Designing Type-1 Fuzzy Logic Controllers via Fuzzy Lyapunov Synthesis for Nonsmooth Mechanical Systems

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***Abstract*— In this paper Fuzzy Lyapunov Synthesis is extended to the design of Type-1 Fuzzy Logic Controllers for nonsmooth mechanical systems. The output regulation problem for a servomechanism with nonlinear backlash is proposed as a case of study. The problem at hand is to design a feedback controller so as to obtain the closed-loop system in which all trajectories are bounded and the load of the driver is regulated to a desired position while also attenuating the influence of external disturbances. The servomotor position is the only measurement available for feedback; the proposed extension is far from trivial because of the nonminimum phase properties of the system. Performance issues of the Type-1 Fuzzy Logic Regulators that were designed are illustrated in a simulation study.**

Keywords: Fuzzy Control, Fuzzy Lyapunov Synthesis, Stability, Nonsmooth systems.

I. Introduction

A major problem in control engineering is a robust feedback design that asymptotically stabilizes a plant while also attenuating the influence of parameter variations and external disturbances. In the last decade, this problem was heavily studied and considerable research efforts have resulted in the development of systematic design methodologies for nonlinear feedback systems. A survey of these methods, fundamental in this respect is given in (Isidori, 2000).

The design of Fuzzy Logic Systems (FLSs) is a heavy task that FLSs practitioners face every time that they try to use Fuzzy Logic (FL) as a solution to some problem, the design of FLSs implies at least two stages: design of the rule-base and design of the Membership Functions (MFs).

There has been some publications in the design of Type-1 Fuzzy Logic Systems (T1FLS), for example (Grefenstette, 1986) presents Genetic Algorithms (GA) as an optimization method for control parameters, they optimize parameters of the closed-loop system but not of the T1FLS. In (Lee and Takagi, 1993) GAs are used to optimize all the parameters of a T1FLS. In (Melin and Castillo, 2001) a hybridizing of Neural Networks and GAs are presented to optimize a T1FLSs. A Hierarchical Genetic Algorithms is proposed in (Castillo *et al.*, 2004) to optimize rules and MFs parameters of a T1FLS.

In the present paper the output regulation problem is studied for an electrical actuator consisting of a motor part driven by a DC motor and a reducer part (load) operating under uncertainty conditions in the presence of nonlinear backlash effects. The objective is to drive the load to a desired position while providing the roundedness of the system motion and attenuating external disturbances. Because of practical requirements (see e.g., (Lagerberg and Egardt, 1999)), the motor's angular position is assumed to be the only information available for feedback.

This problem was first reported in (Aguilar *et al.*, 2007), where the problem of controlling nonminimum phase systems was solved by using nonlinear H_∞ control, but the reported results do not provide robustness evidence. In (Cazarez-Castro *et al.*, 2008*a*), authors report a solution to the regulation problem using a T1FLS. In (Cazarez-Castro *et al.*, 2008*c*) and (Cazarez-Castro *et al.*, 2008*b*), authors report solutions using Type-2 Fuzzy Logic Systems Controllers, and making a genetic optimization of the membership function's parameters, but do not specify the criteria used in the optimization process and GA design. In (Cazarez-Castro *et al.*, 2008*d*), a comparison of the use of GAs to optimize Type-1 and Type-2 FLS Controllers is reported, but a method to achieve this optimization is not provided.

The solution that we propose is to design Type-1 FLSs (Fuzzy Logic Controllers -FLCs-) extending the Fuzzy Lyapunov Synthesis (Margaliot and Langholz, 1998), a concept that is based on the Computing with Words (Mendel, 2007)(Zadeh, 1996) approach of the Lyapunov Synthesis (Khalil, 2002), this approach was reported in (Castillo *et al.*, 2006) and (Castillo *et al.*, 2008) for the design of stable FLCs.

The contributions of this paper are as follows:

- We propose a systematic methodology to design Type-1 FLCs via the Fuzzy Lyapunov Synthesis.
- With the proposed methodology we obtain Type-1 FLCs, that due to the nature of designing method, are stable.
- We solve the output regulation problem for an electrical actuator operating under uncertainty conditions in the presence of nonlinear backlash effects.
- We show via simulations, that the resulting FLSs are so robust that they can deal with the proposed problem.

The paper is organized as follows. The dynamic model of the nonminimum phase servomechanism with nonlinear backlash an the problem statement are presented in Sections II and III respectively. Section IV addresses fuzzy sets and systems theory. The design of Fuzzy Logic Controllers using the Fuzzy Lyapunov Synthesis is presented in Section V. The numerical simulations for the designed FLSs are presented in Section VI. Conclusions are presented in Section VII.

II. Dynamic model

The dynamic model of the angular position $q_i(t)$ of the DC motor and the $q_o(t)$ of the load are given according to

$$\begin{array}{rcl} J_0 N^{-1}\ddot{q}_0 + f_0 N^{-1}\dot{q}_0 & = & T + w_0 \\ J_i\ddot{q}_i + f_i\dot{q}_i + T & = & \tau_m + w_i, \end{array} \tag{1}$$

hereafter, J_0, f_0, $\ddot{q}_0$ and $\dot{q}_0$ are, respectively, the inertia of the load and the reducer, the viscous output friction, the output acceleration, and the output velocity. The inertia of the motor, the viscous motor friction, the motor acceleration, and the motor velocity are denoted by J_i, f_i, $\ddot{q}_i$ and $\dot{q}_i$, respectively. The input torque τ_m serves as a control action, and T stands for the transmitted torque. The external disturbances $w_i(t)$, $w_0(t)$ have been introduced into the driver equation (1) to account for destabilizing model discrepancies due to hard-to-model nonlinear phenomena, such as friction and backlash.

The transmitted torque T through a backlash with an amplitude j is typically modeled by a dead-zone characteristic (Nordin *et al.*, 2001, p. 7):

$$T(\Delta q) = \begin{cases} 0 & |\Delta q| \le j \\ K\Delta q - Kj\text{sign}(\Delta q) & \text{otherwise} \end{cases} \tag{2}$$

with

$$\Delta q = q_i - N q_0, \tag{3}$$

where K is the stiffness, and N is the reducer ratio. Such a model is depicted in Fig. 1. Provided the servomotor position $q_i(t)$ is the only available measurement on the system, the above model (1)–(3) appears to be non-minimum phase because along with the origin the unforced system possesses

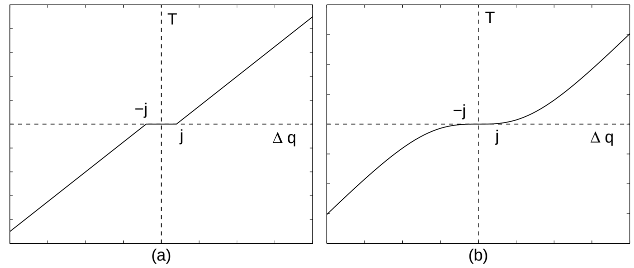

Fig. 1. Dead-zone model of backlash and its monotonic approximation

a multivalued set of equilibria (q_i, q_0) with $q_i = 0$ and $q_0 \in [-j, j]$.

To avoid dealing with a non-minimum phase system, we replace the backlash model (2) with its monotonic approximation:

$$T = K\Delta q - K\eta(\Delta q) \tag{4}$$

where

$$\eta = -2j\frac{1 - \exp\left\{-\frac{\Delta q}{j}\right\}}{1 + \exp\left\{-\frac{\Delta q}{j}\right\}}. \tag{5}$$

The present backlash approximation is inspired from (Merzouki *et al.*, 2004). Coupled to the drive system (1) subject to motor position measurements, it is subsequently shown to continue a minimum phase approximation of the underlying servomotor, operating under uncertainties $w_i(t)$, $w_0(t)$ to be attenuated. As a matter of fact, these uncertainties involve discrepancies between the physical backlash model (2) and its approximation (4) and (5).

III. Problem Statement

To formally state the problem, let us introduce the state deviation vector $x = [x_1, x_2, x_3, x_4]^T$ with

$$\begin{aligned} x_1 &= q_0 - q_d, \\ x_2 &= \dot{q}_0, \\ x_3 &= q_i - N q_d, \\ x_4 &= \dot{q}_i, \end{aligned}$$

where x_1 is the load position error, x_2 is the load velocity, x_3 is the motor position deviation from its nominal value, and x_4 is the motor velocity. The nominal motor position Nq_d has been pre-specified in such a way to guarantee that $\Delta q = \Delta x$, where

$$\Delta x = x_3 - N x_1.$$

Then, system (1)–(5), represented in terms of the deviation vector x, takes the form

$$\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= J_0^{-1}[KNx_3 - KN^2x_1 - f_0x_2 + KN\eta(\Delta q) + w_o], \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= J_i^{-1}[\tau_m + KNx_1 - Kx_3 - f_ix_4 + K\eta(\Delta q) + w_i].
\end{aligned} \tag{6}$$

The zero dynamics

$$\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= J_0^{-1}[-KN^2x_1 - f_0x_2 + KN\eta(-Nx_1)],
\end{aligned} \tag{7}$$

of the undisturbed version of system (6) with respect to the output

$$y = x_3 \tag{8}$$

is formally obtained (see (Isidori, 1995) for details) by specifying the control law that maintains the output identically to zero.

The objective of the Fuzzy Control output regulation of the nonlinear driver system (1) with backlash (4) and (5), is thus to design a Fuzzy Controller so as to obtain the closed-loop system in which all these trajectories are bounded and the output $q_0(t)$ asymptotically decays to a desired position q_d as $t \to \infty$ while also attenuating the influence of the external disturbances $w_i(t)$ and $w_0(t)$.

IV. Type-1 Fuzzy Sets and Systems

A Type-1 Fuzzy Set (T1FS), denoted A is characterized by a Type-1 membership function (T1MF) $\mu_A(z)$ (Castillo and Melin, 2008), where $z \in Z$, and Z is the domain of definition of the variable, i.e,

$$A = \{(z, \mu(z)) | \forall z \in Z\} \tag{9}$$

where $\mu(z)$ is called a Type-1 *membership function* of the T1FS A. The T1MF maps each element of Z to a membership grade (or membership value) between 0 and 1.

Type-1 Fuzzy Logic Systems (T1FLS) - also called Type-1 Fuzzy Inference Systems (T1FIS)-, are both intuitive and numerical systems that map crisp inputs into a crisp output. Every T1FIS is associated with a set of rules with meaningful linguistic interpretations, such as:

$$R^l : \text{IF } y \text{ is } A_1^l \text{ AND } \dot{y} \text{ is } A_2^l \text{ THEN } u \text{ is } B_1^l, \tag{10}$$

which can be obtained either from numerical data, or from experts familiar with the problem at hand. In particular

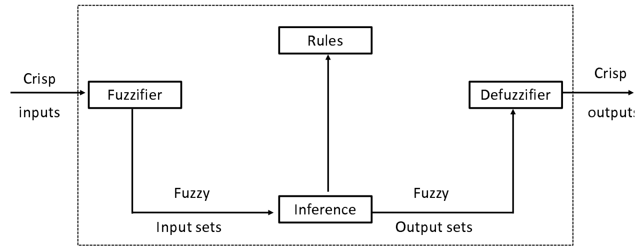

Fig. 2. Type-1 Fuzzy Inference System

(10) is in the form of Mamdami fuzzy rules (Mamdani and Assilian, 1975)–(Mamdani, 1976). Based on this kind of statements, actions are combined with rules in an antecedent/consequent format, and then aggregated according to approximate reasoning theory to produce a nonlinear mapping from input space $U = U_1 \times U_2 \times \cdots \times U_n$ to output space W, where $A_k^l \subset U_k$, $k = 1, 2, \ldots, n$, and the output linguistic variable is denoted by τ_m.

A T1FIS consists of four basic elements (see Fig. 2): the ***Type-1 fuzzifier***, the ***Type-1 fuzzy rule-base***, the ***Type-1 inference engine***, and the ***Type-1 defuzzifier***. The ***Type-1 fuzzy rule-base*** is a collection of rules in the form of (10), which are combined in the ***Type-1 inference engine***, to produce a fuzzy output. The ***Type-1 fuzzifier*** maps the crisp input into a T1FS, which are subsequently used as inputs to the ***Type-1 inference engine***, whereas the ***Type-1 defuzzifier*** maps the T1FSs produced by the ***Type-1 inference engine*** into crisp numbers.

In this paper, to get the crisp output of Fig. 2, we compute a Centroid of Area (COA) (Castillo and Melin, 2008) as a *Type-1 defuzzifier*. The COA is defined as follows:

$$\tau_m = u_{COA} = \frac{\int_u \mu_A(u) u du}{\int_u \mu_A(u) du} \tag{11}$$

where $\mu_A(u)$ is the aggregated output T1MF. This is the most widely adopted defuzzification strategy, which is reminiscent of the calculation of expected values of probability distributions.

V. Design of the Fuzzy Logic Controllers

To apply the Fuzzy Lyapunov Synthesis, we assume the following:

1. The system may have really two degrees of freedom referred to as x_1 and x_2, respectively. Hence by (6), $\dot{x}_1 = x_2$.
2. The states x_1 and x_2 are the only measurable variables.
3. The exact equations (1)–(5) are not necessarily known.

4. The angular acceleration $\dot{x}_2$ is proportional to τ_m, that is, when τ_m increases (decreases) $\dot{x}_2$ increases (decreases).
5. The initial conditions $x(0) = (x_1(0), x_2(0))^T$ belong to the set $\aleph = \{x \in \mathbf{R}^2 : \|x - x^*\| \leq \varepsilon\}$ where x^* is the equilibrium point.

The *control objective* is to design the rule-base as a fuzzy controller $\tau_m = \tau_m(x_1, x_2)$ to stabilize the system (1)–(5).

Theorem 1 that follows establish conditions that help in the design of the fuzzy controller to ensure asymptotic stability. The proof can be found in (Khalil, 2002).

Theorem 1 (Asymptotic stability (Khalil, 2002)): Consider the nonlinear system (1)–(5) with an equilibrium point at the origin, i.e., f(0)=(0), and let $x \in \aleph$, then the origin is asymptotically stable if there exists a scalar Lyapunov function $V(x)$ with continuous partial derivatives such that

- $V(x)$ is positive definite
- $\dot{V}(x)$ is negative definite.

The fuzzy controller design proceeds as follows. Let us introduce the Lyapunov function candidate

$$V(x_1, x_2) = \frac{1}{2}\left(x_1^2 + x_2^2\right), \tag{12}$$

which is positive-definite and radially unbounded function. The time derivative of $V(x_1, x_2)$ results in:

$$\dot{V}(x_1, x_2) = x_1\dot{x}_1 + x_2\dot{x}_2 = x_1x_2 + x_2\dot{x}_2. \tag{13}$$

To guarantee stability of the equilibrium point $(x_1^*, x_2^*)^T = (0, 0)^T$ we wish to have:

$$x_1x_2 + x_2\dot{x}_2 \leq 0. \tag{14}$$

We can now derive sufficient conditions so that inequality (14) holds: If x_1 and x_2 have opposite signs, then $x_1x_2 < 0$ and (14) will hold if $\dot{x}_2 = 0$; if x_1 and x_2 are both positive, then (14) will hold if $\dot{x}_2 < -x_1$; if x_1 and x_2 are both negative, then (14) will hold if $\dot{x}_2 > -x_1$.

We can translate these conditions into the following fuzzy rules:

- If x_1 is *positive* and x_2 is *positive* then $\dot{x}_2$ must be *negative big*.
- If x_1 is *negative* and x_2 is *negative* then $\dot{x}_2$ must be *positive big*.
- If x_1 is *positive* and x_2 is *negative* then $\dot{x}_2$ must be *zero*.
- If x_1 is *negative* and x_2 is *positive* then $\dot{x}_2$ must be *zero*.

However, using our knowledge that $\dot{x}_2$ is proportional to u, we can replace each $\dot{x}_2$ with u to obtain the following fuzzy rule-base for the stabilizing controller:

TABLE I

FUZZY IF-THEN RULES

No.	error	change of error	control
1	positive	positive	negative big
2	negative	negative	positive big
3	positive	negative	zero
4	negative	positive	zero

- If x_1 is *positive* and x_2 is *positive* then u must be *negative big*.
- If x_1 is *negative* and x_2 is *negative* then u must be *positive big*.
- If x_1 is *positive* and x_2 is *negative* then u must be *zero*.
- If x_1 is *negative* and x_2 is *positive* then u must be *zero*.

This fuzzy rule-base can be represented as in Table I.

It is interesting to note that the fuzzy partitions for x_1, x_2, and u follow elegantly from expression (13). Because $\dot{V} = x_2(x_1 + \dot{x}_2)$, and since we require that $\dot{V}$ be negative, it is natural to examine the signs of x_1 and x_2; hence, the obvious fuzzy partition is *positive*, *negative*. The partition for $\dot{x}_2$, namely *negative big*, *zero*, *positive big* is obtained similarly when we plug the linguistic values *positive*, *negative* for x_1 and x_2 in (13). To ensure that $\dot{x}_2 < -x_1$ ($\dot{x}_2 > -x_1$) is satisfied even though we do not know x_1's exact magnitude, only that it is *positive* (*negative*), we must set $\dot{x}_2$ to *negative big* (*positive big*). Obviously, it is also possible to start with a given, predefined, partition for the variables and then plug each value in the expression for $\dot{V}$ to find the rules. Nevertheless, regardless of what comes first, we see that fuzzy Lyapunov synthesis transforms classical Lyapunov synthesis from the world of exact mathematical quantities to the world of computing with words (Zadeh, 1996), (Mendel, 2007).

To complete the controller's design, we must model the linguistic terms in the rule-base using fuzzy membership functions and determine an inference method. Following (Castillo *et al.*, 2008), we characterize the linguistic terms *positive*, *negative*, *negative big*, *zero* and *positive big*. The T1MFs are depicted in Fig. 3. To this end, we had systematically designed a FLC following the Lyapunov stability criterion.

VI. Simulation Results

To perform simulations we use the dynamical model (1)–(5) of the experimental testbed installed in the Robotics & Control Laboratory of CITEDI-IPN (see Fig. 4), which involves a DC motor linked to a mechanical load through an imperfect contact gear train (Aguilar *et al.*, 2007). The parameters of the dynamical model (1)–(5) are given in Table II, while $N = 3$, $j = 0.2$ [rad], and $K = 5$ [N-m/rad]. These paremeters are taken from the experimental testbed.

Performing a simulation of the closed-loop system (1)–(4) with the Type-1 FLC designed in Section V we have

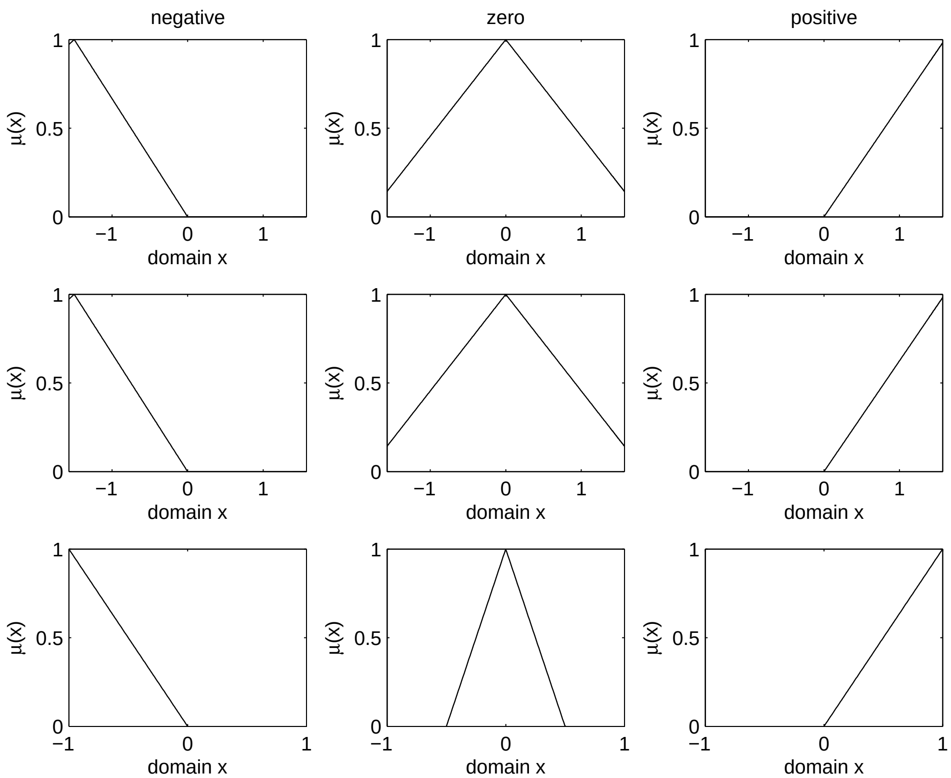

Fig. 3. Set of type-1 membership functions.

TABLE II
NOMINAL PARAMETERS.

Description	Notation	Value	Units
Motor inertia	J_i	$2{,}8 \times 10^{-6}$	Kg-m^2
Load inertia	J_o	1.07	Kg-m^2
Motor viscous friction	f_i	$7{,}6 \times 10^{-7}$	N-m-s/rad
Load viscous friction	f_o	1.73	N-m-s/rad

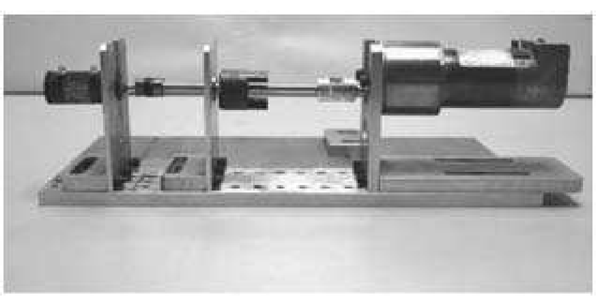

Fig. 4. Experimental testbed

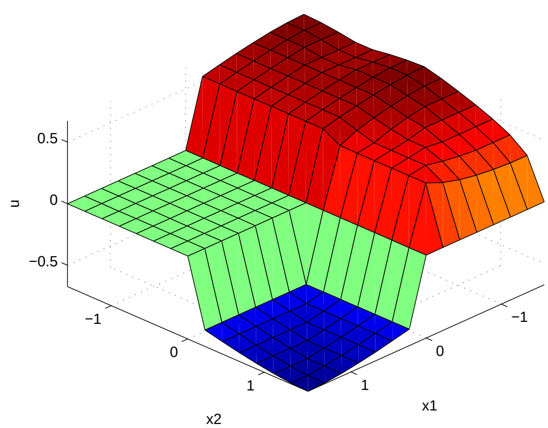

Fig. 5. Control surface for the T1FLC.

the following results: the control surface of Fig. 5 and we obtain the system's response of Fig. 6 showing that the load reaches the desired position, although we only have feedback from the motor position, x_1 and x_2 trajectories are in Fig. 7, in which can seen that $q_0 \to q_d$ while $x_0 \to 0$. In Fig. 8 the trajectories for (12) and (13) are depicted, satisfying Theorem 1.

VII. CONCLUSION

The main goal of this paper was to propose a systematic methodology to design T1FLCs to solve the output regulation problem of a servomechanism with nonlinear backlash. The proposed design strategy results in two controllers that guarantee that the load reaches the desired position q_d. The regulation problem was solved as was predicted, this affirmation is supported with simulations where the

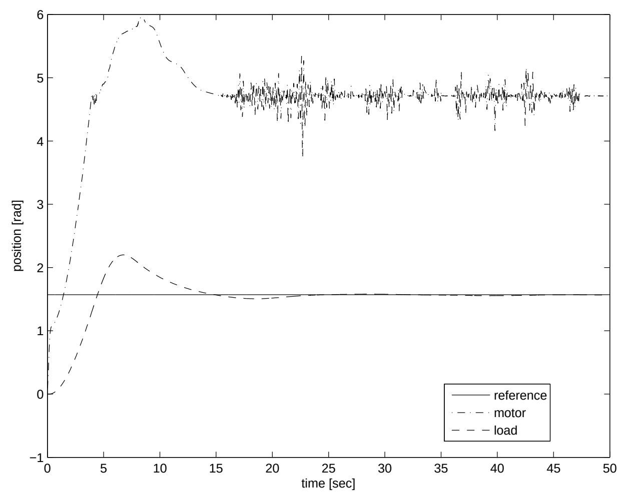

Fig. 6. System's response for the T1FLC.

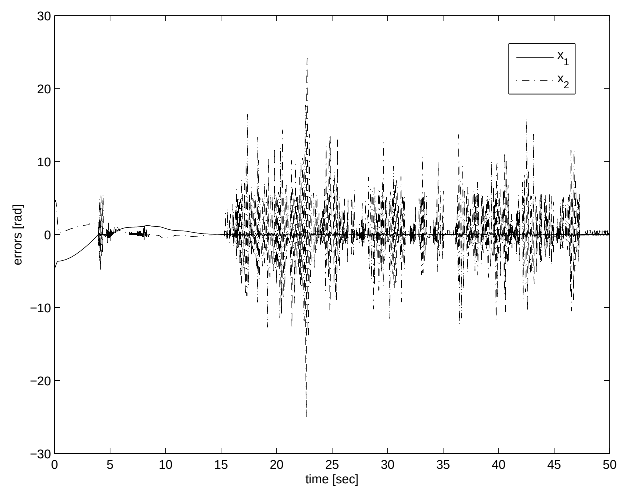

Fig. 7. x_1 and x_2 trajectories for the T1FLC.

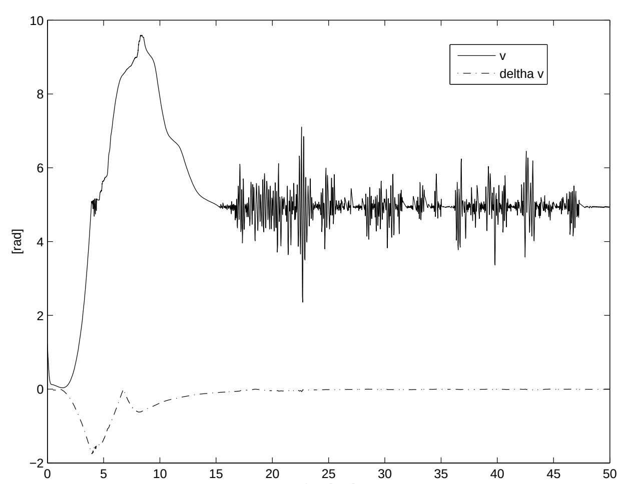

Fig. 8. V and $\dot{V}$ trajectories for the T1FLC.

T1FLC designed by following the Fuzzy Lyapunov Synthesis achieve the solution to the regulation problem, however, the settlement time must be reduced to achieve results like those reported in (Aguilar *et al.*, 2007).

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