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Projectional observers: Analysis and Examples

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***Abstract*— A new type of the, so-called, projectional observers is suggested. The main property of such observers is that they keep the obtained (generated) state estimates of a dynamic model within a compact set, defined by given constraints (usually, having physical meaning) independently of a measurement noise effect. The obtained estimated trajectories turns out to be non-smooth (may be non-differentiable at some times of the process). That's why, to analyze its stability, the standard Lyapunov functions are not applicable. Here we suggest the Lyapunov-Krasovski functional (which is an integral of a standard Lyapunov function) to overcome this problem and to realize the stability analysis for state estimation error. The upper bound for the estimation error is obtained which turns out to be rationally dependent on the lag of the filter and linear with respect to the noise power. Two example (the Chua's circuit and the chemical model) illustrate the workability of the suggested observers.**

Keywords: State observers, Lyapunov stability, projectional operator.

I. Introduction and problem formulation

I-A. State estimation: a brief survey on observation problem

The *state estimation (observation) problem* arrises during *Identification* or *Feedback Control* when the current system states can not be directly measured and the only available information at each time instant is the output of the system which is a function of the current state usually corrupted by "output noise". Modern *Identification Theory* (Eykhoff and Parks, 1990), and (Ljung, 1979) basically deals with the problem of the efficient extraction of signal and systems dynamic properties based on available data measurements. Nonlinear system identification is traditionally concerned with two issues: *a)* estimation of parameters based on direct and complete state space measurements, and *b)* state space estimation of completely known nonlinear dynamics. Here we will deal with the *b*-issue. Contribution on the observer construction problem for nonlinear systems in the presence of *complete information* about the nonlinear dynamics, was performed by (Williamson, 1977), (Krener and Isidori, 1983), (Krener and Respondek, 1985), and (Xia and Gao, 1989). Most of these results deal with the situation where it is possible to obtain a set of rather restrictive conditions when the dynamics of the observation errors is linear and there is no observation noise. In (Walcott and Zak, 1987) a class of observers for nonlinear systems subjected to bounded nonlinearities or uncertainties was suggested. A canonical form and a necessary and sufficient observability condition for a class of nonlinear systems which are linear with respect to the inputs was established by (Gauthier and Bornard, 1981). The extended Luenberger observer for a class of SISO nonlinear systems was designed by (Zeitz, 1987). These results were extended in (Birk and Zeitz, 1988) for a class of MIMO nonlinear systems. An exponentially convergent observer was derived in . (Gauthier, Hammouri and Othman, 1992) for nonlinear systems that are observable for any input signal. More advanced results were obtained in (Ciccarella, Dalla and Germani, 1993) where based on simple assumptions of regularity, global asymptotic convergence of the estimated states to the true states was shown. A further line of investigation relates to the observation problem subjected to bounded nonlinearities or *uncertainties* was developed in (Walcott and Zak, 1987) and (Walcott, Corless and Zak, 1987). In the situation when the plant model is incomplete or uncertain, the implementation of high-gain observers seems to be convenient (Tornambè, 1989), (Dabroom and Khalil, 1997), (Nicosia, Tomei and Tornambe, 1988), (Bullinger and Allgower, 1997)). In (Yaz and Azemi, 1994) a robust/adaptive observer is presented for state reconstruction of nonlinear systems with uncertainty having unknown bounds. A robust adaptive observer for a class of nonlinear systems is proposed in (Ruijun,Tianyou and Cheng, 1997) based on generalized dynamic recurrent neural networks. A robust nonlinear observer is considered in (Shields, 1996) for a class of singular nonlinear descriptor systems subject to unknown inputs. This class is partly characterized by globally Lipschitz nonlinearities. A suboptimal robust filtering of states for finite dimensional linear systems with time-varying parameters under nonrandom disturbances was considered in P (Poznyak and Ososrio, 1994). Sliding mode observers were studied in (Utkin, 1992). The approach described in the book of (Edwards and Spurgeon, 1988) is conceptually similar to that proposed by (Slotine, 1984). The papers of (Walcott and Zak, 1987) and (Walcott and

Zak, 1988) seek global error convergence for a class of uncertain systems using some algebraic manipulations to effectively solve an associated constrained Lyapunov problem for systems of reasonable order. This approach is discussed in detail in (Zak and Walcott, 1990). This collection also describes a hyperstability approach to observer design by (Ballestrino and Innocenti, 1990), based on the concept of positive realness. The comprehensive survey on these problems can be found in (Poznyak, 2004).

An interesting topic related to state estimation arises from the practical implementations, such cases where the state vector belongs to a priori known set (even in the noise presence), these restrictions have been involved into the design of structure observers, we can find as mayor importance examples: the *interval observers* and the *membership function* approach; in the first case is necessary to calculate an upper and a lower bound for the observation process considering a linear form of the system around an equilibrium point; many practical implementations have been implemented in the chemical and biotechnological areas (Dochain, 2003)(Bernard and Gouzé , 2004)(Raïssi, Ramdani and Candau , 2005). In the membership function approach is considered as assumption that the state belongs to a convex set, the most important research in this area involves to discrete systems or continuos linear systems (Alamo, Bravo and Camacho , 2005)(Durieu, Walter and Polyak , 2001), some attempts to extend the results to the continous nonlinear case have been presented by (Chernousko, 2005).

I-B. Observers design problems under known state constraints: motivation

Let us consider the nonlinear continuous-time system with measured output which is given by the following ODE

$$\begin{array}{c} \dot{x}_t = f\left(x_t, t\right) + \xi_t, \ x_0 \text{ is fixed} \\ y_t = C x_t + \eta_t \end{array} \tag{1}$$

where $x_t \in \mathbb{R}^n$ is the state-vector at time $t \geq 0$, $y_t \in \mathbb{R}^m$ is the corresponding output, available for a designer at any time, $C \in \mathbb{R}^{m \times n}$ is the state-output transformation (matrix), ξ_t and η_t are noise in the state dynamics and in the output, respectively.

In many practical problems there is know *a* priory that the state-vector x_t always belongs to a given *compact set* X (even in the presence of noise) which has a concrete physical sense. For example, the dynamic behavior of some reagents participating in chemical reactions always keeps nonnegative the current values of these components which, in fact, are the corresponding concentrations of those reagents and, hence, can not be negative. The same comment seems to be true for another physical variables and there dynamics such as temperature, pressure, heat and etc.

The *state observation problem* consists in designing a vector-function $\hat{x}_t = \hat{x}_t\left(y_{\tau\in[0,t]}\right) \in \mathbb{R}^n$ depending only on the available data $y_{\tau\in[0,t]}$ available up to the time t in such a way that it would be respectively close to its real (but non-measurable) value x_t. The measure of that çloseness"depends on the accepted assumptions concerning the state dynamics as well as the noise effects. The most of filters solving this problem are presented also as an ODE, namely, they are given by

$$\frac{d}{dt}\hat{x}_t = F\left(t, \hat{x}_t, y_{\tau\in[0,t]}\right), \ \hat{x}_0 \text{ is a fixed vector} \tag{2}$$

It seems to be very important to keep the generated state estimates $\hat{x}_t$ always remaining within a given compact set X, that is,

$$\hat{x}_t \in X \tag{3}$$

Indeed, if the obtained state estimated are supposed to be used in some (in fact, feed-back) control construction, for example, $u_t = K\hat{x}_t$, then any changing of a sign in $\hat{x}_t$ may provide significant instability effect of the corresponding close-loop dynamics. One of (seemed to be evident) feasible solutions consists in the introduction of some projection operator $\pi_S\left\{\cdot\right\}$ acting to the right-hand side of (2) keeping the property $\hat{x}_t \in X$, i.e.,

$$\frac{d}{dt}\hat{x}_t = \pi_S\left\{F\left(t, \hat{x}_t, y_{\tau\in[0,t]}\right)\right\} \tag{4}$$

But such "evident"solution immediately provides many constructive problems of the set S, where one needs to project the right-hand side of (2), it´s shape should be depends on significantly nonlinear and non-stationary information, that is, there should be $S = S\left(t, \hat{x}_t, \frac{d}{dt}\hat{x}_t\right)$ since the scheme (4) realizes the projection of the estimates of the state-derivative, but not the state-vector $\hat{x}_t$. That's why the problem of designing of other observers structures, verifying (3), presents a real challenge for the engineering community.

I-C. Basic assumptions

Hereafter to show the error estimarion stability we will assume that

A1) The function $f\left(x,t\right)$ is uniformly (on $t \geq 0$) A-Lipschitz continuous in $x \in X$, that is, for all $t \geq 0$ and all $x, x' \in X$ there exist a matrix $A \in \mathbb{R}^{n \times n}$ and a constant $L_f < \infty$ such that

$$\left\| f\left(x,t\right) \text{-} f\left(x',t\right) \text{-} A\left(x\text{-}x'\right)\right\| \leq L_f \left\|x\text{-}x'\right\| \tag{5}$$

A2) $x_t, \hat{x}_t \in X \subset \mathbb{R}^n$, and X is compact and convex. It follows, there exists a constant $L_\delta < \infty$ such that for all $t \geq h$

$$\left\|\delta_t - \delta_{t-h}\right\| \leq L_\delta h \tag{6}$$

where $\delta_t := \hat{x}_t - x_t$ is the state observation error at time t.

A3) The pair (C, A) is observable (with A as defined in A1), that is, there exists the gain matrix K of the projectional filter (10) - (12) such that

$$\tilde{A} := A - KC = -\lambda T \qquad \lambda > 0 \tag{7}$$

where

$$T = P^{-1}DP$$

P is an appropriate transformation matrix with D triangular matrix given by:

$$D = \begin{bmatrix} 1 & -\frac{1}{\lambda} & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{\lambda} & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 & -\frac{1}{\lambda} \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \tag{8}$$

A4) The noises ξ_t and η_t acting to the system (1) are uniformly (on t) bounded such that

$$\begin{gathered} \|\xi_t\|^2_{\Lambda_\xi} := \xi_t^\intercal \Lambda_\xi \xi \leq 1 \\ \|\eta_t\|^2_{\Lambda_\eta} := \eta_t^\intercal \Lambda_\eta \eta_t \leq 1 \end{gathered} \tag{9}$$

where Λ_ξ and Λ_η are known "normalizing"non-negative definite matrices which permits to operate with vectors having components of different physical nature (for, example, meters, $mole/l$, voltage and etc.).

I-D. *Projectional observers*

Instead of (4) let us consider the following observer called the projectional observer ($t \geq h$) wich involves the state vector in an integral form:

$$\hat{x}_t = \pi_X \left\{ \hat{x}_{t-h} + \int_{\tau=t-h}^{t} F\left(\tau, \hat{x}_\tau, y_{s\in[0,\tau]}\right) d\tau \right\} \tag{10}$$

Here $\pi_X\{\cdot\}$ is the projector to the given convex compact X satisfying the condition

$$\|\pi_X\{x\} - z\| \leq \|x - z\| \tag{11}$$

for any $x \in \mathbb{R}^n$ and any $z \in X$. Heret the set X is known a priori, the projector operator lets consider the estimated stated $\hat{x}_t \,\epsilon\, X$ at each time in a unique form because of the convex nature of X.

Remark 1: In this paper the projectional operator $\pi_X\{\cdot\}$ will be considered as a saturation function for each state component, it is worth notice that under the projection effect the trajectories $\{\hat{x}_t\}$ are not differentiable for any $t \geq h > 0$.

I-E. *Main contributions*

In this paper

- To demonstrate the workability of the *projectional observer* (10), when

$$F\left(t, \hat{x}_t, y_{\tau\in[0,t]}\right) = f(\hat{x}_t, t) + K(y_t - C\hat{x}_t), \quad K \in \mathbb{R}^{n\times m} \tag{12}$$

(that corresponds to the standard Luenberger observer with a linear correction term), we have suggested to implement the "nonstandard"Lyapunov function

$$V_t := \int_{\tau=t-h}^{t} \|\hat{x}_\tau - x_\tau\|^2 d\tau \tag{13}$$

(which is, in fact, the Lyapunov -Krasovski functional) instead of the standard Lyapunov function $V_t = \frac{1}{2}\|\hat{x}_t - x_t\|^2$ which is not differentiable on the trajectories of the *projectional observer* (10) accordindg to *Remark 1*.

- We have found the upper bound for the averaged observation error corresponding to (10) with (12).

- To show the effectiveness of the suggested observer we have presented two illustrative example (the Chua's circuit and the chemical reactor) where two observers have been compared: without projection (2) and with the projection (10). In both examples the *projectional observer* (10) shows the significant advantages.

II. Upper Bound for State Estimation Error

Theorem 1: Under the assumptions A1-A4, for any $\lambda_0 > 0$ and a sufficiently small positive h such that

$$\lambda_0 - 3h(L_f + \lambda_0)^2 > 0$$

with

$$\lambda := 2L_f + \lambda_0$$

in (7), the *projectional observer* (10) - (12) provides the following upper bound for the .ªveraged"estimation error:

$$\lim_{t\to\infty}\sup \frac{1}{t}\int_{\tau=0}^{t} \|\hat{x}_\tau - x_\tau\| d\tau \leq \frac{b}{a} + \sqrt{\frac{b^2}{a^2} + \frac{c}{a}} \tag{14}$$

where

$$\begin{gathered} a = \lambda_0 - 3h(L_f + \lambda_0)^2 > 0 \\ b = \varsigma + \frac{h}{2}\left[L_\delta(4L_f + \lambda_0)\right] \\ c = 3L_\delta^2 h^3 + 3\zeta^2 h \\ \varsigma = \|K\|\left\|\Lambda_\eta^{-1}\right\|^{1/2} + \left\|\Lambda_\xi^{-1}\right\|^{1/2} \end{gathered} \tag{15}$$

Demostración: Using the functional (13), in view of the property (11) and taking into account that $x_t \in X$ by A2, we have

$$\dot{V}_t = \|\delta_t\|^2 - \|\delta_{t-h}\|^2 = -\|\delta_{t-h}\|^2 + \left\| \pi_X\left\{ \hat{x}_{t-h} + \int_{\tau=t-h}^{t} F\left(\tau, \hat{x}_\tau, y_{s\in[0,\tau]}\right) d\tau \right\} - x_t \right\|^2 \leq \alpha_t + \beta_t$$

where

$$\alpha_t = \left\| \int_{\tau=t-h}^{t} \left[f(\hat{x}_\tau, \tau) - f(x_\tau, \tau) - KC\delta_\tau + K\eta_\tau - \xi_\tau\right] d\tau \right\|^2$$

$$\beta_t = 2\left(\delta_{t-h}, \int_{t-h}^{t} \left[f(\hat{x}_\tau, \tau) - f(x_\tau, \tau) - KC\delta_\tau + K\eta_\tau - \xi_\tau\right] d\tau\right)$$

Since

$$\|\eta_\tau\| = \sqrt{\left(\Lambda_\eta^{1/2}\eta_\tau, \Lambda_\eta^{-1}\Lambda_\eta^{1/2}\eta_\tau\right)} \leq \sqrt{\left\|\Lambda_\eta^{-1}\right\| \|\eta_\tau\|^2_{\Lambda_\eta}} \leq \left\|\Lambda_\eta^{-1}\right\|^{1/2}$$

the following upper estimate holds:

$$\beta_t \leq 2\left(\delta_{t-h}, \tilde{A} \int_{\tau=t-h}^{t} \delta_\tau d\tau\right) + 2h\left\|\delta_{t-h}\right\|\varsigma + 2L_f\left\|\delta_{t-h}\right\| \int_{\tau=t-h}^{t} \left\|\delta_\tau\right\| d\tau$$

By (6) the right-hand side in the last inequality may be estimated as

$$\beta_t \leq h\left[(2L_f\text{-}\lambda)\left\|\delta_{t-h}\right\|^2 + \left\|\delta_{t-h}\right\|\left(2\varsigma{+}h\left[L_\delta\left(\lambda{+}2L_f\right)\right]\right)\right]$$

Analogously,

$$\alpha_t \leq \left((L_f+\lambda)\int_{\tau=t-h}^{t}\left\|\delta_\tau\right\| d\tau + \zeta h\right)^2 \leq 3h^2\left((L_f+\lambda)^2\left\|\delta_{t-h}\right\|^2 + L_\delta^2 h^2 + \zeta^2\right)$$

that finally implies

$$\dot{V}_t \leq h\left[-a\left\|\delta_{t-h}\right\|^2 + 2\left\|\delta_{t-h}\right\| b + c\right] = \left[\left(-a\left[\left\|\delta_{t-h}\right\| - \frac{b}{a}\right]^2 + \frac{b^2}{a}\right) + c\right] h \tag{16}$$

Rearranging and integrating (16), by the Jensen inequality

$$\frac{1}{t}\int_{t=h}^{t}\left[\left\|\delta_{t-h}\right\| - \frac{b}{a}\right]^2 dt \geq \left(\frac{1}{t}\int_{t=h}^{t}\left[\left\|\delta_{t-h}\right\| - \frac{b}{a}\right] dt\right)^2$$

we get

$$\frac{1}{t}\int_{t=h}^{t}\left\|\delta_{t-h}\right\| dt - \frac{b}{a}\frac{t-h}{t} \leq \left|\frac{1}{t}\int_{t=h}^{t}\left[\left\|\delta_{t-h}\right\| - \frac{b}{a}\right] dt\right| \leq \frac{1}{t}\int_{t=h}^{t}\left[\left\|\delta_{t-h}\right\| - \frac{b}{a}\right]^2 dt \leq \sqrt{\frac{V_0}{ah\cdot t} + \frac{b^2}{a^2} + \frac{c}{a}}$$

that leads to (14). ■

Remark 2: In absence of noises ($\xi_t = \eta_t = 0$) it is posible to take $\left\|\Lambda_\eta^{-1}\right\|$ and $\left\|\Lambda_\xi^{-1}\right\|$ less than any positive value ε, then, if we consider small enougth h the state estamitation error is asympotical estable:

$$\lim_{t\to\infty}\left\|\hat{x}_\tau - x_\tau\right\| \to 0 \tag{17}$$

III. Numerical Examples

Example 1 (Chua's circuit): The mathematical model ((Wang, Duan and Huang, 2006)) for the considered circuit is given by $\dot{x}_{1,t}$ (Volts), $\dot{x}_{2,t}$ (Volts) y $\dot{x}_{3,t}$ (Amperes) involved in the next system of ODE:

$$\left.\begin{array}{c} \dot{x}_{1,t} = \alpha\left(x_{2,t} - x_{1,t} - f(x_{1,t})\right) \\ \dot{x}_{2,t} = x_{1,t} - x_{2,t} + x_{3,t} \\ \dot{x}_{3,t} = -\beta x_{2,t} \\ f(x_{1,t}) = b(x_{1,t}) + \frac{a-b}{2}((|x_{1,t}+1| - |x_{1,t}-1|) \end{array}\right\} \tag{18}$$

The parameters of the circuit (18) are

$$a = -1{,}05, b = -0{,}711, \alpha = 40 \text{ and } \beta = 93{,}333$$

that corresponds to a specific case, such circuit works as a simple oscillator. We consider

$$y_t = x_{2,t} + \eta_t$$

(voltage measurements) with white noise presence ($\eta_t = 1\,\%$ of $x_{2,t}$ magnitude) and $\lambda = 20$. The compact set X is considered by:

$$X := \left\{\begin{array}{c} 0 \leq x_{1,t} \leq 1{,}4 \\ -2 \leq x_{2,t} \leq 2 \\ 1 \leq x_{3,t} \leq -2{,}5 \end{array}\right\} \tag{19}$$

The figures 1-2 represent the comparison between observer and the *projectional observer* for each state variable estimated.

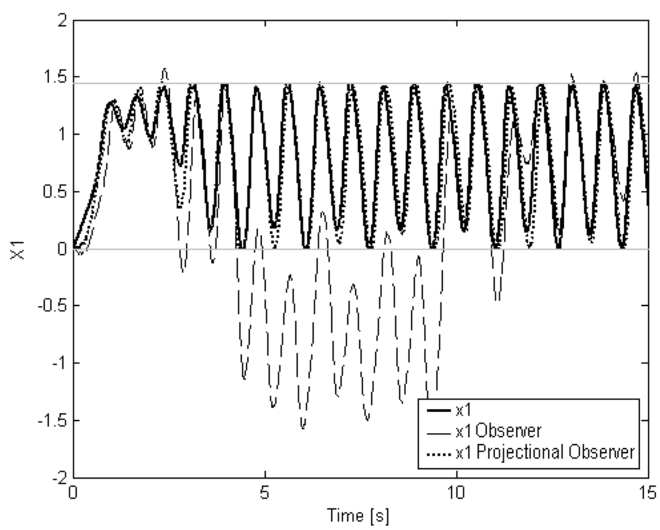

Figura 1. Estimations of $x_{1,t}$.

Example 2 (Ozonation process): High oxidation process employing ozone is one of the most recent approaches in the contaminated soil treatment by chemical agents such as polyaromatic hydrocarbons. The next simplified model ((García, Poznyak, T., Chairez, I., and Poznyak A., 2006)) describes the ozonation process when a contaminant is

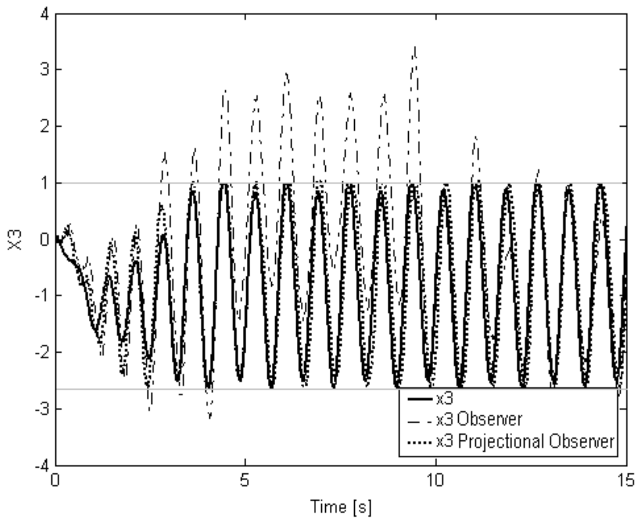

Figura 2. Estimation of $x_{3,t}$.

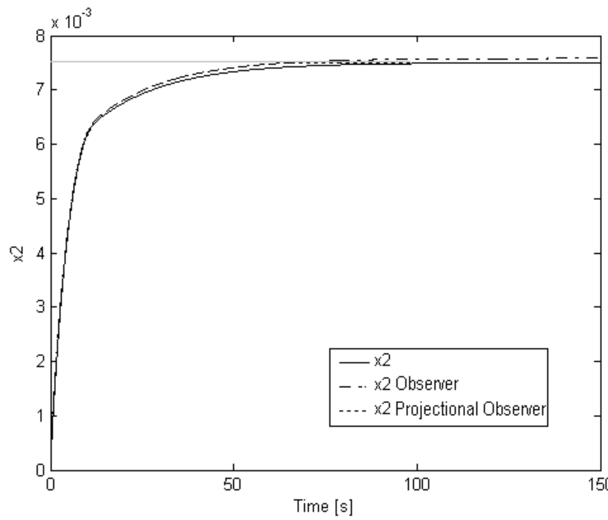

Figura 3. Estimation of $x_{2,t}$.

present in a soil just with solid and gas phase involve (without water presence):

$$\left.\begin{array}{c} V_{gas}\dot{x}_{1,t} = V_{gas}^{-1}\left[W_{gas}C_{\tau}^{in} - W_{gas}x_{1,t} - \right. \\ \left. k_1 S_1 x_{4,t} x_{3,t} - K_t^{abs}\left(Q_{\text{máx}}^{free\_abs} - x_{2,t}\right)\right] \\ \dot{x}_2 = K_t^{abs}\left(Q_{\text{máx}}^{free\_abs} - x_{2,t}\right) \\ \dot{x}_{3,t} = k_1 S_1 x_{4,t} x_{3,t} \\ \dot{x}_{4,t} = -k_1 x_{4,t} x_{3,t} \end{array}\right\} \quad (20)$$

Here in (20)

$$y_t = x_{1,t} + \eta_t$$

is the ozone concentration at the output of the reactor assumed to be measurable, $x_{2,t}$ is the ozone amount absorbed by the soil which is not reacting with the contaminant, $x_{3,t}$ is the ozone amount absorbed by the soil and reacting with the contaminant, and $x_{4,t}$ is the current contaminant concentration. The compact set X is given by:

$$X := \left\{\begin{array}{c} 0 \leq x_{1,t} \leq x_{1,0} \\ 0 \leq x_{2,t} \leq Q_{\text{máx}}^{free\_abs} \\ 0 \leq x_{3,t} \leq V_{gas}C^{in} \\ 0 \leq x_{4,t} \leq x_{4,0} \end{array}\right\} \quad (21)$$

We select $\lambda = 0{,}5$. The next figures 3-5 represents the results of the observation state.

From the curves presented above we may conclude that *projectional observers* suggested in this paper have significantly better quality of state estimation especially in the beginning of a process being compared with traditional (non-projectional) observers.

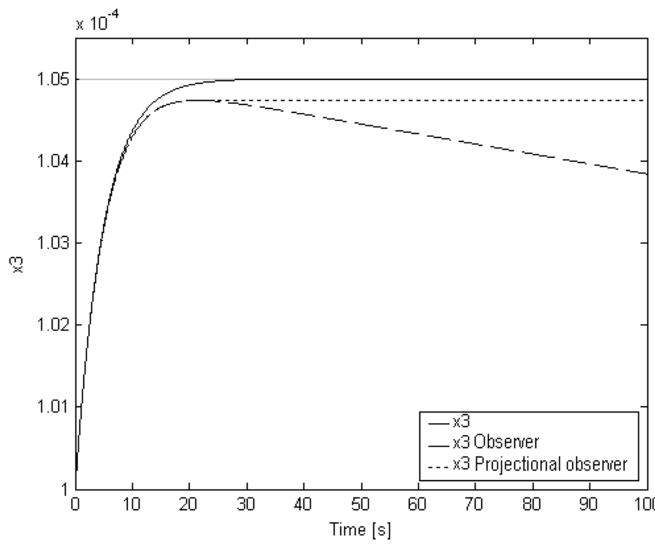

Figura 4. Estimation of $x_{3,t}$.

IV. Conclusion

In this paper a new type of the *projectional observer* is suggested. They permit to keep the obtained state estimates within an *a* priory given compact set even in presence of input and output noises. In most of practical problems to realize the projection operation it is enough to apply the corresponding saturation function. Since the obtained estimated trajectories turns out to be non-smooth, to analyze its stability the Lyapunov-Krasovski method is suggested. The upper bound for the estimation error is obtained which turns out to be rationally dependent on the lag of the filter and linear with respect to a noise power. In presence of output noise η_t this upper bound depends also on the gain matrix K of the filter. If no noise in the output, the obtained upper bound is uniform on K, even in noise absence,

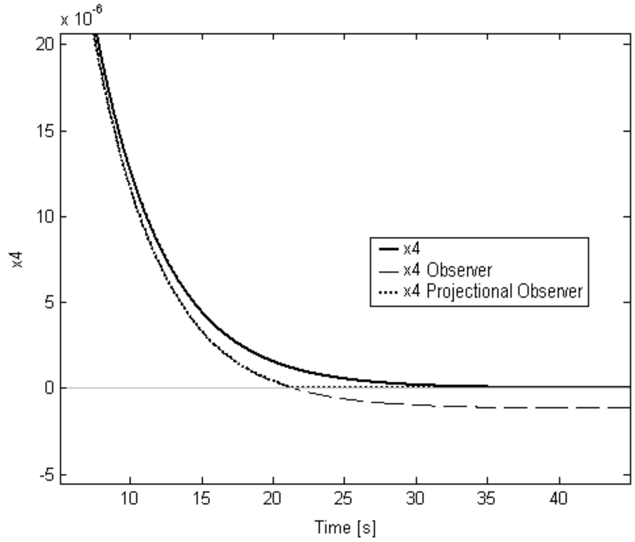

Figura 5. Estimation of $x_{4,t}$.

the asymptotical stability of the state estimation error is achieved.

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