\dot{q}_{2_i}q_{2_i} - 2\mathrm{s}_{2_i}\dot{q}_{2_i}q_{1_i}$ and the estimated parameters are given by

$$\hat{\boldsymbol{\theta}}_i^\top = \begin{bmatrix} \hat{\alpha}_i & \hat{\beta}_i & \hat{\delta}_i & \frac{1}{l_{2_i}}\hat{\delta}_i & \frac{1}{l_{1_i}}(\hat{\alpha}_i - \hat{\delta}_i) \end{bmatrix}. \quad (20)$$

The physical parameters for the manipulators are: the length of links l_{1_i} and l_{2_i}, for both manipulators, is 0.38m; the masses for the links are $m_{1_l} = 3.9473$Kg, $m_{2_l} = 0.6232$Kg, $m_{1_r} = 3.2409$Kg and $m_{2_r} = 0.3185$Kg. The initial conditions are $\ddot{\mathbf{q}}_i(0) = \dot{\mathbf{q}}_i(0) = \mathbf{0}$ and $\mathbf{q}_l^\top(0) = [2/5\pi; 1/3\pi]$, $\mathbf{q}_r^\top(0) = [1/6\pi; -1/4\pi]$. The controllers gains are $K = 3$ and $\boldsymbol{\Gamma}_l = 0.5\mathbf{I}$ and $\boldsymbol{\Gamma}_r = 2\mathbf{I}$. The time-delays in both paths is set to 0.4s.

The first simulations, Figs. 1–3, show the system response when the human operator and the environment do not exert any force on the local and remote manipulators. It can be seen that the parameter estimations converge to constant values and positions asymptotically stabilize at zero, as stated by Proposition 2.

The second set of simulations analyzes the time evolution of the system trajectories when the human operator is modeled as a spring-damper system with gains 25 and 5, respectively. The desired trajectory of the human operator is shown in Fig. 4. In this case, Figs. 5 and 6, the parameter estimation term injects opposite forces that prevent the teleoperator to move from the equilibrium $\mathbf{q}_i = \mathbf{0}$. As in the previous simulations one can clearly see that positions asymptotically converge to zero, Fig. 7.

B. Proposed Adaptive Controller

Matrices $\mathbf{Y}_i$, with $\boldsymbol{\lambda} = \mathbf{I}$, are given by

$$-\begin{bmatrix} \dot{e}_{1_i} & Y_{21_i} & \dot{e}_{2_i} & g\mathrm{c}_{12_i} & g\mathrm{c}_{1_i} \\ 0 & \mathrm{c}_{2_i}\dot{e}_{1_i} + \mathrm{s}_{2_i}\dot{q}_{1_i}e_{1_i} & \dot{e}_{1_i} + \dot{e}_{2_i} & g\mathrm{c}_{12_i} & 0 \end{bmatrix}$$

where $Y_{21_i} = 2\mathrm{c}_{2_i}\dot{e}_{1_i} + \mathrm{c}_{2_i}\dot{e}_{2_i} - \mathrm{s}_{2_i}\dot{q}_{2_i}e_{2_i} - 2\mathrm{s}_{2_i}\dot{q}_{2_i}e_{1_i}$ and the estimated parameters follow (20).

The physical parameters, initial conditions and estimation gains are the same as in the previous sets of simulations. The controllers gains, in (19), are $\mathbf{K}_i = 3\mathbf{I}$ and $\mathbf{B} = \mathbf{I}$.

The simulations with the proposed controller, when the human operator and the environment do not exert any force on the local or remote manipulators, are shown in Figs. 8–10. Notice that position errors asymptotically converge to zero but positions converge to values different than zero, *i.e.*, $q_{l_j} = q_{r_j} \neq 0$. Figs. 11–13 show that, when the human exerts forces, the local manipulator moves consequently and the remote follows the corresponding trajectory.

VI. Conclusions

It has been shown that the adaptive controller proposed in (Chopra *et al.*, 2008) suffers from a practical drawback—that it is basically applicable only to planar manipulators, kinematically identical and moving on the horizontal plane. To overcome this obstacle a new adaptive algorithm is proposed. The new algorithm assures that, in free motion, for any constant time delay, all signals in the system are bounded, and position errors and velocities asymptotically converge to zero. The simulations performed confirm these conclusions.

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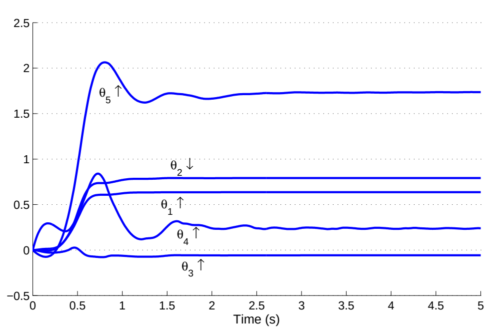

Figura 1. Parameter estimation for the local manipulator when $\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$. Using the scheme of Chopra et al. (2008)

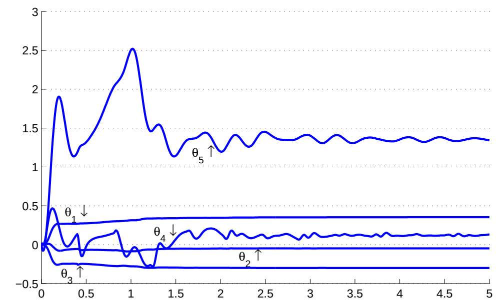

Figura 2. Parameter estimation for the remote manipulator when $\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$. Using the scheme of Chopra et al. (2008)

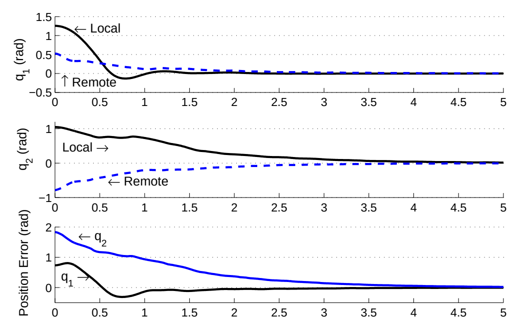

Figura 3. Position of the local and remote manipulators when $\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$. Using the scheme of Chopra et al. (2008)

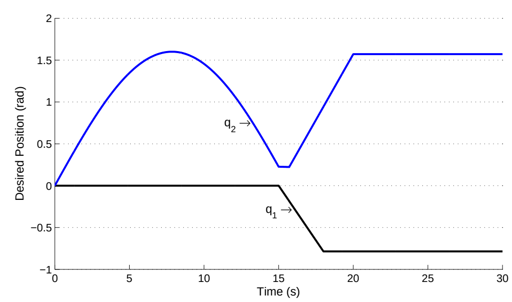

Figura 4. Desired trajectory of the human operator.

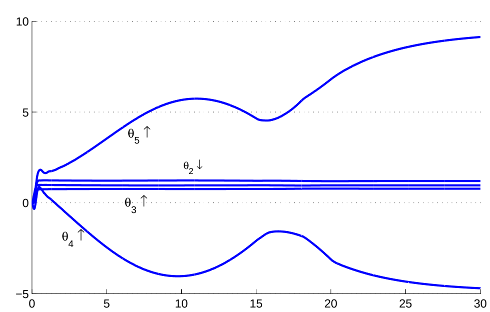

Figura 5. Parameter estimation for the local manipulator. Using the scheme of Chopra et al. (2008)

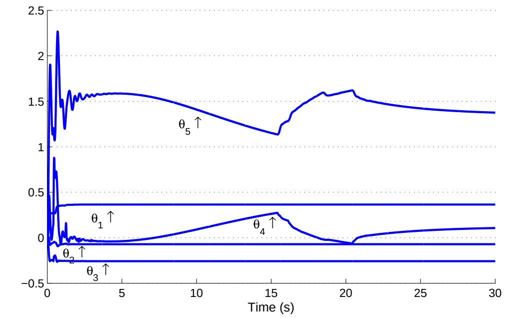

Figura 6. Parameter estimation for the remote manipulator. Using the scheme of Chopra et al. (2008)

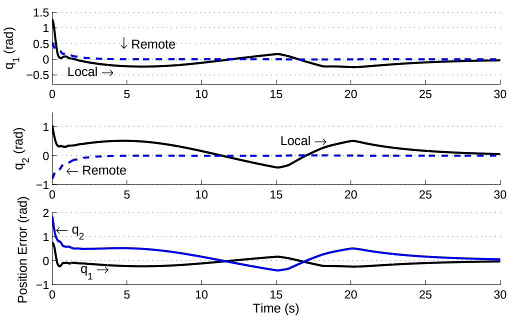

Figura 7. Position of the local and remote manipulators. Using the scheme of Chopra et al. (2008)

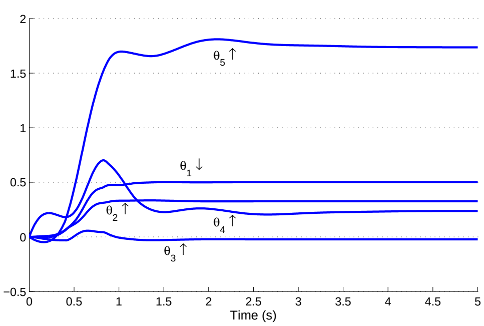

Figura 8. Parameter estimation for the local manipulator when $\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$. Using the proposed controller.

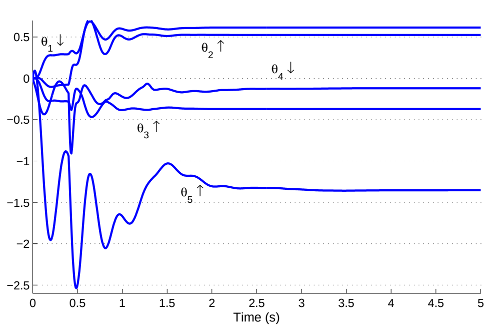

Figura 9. Parameter estimation for the remote manipulator when $\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$. Using the proposed controller.

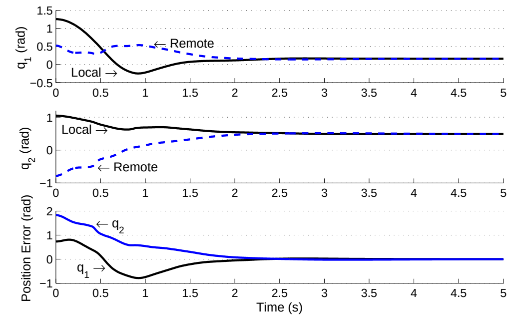

Figura 10. Position of the local and remote manipulators when $\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$. Using the proposed controller.

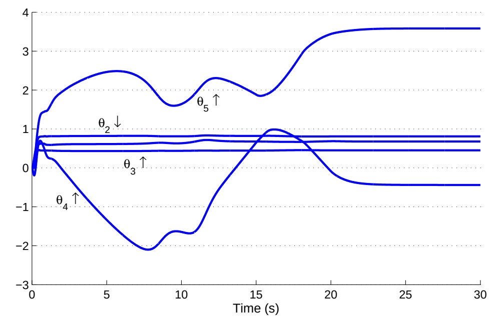

Figura 11. Parameter estimation for the local manipulator. Using the proposed controller.

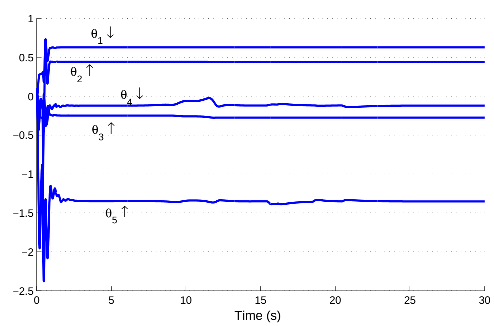

Figura 12. Parameter estimation for the remote manipulator. Using the proposed controller.

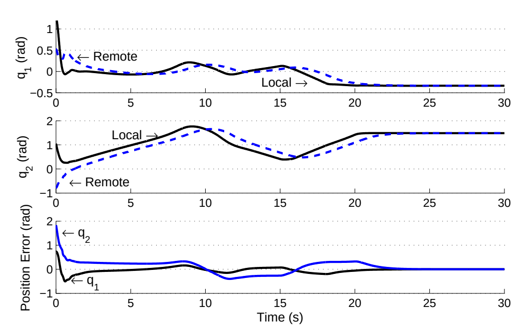

Figura 13. Position of the local and remote manipulators. Using the proposed controller.