Comparison of different control algorithms for pneumatic actuators

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***Abstract*— Air used as power transmission for simple uses or power supply is, as mentioned in [1], known more than two millenniums. For newer applications in the industry, the attractiveness of pneumatic systems, based in the cleanness, low prices, weight to force ratio and the easy assembling are basically the most important matters. For these reasons too it is on the one hand interesting to develop control applications with pneumatic actuators, not only for automation industry but at all in robotic applications too.**

But on the other hand, due to the high no linear nature of pneumatic systems, it is not as easy as for hydraulic systems, which seems comparable. For this reason the investigation in the last years was increasing, but there are still some steps missing to use as good as possible the properties of pneumatic actuators or pneumatic systems.

This paper deals with a valuation of the simulations of some of the most used control techniques like a simple PID of the area linear control, Feedback Linearization from the area nonlinear control and Sliding Mode Control for the area of robust control.

I. INTRODUCTION

Most of the recent papers are using techniques like advanced PID's ([2]), PVA - Controllers ([2], [3]), Exact Linearization ([4], [5]); Flatness based Control ([6]), Sliding Mode Control ([7], [8]), Cascade Control ([9], [10], [11]), Model Reference Adaptive Controller (MRAC) ([12]), Neural Networks ([13]) etc. with growing success to realize the challenge of control pneumatic systems. Due to the strong nonlinearity of the system it is nearly imposible to know well the dynamic model, or to use a model which is reflecting the real behavior of the system in a usable order. In fact some of the above mentioned controllers are in the fine position, that the used model reflects the reality sufficiently exact and that it is in a still usable order.

We won't discuss the differences between all these controllers rather this paper evaluates the simulations of 3 of the common controllers with a focus on its performance on tracking tasks. To get an objective overall idea, an index $I(e,u)$ of error (e) and energy (u) is used. At the end of the paper a conclusion is presented.

The used index $I(e,u)$

$$I(e,u) = \sqrt{\frac{1}{T}\int_0^T (e^2 + u^2)dt}$$

calculated with the trajectory error and the control signal helps to integrate the energy in this contemplation. It is assumed, that the output signal of the controler u is proportional to the necessary energy to control the system. Taking a closer look it is the voltage to control a proportional valve, which is controlling the mass flows m_1 and m_2, which is equivalent to the energy of the system, the compressed aire.

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II. THE DYNAMIC MODEL

The base for all the 3 simulated controllers is the well known and in several papers ([4], [14], [6], [3], [11], [8], [15], [10], [1], [13]) used dynamical model of the pneumatic system,

$$\mathbf{x} = \begin{pmatrix} y \\ \dot{y} \\ p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \quad (1)$$

A simplified block-diagram is shown in figure 1.

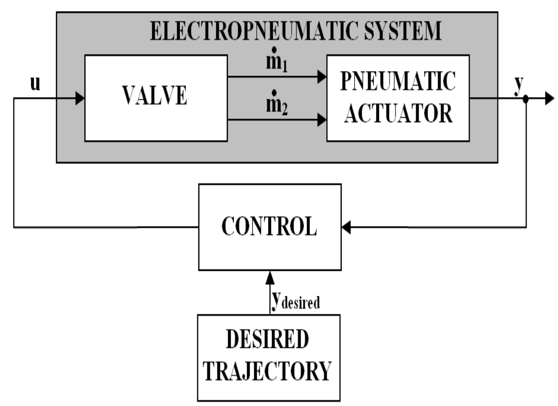

Fig. 1. Block Diagram of the electropneumatic system

Principally based on Newton's second law and the first law of thermodynamics used for an adiabatic processes with the dynamics shown in the following equations.

$$\dot{x}_1 = x_2 \quad (2)$$

$$\dot{x}_2 = \frac{A_1}{M}x_3 - \frac{A_2}{M}x_4 - \frac{A_1 - A_2}{M}p_{atm} - \frac{F_{FRVIS}(x_2)}{M} - \frac{F_{FRNL}(x_2, x_3, x_4)}{M} \quad (3)$$

$$\dot{x}_3 = -\frac{\gamma}{V_{10} + Ax_1}\big(x_2 x_3 A - RT\dot{m}_1(x_3, u)\big) \quad (4)$$

$$\dot{x}_4 = \frac{\gamma}{V_{20} + A(L - x_1)} \left(x_2 x_4 A - RT\dot{m}_2(x_4, u) \right) \quad (5)$$

$$\dot{m}_1 = \rho_0 C_{\max} u \begin{cases} p_s \sqrt{1 - \left(\frac{\left(\frac{x_3}{p_s} - b\right)}{1-b} \right)^2} & , u \geq 0 \\ x_3 & , u \leq 0 \end{cases} \quad (6)$$

$$\dot{m}_2 = -\rho_0 C_{\max} u \begin{cases} p_s \sqrt{1 - \left(\frac{\left(\frac{x_4}{p_s} - b\right)}{1-b} \right)^2} & , u \geq 0 \\ x_4 & , u \leq 0 \end{cases} \quad (7)$$

With the following variables and parameters: $y = x_1$ = displacement (m), $\dot{y} = x_2$ = velocity $(\frac{m}{s})$, $\ddot{y} = \dot{x}_2$ = acceleration $(\frac{m}{s^2})$, $p_1 = x_3$ = pressure chamber 1 (Pa), $p_2 = x_4$ = pressure chamber 2 (Pa), m_1 = mass flow chamber 1 $(\frac{kg}{s})$, m_2 = mass flow chamber 2 $(\frac{kg}{s})$, A_1 = cross section of the piston chamber 1 $(4.9087 * 10^{-4} m^2)$, A_2 = cross section of the piston chamber 2 $(4.9087 * 10^{-4} m^2)$, M = mass $(5.8kg)$, p_{atm} = atmospheric pressure $(1 * 10^5 Pa)$, F_{FRVIS} = Coulomb friction force $(B = 240)$, F_{FRNL} = no linear friction forces (not considered), γ = adiabatic index (1.4), V_{10} = dead zone chamber 1 including tubes $(1.5 * 10^{-5} m^3)$, V_{20} = dead zone chamber 2 including tubes $(1.5 * 10^{-5} m^3)$, L = length of piston rod $(0.6m)$, R = gas constant of aire $(287.05 \frac{J}{kg * K})$, T = temperature $(293.15K)$, u = control signal, ρ_0 = density $(1.204 \frac{kg}{m^3})$, C_{max} = mass flow constant (1), p_s = supply pressure (Pa), b = critical pressure ratio (0.582)

III. Simulations

For the simulation are selected 3 of the most used controllers of the following areas of control: linear control, non linear control and robust control.

The simulations are basically all programmed in the structure of figure 2.

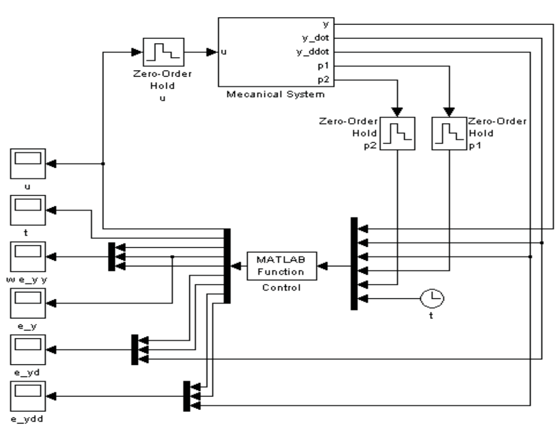

Fig. 2. Structure of Simulations in Simulink

The desired trajectory $y_d, \dot{y}_d, \ddot{y}_d$ is in all the 3 cases a sine wave with the frequency of $1Hz$, the amplitude of $0.15m$ and a bias of $0.25m$. The piston is in all the 3 simulations in it's home position $0.25m$.

A. PID Controller

The first simulation is a simple PID with the controller structure:

$$u(t) = K_p * e_y(t) + K_i * \int_0^t e_y(\tau) d\tau + K_d \frac{de_y(t)}{dt} \quad (8)$$

with $e_y = y_{desired} - y_{real}$ and the manually tuned controller parameters $K_p = 0.0000025$ $K_i = 0.00000018$ $K_d = 0.00000025$.

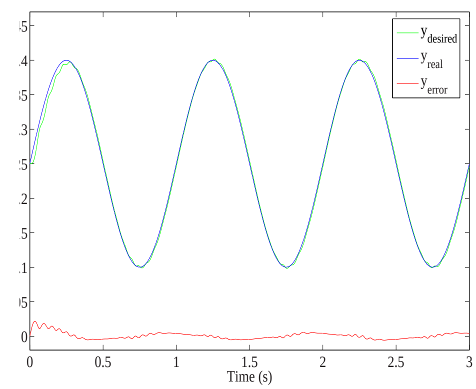

Fig. 3. PID: Desired trajectory (w), real trajectory (y) and error (e_y)

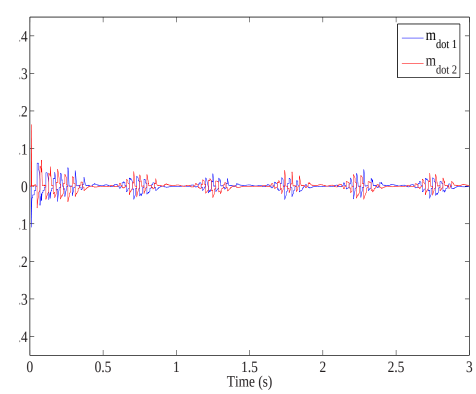

Fig. 4. PID: Mass flows m_1 and m_2

The figure 3 shows the curves of the desired trajectory, the real trajectory and the resulting error. The error e_y has the maximum value of $5mm$. As we can see in the figure 4 and 5, the controller is controlling really chaotically the mass flows and with a result of this, the pressures p_1 and p_2 are chaotic too. With an Index $I(e, u)$ of $5.0308 * 10^{-}3$ the controller has the highes index, what we will discuss later in Conclusions. The last figure (Figure 7) shows the control signal u.

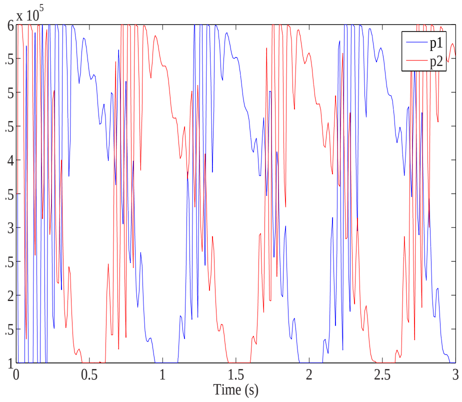

Fig. 5. PID: Pressures p_1 and p_2

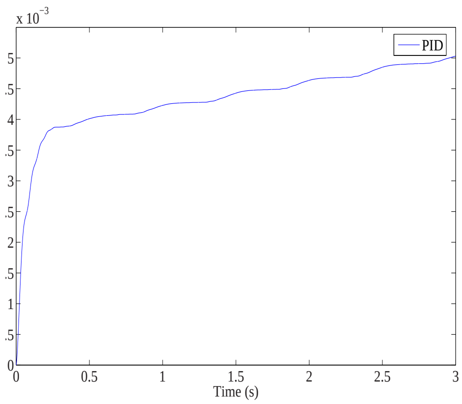

Fig. 6. PID: Index $I(e,u)$

B. Exact Linearization Controller

The next simulated controller is a controller based on exact linearization. This controller was developed in varios papaers ([4] and [5] to mention only two of them).

Using the dynamic model of the system, the controller results in the next equations, following the law of control

$$u = \frac{v - \alpha(z)}{\beta(z)} \tag{9}$$

and the equations for v, α and β. The general idea of exact linearization is to compense, and in the best case eliminate, the non linear terms of the system by it's law of control u. Differentiate equation (3) results

$$\dot{x}_2 = \frac{A1}{M}\dot{x}_3 - \frac{A2}{M}\dot{x}_4 - \frac{1}{M}\dot{F}_{FRVIS} - \frac{1}{M}\dot{F}_{FRNL} \tag{10}$$

Replacing $\dot{x}_3$ with equation (4), $\dot{x}_4$ with equation (5) and separating for terms with u and without u we get

$$\dddot{y} = \alpha(x_1,x_2,x_3,x_4) + \beta(x_1,x_3,x_4)u \tag{11}$$

with

$$\alpha(x_1,x_2,x_3,x_4) = -\frac{A1}{M}\frac{\gamma x_3 x_2}{L_{01}+x_1} - \frac{A2}{M}\frac{\gamma x_4 x_2}{L+L_{02}-x_1} - \frac{1}{M}\dot{F}_{FRVIS} - \frac{1}{M}\dot{F}_{FRNL} \tag{12}$$

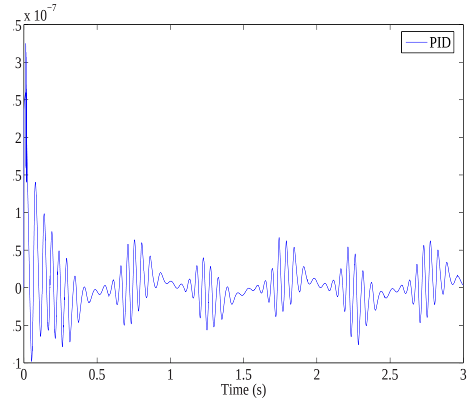

Fig. 7. PID: Control Signal u

$$\beta(x_1,x_3,x_4)u = \left(\frac{A_1}{M}b_3 + \frac{A_2}{M}b_4\right)u \tag{13}$$

and the terms b_3 and b_4

$$b_3 = \frac{\gamma T R \rho_0 C_{\max}}{A_1(L_{01}+x_1)} \begin{cases} p_v\sqrt{1-\left(\frac{\left(\frac{x_3}{p_v}-b\right)}{1-b}\right)^2} & , u \geq 0 \\ x_3 & , u \leq 0 \end{cases} \tag{14}$$

$$b_4 = -\frac{\gamma T R \rho_0 C_{\max}}{A_2(L_{02}+x_1)} \begin{cases} p_v\sqrt{1-\left(\frac{\left(\frac{x_4}{p_v}-b\right)}{1-b}\right)^2} & , u \geq 0 \\ x_4 & , u \leq 0 \end{cases} \tag{15}$$

v will take the form

$$v = \dddot{w} + k_a(\ddot{e}_y) + k_v(\dot{e}_y) + k_y(e_y) \tag{16}$$

As we can see, substituting (9), (12), (13) including (14) / (15) and (16) in (11) our control law (u) is canceling the non linear terms α and β.

The controller parameters (k_a, k_v, k_y) of equation (16) are calculated for poles in -6, -100 -100. The parameters are $k_a = 206$, $k_v = 11200$ and $k_y = 60000$.

In the figures from 8 to 12, we can observe the good response of the controller, with a small error e_y of $1.5 * 10^-3m$ and really smooth control movements, see behavior of mass flows (Figure 9) and pressures (Figure 10). The index $I(e,u)$ reaches a value of $2.5363 * 10^-3$.

C. Sliding Mode Controller

The last simulated controller of this paper is a Sliding Mode Controller. Sliding Mode is recently used in some papers for controlling tasks of force controlling and, like in this case, for trajectory controlling. This controller is published in the paper [2].

The following sliding surface is proposed:

$$S = \dot{x}_2 - \ddot{y}_d + 2\lambda(x_2 - \dot{y}_d) + \lambda^2(x_1 - y_d) \tag{17}$$

As mentioned in the paper and as well known too, the sliding surface has the function to keep the state of the system on it in a "sliding mode" after the state has reached at the surface.

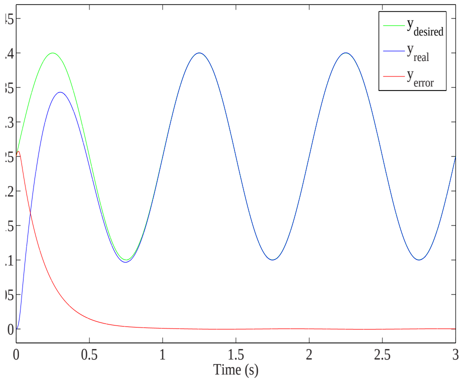

Fig. 8. EX LIN: Desired trajectory (w), real trajectory (y) and error (e_y)

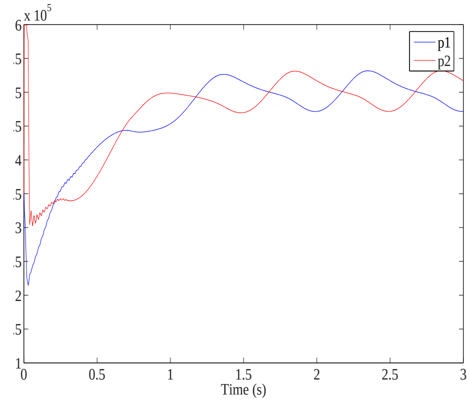

Fig. 10. EX LIN: Pressures p_1 and p_2

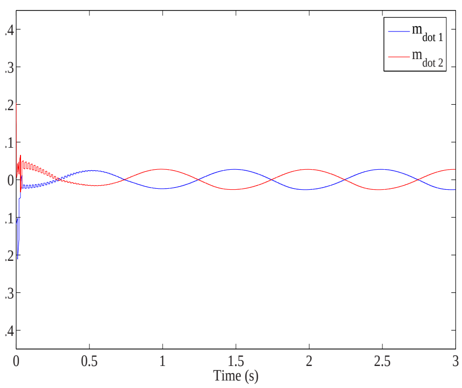

Fig. 9. EX LIN: Mass flows m_1 and m_2

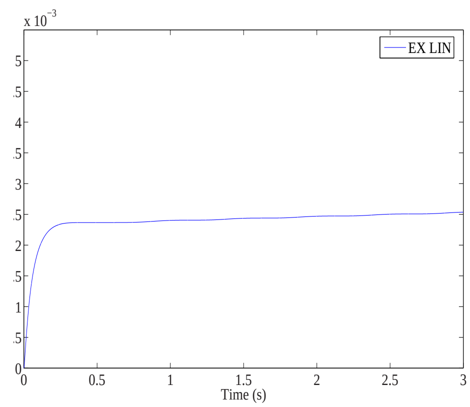

Fig. 11. EX LIN: Index $I(e,u)$

To realize that the state reaches and stay there (bounded by the range of an defined layer) at the sliding surface, 2 control signals u_{eq} and u_s are used, which are added to get the final u.

$$u = u_{eq} + u_s \tag{18}$$

with the switching control signal

$$u_s = -k_s sat(\frac{S}{\phi}) \tag{19}$$

where

$$sat(\frac{S}{\phi}) = \begin{cases} \frac{S}{\phi} & |\frac{S}{\phi}| \leq 1 \\ sign(\frac{S}{\phi}) & |\frac{S}{\phi}| > 1 \end{cases} \tag{20}$$

and the equivalent control signal

$$u_{eq} = \frac{1}{n_0}[\ddot{y}_d + d_2\dot{x}_2 + d_1x_2 - 2\lambda(\dot{x}_2 - \ddot{y}_d) - \lambda^2(x_2 - \dot{y}_d)] \tag{21}$$

To obtain the u_{eq} the equation (17) is derivated, setted to $\dot{S} = 0$ and solved for u. The resulting $\dddot{y} = \ddot{y}_2$ is substituted by

$$\ddot{y}_2 = -d_2\dot{y}_2 - d_1y_2 + n_0u \tag{22}$$

which is the nominal plant of the model.

The parameters $n_0 = 926.8$, $d_1 = 405.8$ and $d_2 = 17.27$ are taken from [2]

The controller parameters are manually tuned to $\lambda = 150$, $k_s = 25$ and $\phi = 4$. To have comparable signals of u, the signal is saturated from $-1 * 10^{-7}$ to $3.5 * 10^7$. Without saturation, the signal u reaches huge values and the Index $I(e,u)$ will not be usable for comparisons. This saturation doesn't effect the magnitude of the error which is $e_y = 4 * 10^-4m$.

As you can see in the figures from 13 to 17, the controller is working well, but with the disadvantages of Sliding Mode Control, for example the chaotic mass flows m_1 and m_2, based on the fast changing control signal u (chattering). The index $I(e,u)$ reaches a value of $1.8213 * 10^-3$ and seems to be the most efficient of the 3 compared controllers.

IV. CONCLUSIONS

Of the results we can conclude various points. The PID is able to control the system in simulation to the small error of $5.7 * 10^-3m$. The was reason enough for the last years

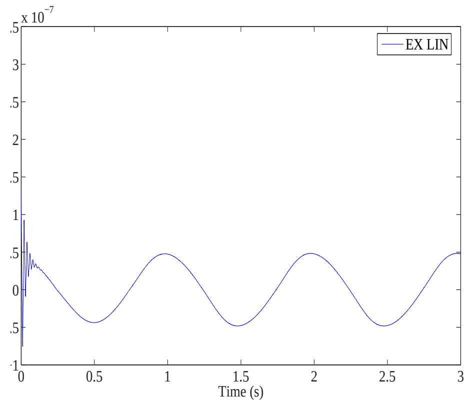

Fig. 12. EX LIN: Control Signal u

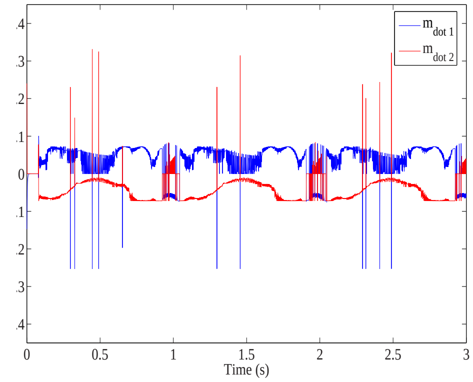

Fig. 14. SMC: Mass flows m_1 and m_2

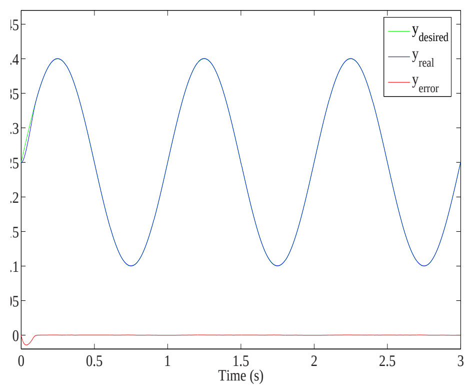

Fig. 13. SMC: Desired trajectory (w), real trajectory (y) and error (e_y)

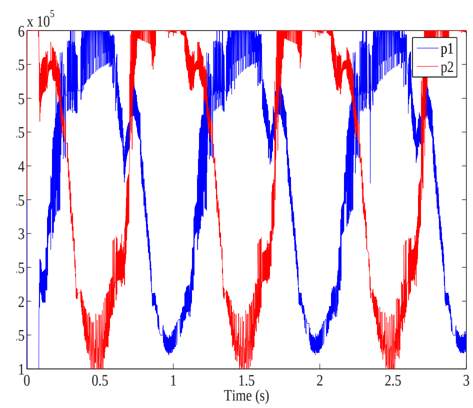

Fig. 15. SMC: Pressures p_1 and p_2

to keep using the PID controller as one of the most used controllers in the industry for trajectory tasks. But with faster signal he reaches it's limits. His response of the control signal to the error is quite good. In few words, on the one hand the controller doesn't save energy in form of compressed aire, but on the other hand it neither doesn't destroy the energy with unnecessary movements. The controller based on exact linearization is working very well, reaches a really good value for the error $e_y = 1.5 * 10^-3m$ and a small index $I(e,u)$ of $2.5363 * 10^-3$ too. The matter of this method is that is necessary to know very well the dynamic model of the system to be able to cancel all non linearities. In the simulation the values are really good, in reality it will be complicated to get these values or to get a good approximation, because of uncertainties in the model, caused of friction etc. But keeping an eye on the smooth signal of control u, this is a very good controller to save the live of equipment, because it is not demanding hard changing's with steep ascent's of signals and of the system, what leads to small energy amounts too . Another problem may be the internal dynamic's of the system. Including more complexly structures like the dynamics of the valve, tubes etc. it may be difficult or nearly impossible to handle the model for an online control process. The Sliding Mode Control shows it's capacity of the group robust controllers and minimizes the error e_y to the lowest value of $e_y = 4*10^-4m$ and the lowest $I(e,u) = 1.8213 * 10^-3$ too. Observing the control signal u the controller shows it's characteristic switching/chattering, what it makes posible to use less expensive on/off high speed valves instead of the expensive proportional valves. Another point is, that it is not necessary to know well the systems dynamics. This helps this kind of controller to get fast and good results, at least in simulations.

In further investigations open topics will be investigated and implemented to a experimental system.

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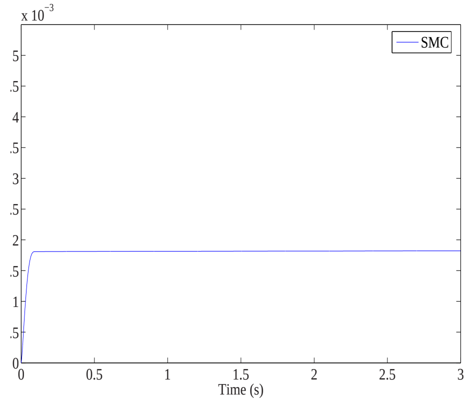

Fig. 16. SMC: Index $I(e, u)$

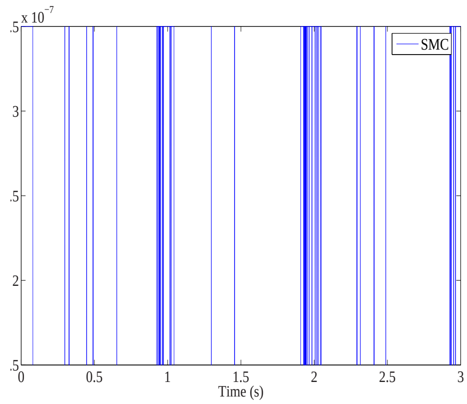

Fig. 17. SMC: Control Signal u

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