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Global Observability and Reduced Order Observers for a Class of Biochemical Process Models

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***Abstract*— A class of mathematical models for biochemical processes is analyzed with respect to its global observability properties. It is shown that, for non-monotonic reaction kinetics, there exist not identical indistinguishable trajectories and thus the system model is not globally observable in this case. However, it is shown that the system is detectable, since indistinguishable trajectories are convergent. Despite of this fact, several observer design methods, as for example the High-Gain method, cannot be used to design an observer for this system. In particular, it is shown in this paper that for a general class of Reduced Order Observers it is impossible to select an output injection, so that the observer converges for all (distinguishable) trajectories, when the kinetics is not monotone. This result is surprising due to the detectability of the system.**

Key Words: Biochemical process models, Global Observability and Detectability, Reduced Order Observes.

I. Introduction

A. General aspects of the work

The importance of observers in technical applications has increased in the last years. Nevertheless the design of observers for nonlinear systems is a challenging task requiring in general a high analytic effort. The possibility of designing an observer is influenced by the observability properties of the system. Loosely speaking, observability represents the possibility to distinguish all system states by knowledge of the input and output signals. If the system's trajectories are not globally distinguishable the system is not observable. In this case a necessary condition for the convergence of an observer is the system's detectability, i.e. the convergence of all indistinguishable trajectories. Thus the first step in the design of an observer has to be the analysis of the observability properties. For nonlinear systems a complete analysis of the observability using conventional methods, such as the Kalman condition or the observability map, is often hard to obtain. Moreover, if these conditions are not satisfied it is not possible to assert the non observability of the system since they are only sufficient. Furthermore, they are of local character and do not permit an analysis of the system's detectability. Recently a method to analyze the global observability and detectability has been proposed (Ibarra, Moreno and Espinosa, 2004), (Moreno and Dochain, 2005), (Schaum, Moreno and Vargas, 2005) using basic definitions of indistinguishability and observability. This method results as a quite natural implication of these definitions and permits a deep understanding of the dynamical nature determining the observability properties of the system. Using this method the dynamics of a class of (bio)chemical processes is analyzed. These processes are determined by the underlying kinetics law. The analysis is performed for two basic types of such kinetics found in process engineering. The observability analysis results in the existence of indistinguishable trajectories, and thus the loss of global observability, and finishes with the demonstration of the system's detectability. This shows that a necessary condition for the existence of globally converging observers is satisfied, so that in principle an observer exists that converges for every system's trajectory. Despite of this it is shown in this paper that there does not exist a Reduced Order Observer (ROO) converging for all (distinguishable) trajectories in the case of a non-monotonic kinetics of Haldane type, although it exists in the case of a monotonic kinetics. Thus the existence of ROOs depends heavily on the type of reaction kinetics determining the process dynamics.

B. State-of-the-Art

As mentioned above, the considered model class describes in a general way the dynamical behavior of some (bio)chemical processes frequently found in process engineering. The main inspiration of the presented work is a biological reactor for the treatment of industrial waste water, actually investigated at the Instituto de Ingeniería of the Universidad Nacional Autónoma de México (UNAM) (Moreno and Buitron, 2002). The model has been used for different controller designs. Due to the fact, that the considered concentration of the substrate is not measurable, the applicability of different observer design methods has been analyzed in former works (see e.g. (Vargas and Moreno, 1999), (Vargas, 1999)). The design of observers turned out to be more difficult than expected and thus a profound analysis of the dynamical nature of the occurring problems is searched. This aspect has been treated in following works. Therein the above mentioned method to analyze the system's detectability has been used (Schaum, Moreno and Vargas, 2005), (Schaum, 2006)). This leads to a deeper understanding of the structural problems obstructing the design of some observers. In the case of a Reduced Order Observer (ROO) it is possible, to show explicitly the influence of the existence of a set of indistinguishable trajectories (and thus the loss of global observability) on the observer error dynamics. Thus the presented analysis offers a deeper understanding of the influences of the observability properties on the design of observers for the presented model class and is a progress in the comprehension of the dynamical interconnections eventually complicating the direct application of certain methods.

The paper is organized as follows. In section II the class of process models, as well as the corresponding observability and detectability analysis, is presented. This is followed in section III by the analysis of the applicability of ROOs. Section IV includes some further remarks and implications and closes the presented analysis.

II. Observability and Detectability

A. Presentation of the mathematical model

In the sequel the considered class of (bio)chemical process models is introduced. We restrict the analysis to a model recently developed for a biochemical process for the treatment of industrial waste water. Note that many (bio)chemical processes are dynamically similar to the one studied here and thus the presented analysis is valid (with some restrictions) for a wide class of nonlinear systems occurring in process engineering. In the following all parameters are assumed to be known and further constant. System's equations are:

$$\begin{aligned} \dot{X} &= \mu(S)X - (K_d + u)X \\ \dot{S} &= -C_1\mu(S)X + u(S_{in} - S) \\ y &= X, \end{aligned} \qquad (1)$$

with the non-negative system's state $\boldsymbol{\eta}(t) := [X(t) \quad S(t)]^T \in \mathbb{R}^2_+$ and the initial condition $\boldsymbol{\eta}(0) = [X_0 \quad S_0]^T$, where X represents the biomass (concentration) used to degrade the toxic substrate (concentration) S. The trivial case of a reactor without biomass ($X = 0$) will be excluded. The input is given by the dilution rate (i.e. inflow divided by the reactor volume) and represented by u. The model parameters are given by the biomass mortality rate K_d and the reciprocal biomass yield coefficient C_1, both constant and positive. The substrate concentration in the inflow is given by $S_{in} > 0$. The specific biomass growth rate $\mu(S)$ can depend on the substrate concentration in different ways. If the dynamics of the reactor is determined by a monotonic kinetics like Monod, it can be represented as follows:

$$\mu_M(S) = \frac{\mu_{M0}S}{K+S}, \qquad (2)$$

with kinetic parameters μ_{M0} and K. A further possibility is given by a non-monotonic Haldane kinetics

$$\mu_H(S) = \frac{\mu_{H0}S}{\frac{S^2}{Ki} + S + K_s}, \qquad (3)$$

with (positive) kinetic parameters $\mu_{H0},\ K_s$ and K_i. Note that, due to the non-monotonicity, the kinetics is not injective. It reaches its maximum $\mu_{max} < \mu_0$ at $S_{max} = \sqrt{K_s K_i}$. The dependence on the substrate concentration S for the Haldane kinetics is illustrated in Fig. 1. For further

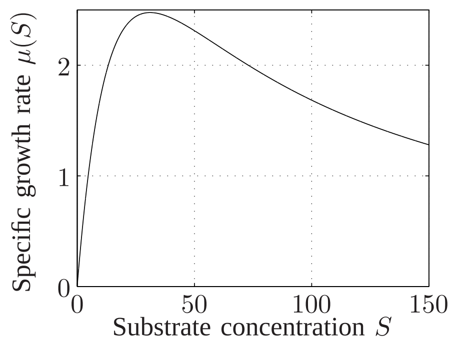

Figure. 1. Specific biomass growth rate determined by a Haldane kinetic $\mu_H(S)$ in dependence on the substrate concentration S.

studies and deduction of the special model for the biological process for the treatment of waste water see (Henze, 1986), (Moreno and Buitron, 2002), (Vargas and Moreno, 1999) and references therein.

B. Analysis of the observability properties

In the sequel a short deduction of the system's observability properties is presented. By a classical observability analysis using the observability map $\Phi = [X, L_f X]^T$ one immediately can see, that the model is observable in the case of a monotonic kinetics, like Monod. For a Haldane kinetics, due to the non-injective behavior of $\mu(S)$, the observability map becomes non-injective except at $S = S_{max}$, and thus can not be used to analyze the observability properties. On the other hand, a linearization of this system leads to a Kalman-Observability matrix which is singular at $S = S_{max}$. Thus the only point where, by use of the observability map observability can be assured, is not locally observable! This demonstrates a certain paradox between these two definitions, whose clarification is not the issue of this paper.

In order to obtain a global analysis of the observability properties, by necessary and sufficient conditions, a method is used which has been proposed in some previous works (see e.g. (Ibarra, Moreno and Espinosa, 2004), (Moreno and Dochain, 2005), (Schaum, Moreno and Vargas, 2005)). This method uses direct implications of the basic definitions of indistinguishability and observability as e.g. given in (Nijmeier and van der Schaft, 1990). Two trajectories are called to be indistinguishable, if they produce the same output signal while guided by the same input. A system then is called observable, if all indistinguishable trajectories are identical, i.e. the internal states are exactly the same for all time. This property is very strong. One can introduce a less restrictive property by defining detectability as the convergence to the same trajectory of all indistinguishable pairs of trajectories. Finally a Bad Input is defined as an input signal producing not identical indistinguishable trajectories. Thus the existence of Bad Inputs implies the non-observability of a dynamical system. From this considerations one may understand the following analysis as the determination of the observability properties of (1) by the determination of all indistinguishable trajectories and their dynamical behavior (see e.g. (Ibarra, Moreno and Espinosa, 2004), (Moreno and Dochain, 2005) for more information). First of all an exact copy of the system is introduced

$$\begin{aligned}\dot{\xi} =&\mu(\sigma)\xi - (K_d + u)\xi\\ \dot{\sigma} =& - C_1\mu(\sigma)\xi + u(S_{in} - \sigma)\\ \rho =&\xi,\end{aligned} \tag{4}$$

with the system state $\boldsymbol{\zeta}(t) = [\xi(t)\ \sigma(t)]^T \in \mathbb{R}^2_+$ and the initial state $\boldsymbol{\zeta}(0) = [\xi_0\ \sigma_0]^T$. Further, the state error is defined by $\boldsymbol{\epsilon} := \boldsymbol{\zeta} - \boldsymbol{\eta}$. A necessary and sufficient condition for the indistinguishability of a pair $(\boldsymbol{\eta}, \boldsymbol{\zeta})$ of trajectories is that the two outputs X and ξ are identical, i.e. $y(t) \equiv \rho(t) \Rightarrow \boldsymbol{\epsilon} \equiv 0$. If this is fulfilled, then $X^{(n)}(t) = \xi^{(n)}(t),\ \forall n \in \mathbb{N}_0,\ t \in \mathbb{R}_+$. For $n = 0$ this equivalence reads obviously $X \equiv \xi$. For $n = 1$ the equality can be reformulated as

$$\mu(S(t) + \epsilon_2(t)) \equiv \mu(S(t)). \tag{5}$$

Thus the indistinguishable trajectories are restricted to a submanifold $\Psi \subset \Xi := \mathbb{R}^2_+ \times \mathbb{R}^2$, defined in the extended state space of the system state and the error, which is determined by the following set of trajectories

$$\begin{aligned}\Psi :=&\{(\eta, \epsilon) \in \Xi := \mathbb{R}^2_+ \times \mathbb{R}^2 | \epsilon_1 \equiv 0 \,\&\, \mu(\eta_2 + \epsilon_2) \equiv \mu(\eta_2)\} \\ =&\{(\eta, \epsilon) \in \Xi | \epsilon_1 \equiv 0 \,\&\, S = \frac{1}{2}\left(-\epsilon_2 + \sqrt{\epsilon_2^2 + 4K_sK_i}\right)\}.\end{aligned} \tag{6}$$

One directly notices, that for a monotonic kinetics $\mu(S)$ as e.g. corresponding to Monod (2) this set is given by all trajectories $(\eta, \epsilon) \equiv (\eta, 0)$, as the equivalence of $\mu_M(S) \equiv \mu_M(S + \epsilon_2)$ directly implies $\epsilon_2 \equiv 0$. Further one can see, that for a non-monotonic kinetics like Haldane (3) there exist in principle $\epsilon_2 \neq 0$ satisfying condition (5). For Bad Inputs and initial conditions of the extended state vector (η, ϵ) on the manifold Ψ the trajectories will stay indistinguishable for all times. Note that the existence of the indistinguishable trajectories requires the existence of a positive Bad Input to be physically realizable. The Bad Input can be obtained from $X^{(n)} \equiv \xi^{(n)}$ with $n = 2$ and corresponds to

$$\begin{aligned}u^* =&\frac{C_1\mu(S)X(\mu'(S + \epsilon_2) - \mu'(S))}{\mu'(S + \epsilon_2)(S_{in} - S - \epsilon_2) - \mu'(S)(S_{in} - S)}\\ =&\frac{C_1\mu(S)XK_sK_i(2S + \epsilon_2)}{(K_sK_i - 1)[(S_{in} - S)(2S + \epsilon_2) + S^2K_sK_i]}.\end{aligned} \tag{7}$$

As one can see from the algebraic constraint on S in (6), it is not possible for S to converge to zero on the manifold Ψ. Thus $\mu(S(t)) > 0\ \forall\ t \in \mathbb{R}_+$ holds. Further $X > 0$ for all $t \in \mathbb{R}_+ \backslash \{\infty\}$. Noting that the only zero of (7) is $\epsilon_2 = -2S$ which contradicts (5) one can see, that for all kinetics with parameters $K_s,\ K_i$ satisfying $K_sK_i > 1$ and $S_{in} > S_{max}$ the input signal is positive and thus feasible, as S is naturally bounded by $S \leq S_{in}$, when $S_0 \leq S_{in}$. At this point of the analysis one can summarize that the system can excite not identical indistinguishable trajectories guided by bad input signals determined by (7). Thus (1) is not globally observable. To see if (1) is detectable one has to prove, that all pairs of indistinguishable trajectories $(\boldsymbol{\eta}, \boldsymbol{\zeta})$ converge. To analyze this condition one has to analyze the stability of the dynamics of the extended state on the manifold Ψ, i.e. the dynamics of $[\boldsymbol{\eta}, \boldsymbol{\epsilon}]^T \in \Psi \subset \Xi$. From the definition of Ψ (6) one can see, that $\epsilon_1 \equiv 0$ on Ψ. This dynamics thus corresponds in principle to the following differential-algebraic system

$$\begin{aligned}\dot{X} =&X\big(\mu(S) - K_d - u^*\big)\\ \dot{\epsilon_2} =& - u^*\epsilon_2\\ S =&\frac{1}{2}\left(-\epsilon_2 + \sqrt{\epsilon_2^2 + 4K_sK_i}\right).\end{aligned} \tag{8}$$

Thus, in order to analyze the detectability of (1), one has to analyze the asymptotic stability of the set $(\tilde{\eta}, 0)^T \in \Psi$ defined by $\epsilon_2 = 0$. Note that (8) is actually a planar system, as the Bad Input (7) only depends on $X,\ \epsilon_2$ and $S(\epsilon_2)$. Under the above mentioned conditions, assuring the positivity of the Bad Input signal, the global stability of $\boldsymbol{\epsilon} = \mathbf{0}$ in Ψ can be assured and the detectability of (1) thus is implied. This result is formalized in the following proposition.

Proposition 1: Consider the dynamics (1) on the differentiable manifold Ψ of indistinguishable trajectories defined by (6). Further suppose that $K_sK_i > 1$ and the substrate concentration in the inflow $S_{in} > S_{max}$ and constant, so that the Bad Input (7) is positive. Then $\epsilon_2(t)$ converges asymptotically to $\epsilon_2 = 0\ \forall\ [X_0\ S_0\ 0\ \epsilon_{20}]^T \in \Psi$. As further $\epsilon_1 \equiv 0$ on Ψ, $\boldsymbol{\epsilon} = \mathbf{0}$ is asymptotically stable on Ψ, i.e. system (1) is detectable.

PROOF: To proof the asymptotic stability of $\epsilon_2 = 0$, i.e. its attractiveness and stability in the sense of *Lyapunov*, we make use of the well-known theorem of *Poincaré-Bendixson*. First note, that due to the positiveness of the Bad Input (7) under the above assumptions $\left|\epsilon_2(t)\right| \leq \left|\epsilon_{20}\right| \; \forall \; t \in \mathbb{R}_+$, i.e. ϵ_2 is not increasing. Further note that the dynamics (8) has at most two equilibrium points located on the X-axis. Thus if the trajectory converges then $\epsilon_2(t) \rightarrow 0$, i.e. $\epsilon_2 \equiv 0$ is attractive. As ϵ_2 is non-increasing it is further stable. Further from (8) one can directly see that if the Bad Input u^* is integrally unbounded asymptotic stability is warranted. Now, if $\left|\left|[X(t) \;\; \epsilon_2(t)]^T\right|\right| \rightarrow \infty$, then $\exists \; \kappa > 0 \;:\; u^*(t) > \kappa \; \forall \; t \in \mathbb{R}_+$, i.e. the input is integrally unbounded and thus $\lim_{t\rightarrow\infty} \epsilon_2(t) = 0$. A limit cycle cannot exist as $\epsilon_2(t)$ is not increasing. Thus $\epsilon_2 \rightarrow 0$ holds under the above assumptions assuring the positivity of the Bad Input and the convergence of all pairs of indistinguishable trajectories to one trajectory is guaranteed. Thus (1) is detectable. $\square$

Figure 2 illustrates the behavior of indistinguishable trajectories of substrate concentrations on $\Psi \subset \Xi$ in this case. One can see, that the substrate concentrations

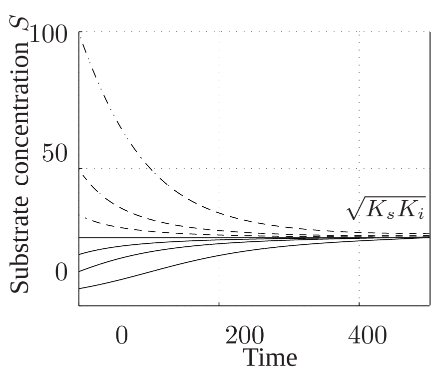

Figure. 2. Behavior of indistinguishable trajectories of the substrate concentration S and $\sigma = S + \epsilon_2$.

converge to the maximum argument of the specific growth rate $\mu(S) \equiv \mu(\sigma)$. As S can not converge to zero on Ψ this is the only possibility for the indistinguishable trajectories to converge to each other.

III. Existence of Reduced Order Observers

In the sequel the existence of a Reduced Order Observer (ROO) for system (1) is analyzed. First of all a short introduction to the basic idea of the design method is given, followed by the illustration that the existence of ROOs for this model class depends heavily on the type of the underlying kinetics.

A. General Introduction

Reduced Order Observers have first been introduced by *Luenberger* (see (Luenberger, 1971)). The general idea is to reduce the order of the observer for a n-dimensional system with m-dimensional output signal to $(n-m)$ observer states. The designed observer thus does not have to reconstruct measured data, i.e. there is no redundancy in the observer states. To illustrate the basic idea consider the following system

$$\begin{aligned}\dot{\boldsymbol{\eta}} &= \boldsymbol{f}_1(\boldsymbol{\eta}, \boldsymbol{y}, u)\\ \dot{\boldsymbol{y}} &= \boldsymbol{f}_2(\boldsymbol{\eta}, \boldsymbol{y}, u) + \boldsymbol{f}_3(\boldsymbol{y}, u),\end{aligned}$$

with initial condition $\boldsymbol{\eta}(0) = \boldsymbol{\eta}_0 \in \Xi \subseteq \mathbb{R}^n$ and $\boldsymbol{y}(0) = \boldsymbol{y}_0 \in \mathbb{R}^m$ and all $\boldsymbol{f}_i, \; i = 1, 2, 3$ assumed to fulfill a Lipschitz condition. One assumes the time derivative of the output $\boldsymbol{y}$ to be measurable and defines the new fictive output

$$\boldsymbol{\gamma} := \dot{\boldsymbol{y}} - \boldsymbol{f}_3(\boldsymbol{y}, u) = \boldsymbol{f}_2(\boldsymbol{\eta}, \boldsymbol{y}, u).$$

Thus one can restrict the system representation to the following

$$\begin{aligned}\dot{\boldsymbol{\eta}} &= \boldsymbol{f}_1(\boldsymbol{\eta}, \boldsymbol{y}, u)\\ \boldsymbol{\gamma} &= \boldsymbol{f}_2(\boldsymbol{\eta}, \boldsymbol{y}, u),\end{aligned}$$

and can define an $(n-m)$ dimensional observer according to

$$\begin{aligned}\dot{\hat{\boldsymbol{\eta}}} &= \boldsymbol{f}_1(\hat{\boldsymbol{\eta}}, \boldsymbol{y}, u) + \boldsymbol{K}(\boldsymbol{y})[\boldsymbol{\gamma} - \hat{\boldsymbol{\gamma}}] \qquad (9)\\ \hat{\boldsymbol{\gamma}} &= \boldsymbol{f}_2(\hat{\boldsymbol{\eta}}, \boldsymbol{y}, u),\end{aligned}$$

with initial state $\hat{\boldsymbol{\eta}}(0) = \hat{\boldsymbol{\eta}}_0$. In practical applications the time derivative of the output y in γ has to be eliminated, as, in general, it can only be approximatively determined. This can be realized in some cases by a state transformation. Thus remark that (9) can be rewritten as

$$\dot{\hat{\boldsymbol{\eta}}} = \boldsymbol{f}_1(\hat{\boldsymbol{\eta}}, \boldsymbol{y}, u) + \boldsymbol{K}(\boldsymbol{y})\Big[\underbrace{\dot{\boldsymbol{y}} - \boldsymbol{f}_3(\boldsymbol{y}, u)}_{\boldsymbol{\gamma}} - \boldsymbol{f}_2(\hat{\boldsymbol{\eta}}, \boldsymbol{y}, u)\Big].$$

Now define the new state $\boldsymbol{z} := \hat{\boldsymbol{\eta}} - \Phi(\boldsymbol{y})$, with a $\mathcal{C}^1$–function $\Phi(\boldsymbol{y})$ to be defined in the following. The time derivative of $\boldsymbol{z}$ is governed by

$$\begin{aligned}\dot{\boldsymbol{z}} =& \boldsymbol{f}_1(\boldsymbol{z} + \Phi(\boldsymbol{y}), \boldsymbol{y}, u) + \boldsymbol{K}(\boldsymbol{y})\Big[\dot{\boldsymbol{y}} - \boldsymbol{f}_3(\boldsymbol{y}, u) - \qquad (10)\\ &- \boldsymbol{f}_2(\boldsymbol{y}, u)\big(\boldsymbol{z} + \Phi(\boldsymbol{y})\big)\Big] - \frac{\partial \Phi(\boldsymbol{y})}{\partial \boldsymbol{y}}\dot{\boldsymbol{y}}.\end{aligned}$$

Thus the influence of $\dot{\boldsymbol{y}}$ is eliminated if $\frac{\partial \Phi(\boldsymbol{y})}{\partial \boldsymbol{y}} = \boldsymbol{K}(\boldsymbol{y})$, such that the original state has to be reconstructed corresponding $\hat{\boldsymbol{\eta}} = \boldsymbol{z} + \Phi(\boldsymbol{y})$. To apply this method it has to be assured at least that the transformed observer state $\boldsymbol{z}$ converges to the transformed original state which is equivalent to the convergence of the observer corresponding to (9). This is very important as it represents a fundamental condition for the applicability of this strategy.

In the following the applicability of this approach to the model class is analyzed. The main result is that for a non-monotonic kinetics of Haldane type due to the existence of a submanifold of indistinguishable trajectories $\Psi \subset \Xi$ it is not possible to design the observer gain in such a way, that for all initial states and inputs all distinguishable trajectories converge. Thus for a non-monotonic Haldane type kinetics no ROO exist for (1). This shows the dependence of the existence on the kinetics and thus the observability properties of the regarded process.

B. Existence of ROO for the model class

In the following the existence of ROOs for the model class defined by (1) is analyzed. Therefore assume that the time derivative of the output X, i.e. $\dot{X}$ is known at each time-instant. Thus one can construct the output $\gamma := \dot{X} + (u + K_d)X = \mu(S)X$. A ROO thus has to be designed for the substrate concentration and has the form

$$\begin{aligned}\dot{\hat{S}} =& - C_1\mu(\hat{S})X + u(S_{in} - \hat{S}) + K(X)(\gamma - \hat{\gamma}) \\ \hat{\gamma} =& \mu(\hat{S})X,\end{aligned} \quad (11)$$

according to the introductory explanations to this section. Defining the observation error as $\epsilon := \hat{S} - S$ and considering $\gamma = \mu(S)X$, the error dynamics can be written as

$$\dot{\epsilon} = -\big(\mu(S) - \mu(S+\epsilon)\big)K(X)X - u\epsilon. \quad (12)$$

In the sequel assume that the kinetics $\mu(S)$ is of Haldane type. Sign and value of the first term in (11) are influenced by the initial error, because of the non-monotonicity. The difference between observed and real growing factor, i.e. $\Delta(S,\epsilon) := \big(\mu(S+\epsilon) - \mu(S)\big)$ vanishes on the submanifold of indistinguishable trajectories Ψ. As $\epsilon_1 \equiv 0$ is considered, this manifold can be represented by all pairs $(\epsilon,\ S_\Psi)$ fulfilling $S_\Psi = \frac{1}{2}\big(-\epsilon + \sqrt{\epsilon^2 + 4K_sK_i}\big)$. There exist three possibilities fixing $\epsilon > 0$ as illustrated

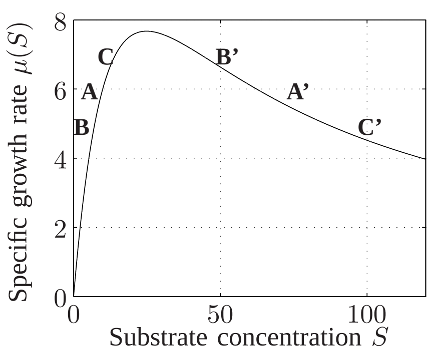

Figure. 3. Possible constellations for positive observation error: **A–A'**: $S = S_\Psi$, **B–B'**: $S < S_\Psi$, **C–C'**: $S > S_\Psi$.

in Fig. 3. Thus for $S < S_\Psi, \ \Rightarrow \ \Delta(S,\epsilon) > 0$ and for $S > S_\Psi, \ \Rightarrow \ \Delta(S,\epsilon) < 0$. If $\epsilon < 0$ these results are exactly reverse, i.e. $S < S_\Psi, \ \Rightarrow \ \Delta(S,\epsilon) < 0$ and $S > S_\Psi, \ \Rightarrow \ \Delta(S,\epsilon) > 0$. Due to the non-monotonic behavior Ψ forms a separating curve in the (S,ϵ)-plane segmenting the plane, together with the coordinate axes, in 4 regions of different sign of the first term, i.e. $\Delta(S,\epsilon)K(X)X$, as illustrated in Fig. 4 for a positive K. The following proposition emphasizes a contra-intuitive result caused by the difference of observed and real growth rate. It clarifies that it is not possible to design the observer gain in such a way, that the observer converges for all system trajectories produced by inputs which are not bad. This phenomenon is unexpected as it has been shown that the system is detectable. But the influence of the non-monotonicity yields a destabilization of certain

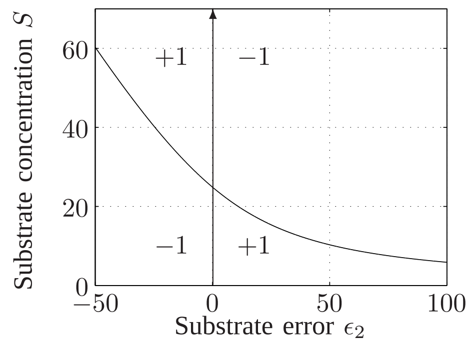

Figure. 4. Phase plane of the error dynamics. Segmentation in 4 regions of different sign by the *Haldane* kinetic for $K > 0$. The curve represents the phase-pairs for which $\mu(S) = \mu(S+\epsilon)$ and thus corresponds the manifold Ψ.

observer error trajectories. Thus there does not exist a ROO for (1) determined by a non-monotonic kinetics of Haldane type. The fundamental idea of the proof is to derive necessary conditions for the convergence and to show that they can not be satisfied in some special cases. Therefore a case distinction of the sign of $K(X)$ is used.

Proposition 2: Independently on the observer gain $K(X) \in \mathbb{R}$ of the ROO (11) there exist initial conditions $[S_0\ \epsilon_0]^T \in \mathbb{R}_+ \times \mathbb{R}$ as well as $u : \mathbb{R}_+ \to \mathbb{R}_{\geq 0}$ such that the observation error $\epsilon(t) = \hat{S}(t) - S(t)$ with dynamics according to (12) and (1) can not converge for all distinguishable trajectories, i.e. trajectories $[X(t)\ S(t)\ 0\ \epsilon(t)]^T \in \Xi \backslash \Psi$. Thus no ROO exist for (1).

PROOF: Remind that the error dynamics is given according (12). Note that the observer gain $K(X)$ can change its sign only in dependence of X as from X the substrate concentration S can not be directly deduced. Thus consider in the following the three cases of different sign possible for $K(X)$.

(A) Let $K(X) > 0$ and $u = 0$. In (Schaum, 2006) it is shown, that the substrate concentration converges to $S = 0$, at least in the absence of the input. Thus all trajectories in the $(S,\ \epsilon)$-phase plane are attracted by the line $S = 0$. Therefore if $S < S_\Psi$ while $\epsilon \neq 0$, ϵ always is repelled from $\epsilon = 0$ as can be seen by the qualitative analysis in Fig. 4.

(B) Now assume that $K(X) \equiv 0$. Then the observer error dynamics reduces to $\dot{\epsilon} = -u\epsilon$. Thus if $u \equiv 0$ the observation error keeps constant. Note that $u \equiv 0$ is not a Bad Input.

(C) Last of all let $K(X) < 0$. A necessary and sufficient condition for the stability of the time-variant error equation $\dot{\epsilon} = f(t,\epsilon)$ is that $\exists \mathcal{T} := [t_s, \infty) \subset \mathbb{R}_+$ with t_s arbitrarily large, such that $\epsilon f(t,\epsilon) < 0,\ \forall\, t \in \mathcal{T}$. This condition yields the following assessment for $K(X)$:

$$\epsilon\Delta(S,\epsilon)K(X)X - u\epsilon^2 < 0. \quad (13)$$

Solving this inequality for $K(X)$ and assuming that the pair (ϵ, S) fulfills $\epsilon\Delta(S,\epsilon) > 0$ and $\Delta(S,\epsilon) < 0$ (which is

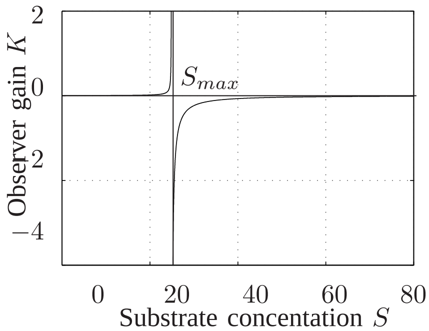

Figure. 5. Qualitative behavior of the right hand side of assessment on K in dependence on the substrate concentration S for $\epsilon \ll 1$.

satisfied for pairs (S, ϵ) in the first quadrant above the curve defined by S_Ψ). In this case (13) yields $K < -\frac{u\epsilon}{X|\Delta(S,\epsilon)|}$. Thus for crossing the indistinguishable manifold Ψ in the first quadrant from the the right to the left, $K(X)$ has to be less than minus infinity, as $\Delta(S, \epsilon) \to 0$ as $(\epsilon, S) \to \Psi$. Thus such a trajectory can not converge for a finite observer gain. □

One can see that for all of the three cases of sign(K) there exist trajectories for which it is impossible to design $K(X)$ in such a way that the error is globally converging for distinguishable trajectories. Note that the dependence of $K(X)$ in the last mentioned case can be illustrated as $\epsilon \to 0$ applying the law of *L'Hospital* for $\epsilon\Delta(S, \epsilon) > 0$ and $0 < \epsilon \ll 1$, yielding $K < -\frac{u}{X\mu'(S)}$. Thus for $S \to S^+_{max}$ this term tends to minus infinity. This illustrates that the manifold Ψ is obstructing the convergence of the observation error as it produces a separating submanifold which in some cases can not be crossed with a finite observer gain. The dependence of the right-hand-side of this assessment for $\epsilon \ll 1$ is illustrated in Fig. 5. Note that the non-applicability results from the separation in the (S, ϵ_2) plane by the indistinguishable submanifold Ψ. For a monotonic *Monod* kinetics this separation does not occur as it results in the previous section that for an equivalent system with *Monod* kinetic the set of indistinguishable not identical trajectories is empty. Thus this observer design method is applicable to an equivalent system determined by such a kinetics. This shows, that for a change in the type of the kinetics ROO can lose their applicability. Furthermore, it illustrates the influence of the non-monotonicity of the kinetics on the design of observers. Moreover it shows, that the detectability of a nonlinear system is not sufficient for the existence of a ROO.

IV. Some remarks on other observer design methods and implications

The presented system model has been analyzed with respect to the applicability of many observer design methods (Schaum, 2006). Surprisingly some of them resulted to be not applicable, as illustrated in this paper for the ROO. Due to the loss of observability it is easily to see that a classical High-Gain Observer can not be designed. Further it results impossible to find a (at least practically applicable) extension of the system dynamics to a state affine form, which is a classical method to treat systems with reduced observability properties. Recently an observer design based on dissipativity has been applied resulting in satisfying observer convergence. This result further assures the global exponential convergence of the observer trajectories (Schaum and Moreno, 2006). Thus there exists a Full Order Observer (FOO). In the case of linear systems, the existence of a FOO implies the existence of an ROO. The presented example thus provides a system type for which this implication does not hold in the case of nonlinear systems (at least not in general).

V. Conclusions

The mathematical model of a biochemical reactor process is analyzed with respect to its observability properties. It is shown that the system is not globally observable due to the existence of not identical indistinguishable trajectories. Sufficient conditions for the system's detectability are given and the detectability under this conditions is proven. Further, the applicability of a Reduced Order Observer (ROO) to this model is analyzed resulting in the impossibility to design the observer gain so that all possible distinguishable trajectories can be observed.

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