Stochastic Petri Net Model for an "English" Turn

Alejandro Malo-Tamayo & Alexander Poznyak
Automatic Control
Cinvestav-IPN
Av. IPN 2508, Zacatenco, D.F.
e-mail: {alexmalo,apoznyak}@ctrl.cinvestav.mx

Abstract

As cities become bigger so have their bottlenecks. The urban traffic average speed has decreased to a crawling pace. In México city, during peak hours usually we can find cars slowly moving in every avenue or highway. Each intersection has become a bottleneck. The chaos due to un-existent enforcement of the law and usual lack of the driver's education or patience, is an everyday incident. In recent years, the authorities of México city have modified certain conflicting intersections, to avoid having a bottleneck at the crossing, this has been done with what we call "English" turns, where turning left (i.e. crossing the flow in opposite direction) is decoupled from going straight or turning right. Here we present a model of this kind of turn that will serve as starting point to further work on the analysis, identification and optimization of the flow in it.

Keywords: urban traffic control, stochastic processes, stochastic petri nets .

1 Introduction

The problem of traffic control has been of interest for a long time and can not be considered solved to everyone's satifaction[1]; different approaches have been used centralized[2], decentralized[3], intelligent[5]. Here we use Petri Nets to Model [6, 7] the system, our starting point to further developments.

For our purpose we take the intersection at Politécnico and Juan de Dios Bátiz Avenues. This is not a symmetrical system, since only North, East and West bound traffic can turn left. The physical model can be seen in figure 1.

2 Petri Nets

We will define what is a Generalized Petri net. This description will be enough to understand the basic behavior of the model. To this basic model we will associate stochastic processes, time or external control to its transitions [6, 7].

Definition 1 ***Petri Net*** *A Petri Net is a 3-tuple* $N = \langle P, T, F\rangle$*, where* P *is a finite set, whose elements are called places,* T *is a finite set, whose elements are called transitions, and* F *defines the input-output relation between places and transitions;* $F : (P\times T)\cup(T\times P) \to \mathbb{N}$*. Where* $P \cap T = \emptyset$ *and* $P \cup T \neq \emptyset$*. And* $F(p, t)$ *defines the weight of the arc that goes from the place* p *to the transition* t *(input arc) and* $F(t, p)$ *defines the weight of the arc that goes from the transition* t *to the place* p*.*

The Petri net is a bipartite digraph, i.e. it has a graphical representation. Usually the elements of P, the places, are drawn as empty circles and the elements of T, the transitions, are drawn as bars or rectangles. The net represents the structure of the system.

Definition 2 ***Marking*** *The marking of a Petri net is the function* M *that associates tokens or marks to the places* $M : P \to \mathbb{N}$*.*

The current marking of a net represents its current state. A system is represented by a marked Petri net

Definition 3 ***System or Marked Petri Net*** *A system is a marked petri net* $S = \{N, M\}$.

The behavior of the system is given by the token movement. The transitions represents the events and the places the states of the system.

Definition 4 ***Enabled Transitions*** *A transition* t *is enabled when all its inputs places have a marking at least the weight of the arc connecting to the place with the transition.*

$$\forall p \in P : M(p) \geq F(p, t)$$

An enabled transition can be fired.

Definition 5 ***Firing of a Transition*** *An enabled transition* t *can be fired, and in this way the marking of the net is modified, in the following way*

$$\forall p \in P : M_{k+1}(p) = M_k(p) - F(p, t) + F(t, p)$$

marks are remove from the transition's input places and placed on the transition's output places.

Decisions or conflict situations have a structural representation in the net.

Definition 6 ***Conflict*** *A conflict is a place whose marking enables several output transitions.*

A conflict represents a point or state where a choice must be made.

3 Logical Model

In the model proposed, we identified several typical elements.

- Arrival transition or Process,
- Exit transition or process,
- Decision Structure,
- Semaphores controlled exclusion set, i.e., collision avoidance processes.

which we describe more thoroughly in the following sections.

3.1 Typical Arrival Process or Transition

The arrival of the vehicles to the system is done from source transitions (transitions without input places). Since the control elements of the system, the semaphores, usually are operated considering time as discrete, we considere a discrete behavior of the system, where the events occur at each and every second. The upper limit on the number of vehicles that can arrive to the system per second, n, depends on the maximum speed considered and on the number of lanes. A defensive driving approach considers a two second separation between vehicles, however, this situation is rare in practice. For three lanes we have the following educated guesses:

$$n = \begin{cases} 6.66 \text{ vehicles}/s \text{ for } V_{\max} = 80km/hr \\ 4.99 \text{ vehicles}/s \text{ for } V_{\max} = 60km/hr \\ 3.33 \text{ vehicles}/s \text{ for } V_{\max} = 40km/hr \\ 1.66 \text{ vehicles}/s \text{ for } V_{\max} = 20km/hr \end{cases}$$

In this case the marking of the transition's output place will evolve on time according to the following expressions

$$\begin{aligned} &P(M_{t+1} = j | M_t = i, C = *) = P(j, i) \\ &\text{where} \quad \sum_{i=0,\ldots,n} P(j, i) = 1 \\ &\text{and} \quad i, j \in \{0, \ldots, n\} \end{aligned}$$

where $P(j, i)$ is the probability after i vehicles arrived at the previous instant, j vehicles arrive in the current instant, regardless of the control C.

3.2 Typical Decision Structure or Conflict

Inside the system, the drivers must take several decisions, according how deep they are in the system. So the sum of the probabilities at each place must be one. So we have:

- at the input place

$$P(M_{forward}|M_{inbound}) + P(M_{left}|M_{inbound}) = 1$$

- for those waiting for the first left turn

$$P(M_{return}|M_{inbound}) + \quad P(M_{left}|M_{inbound}) = 1$$

- for those within the intersection

$$P(M_{forward}|M_{inbound}) + P(M_{left}|M_{inbound}) + P(M_{right}|M_{inbound}) = 1$$

the left turn at this point is used by lost, confused or rogue drivers and ambulances, since it is the shortest route.

3.3 Typical Exit Process or Transition

At the output of the system we suppose that they are no constraints, that is the vehicles exit the system at top speed, so the firing of the output transitions (sink transitions or transitions without output places) will depend on the current marking of its input place according to:

$$M_{t+1} = M_t - n \text{ when } M_t \geq n$$
$$M_{t+1} = M_t - c \text{ when } M_t = c < n$$

so a soon a mark arrives to these places it enables the transition, with the only restriction caused by the maximum output speed. This is only true in an uncoupled system.

3.4 Typical Semaphore Controlled Transition or Exclusion set

The function of the semaphores is to give the right of way to certain parts of the intersection that are shared. They work sequentially, with a fixed schedule. They are fair, since they allow the crossing of the intersection to everyone that access it at one point in time. Frequently this is not the optimal behaviour, since the flow in certain senses depends on the time of the day, but it is a fair one. A better aim could be to minimize the waiting time of every vehicle in the system.

The following expressions try to model the marking change under the described behavior and according to the control input C (color of the light)

- **C=Red light**
  - Input place marking change

$$P(M_{t+1} = M_t + r_t | M_t, C = \text{“red”}) \overset{a.s.}{=} 1$$
$$\text{where} \quad 0 \leq r_t \leq n$$
$$\text{and} \quad t \in [t_{\text{red}_0} + \delta_{\text{red}}, t_{\text{red}_0}]$$

- **C=Green light** or **C=Yellow light**
  - Input and Output place marking change

$$1 \geq P(M_{t+1} = M_t \pm c_t | M_t, C = \text{“c”} \geq 0$$
$$\text{where} \quad 0 \leq c_t \leq n$$
$$\text{and} \quad t \in [t_{\text{c}_0} + \delta_{\text{c}}, t_{\text{c}_0}]$$

$$\delta_{\text{green}} + \delta_{\text{yellow}} = \delta_{\text{red}}$$

3.5 Timed Places

In order to have a simpler net, we consider that each place for each mark in the net has associated a distance d_p and a speed v_p that are used to model the time $t_p = d_p/v_p$ the vehicles spent in the state p before being able to enable the output transitions. Where d_p is fixed and $0 \geq v \geq V_{\max}$.

4 Conclusions and Further Work

This model is only starting point. Since traffic changes a lot during the day and night hours, a fixed schedule will annoy drivers and will tempt them to ignore the signals. To avoid identifying the different parameters, like in[5], of the system and have an intelligent system, a state of the art system could be proposed, that uses cameras to identify the load in each part of the system.

Figure 1: Physical model of the intersection analysed (North at the top)

References

[1] Thomas R. Horton, *Traffic Control - Theory and Instrumentation*, Plenum Press, New York, 1965

[2] V. Van Breusegem, G. Campion and G. Bastin, *Traffic Modeling and State Feedback Control for Metro Lines*, IEEE Trans. on Automatic Control, Vol.36, No.7, pp.770-784, July 1991

[3] Philip E. Sarachik, Umit Ozguner, *On DecentralizedDynamnic Routing for Congested of traffic networks*, IEEE Trans. Automatic Control, Vol.27, pp.677-688, June 1982

[4] John S. Baras, William Levine and Tahsin L. Lin, *Discrete-Time Point Processes in Urban Traffic Queue Estimation*, IEEE Trans. on Automatic Control, Vol.24, No.1, pp.12-27, 1979

[5] D. Xue and Z. Dong,*An Intelligent Contraflow Control Method for Real-Time Optimal Traffic Scheduling using Artificial Neural Network, Fuzzy Patter Recognition, and Optimization*, IEEE Trans. on Control Systems Technology, Vol. 8. No. 1,pp.183-191 January 2000

[6] N. Viswanadham and Y Narahari,*Performance Modeling of Automated Manufacturing Systems*, Prentice Hall, Englewood Cliffs, 1992

[7] Jiacun Wang, *Timed Petri Nets: Theory and Applications*, Kluwer Academic Publishers, Boston, 1998

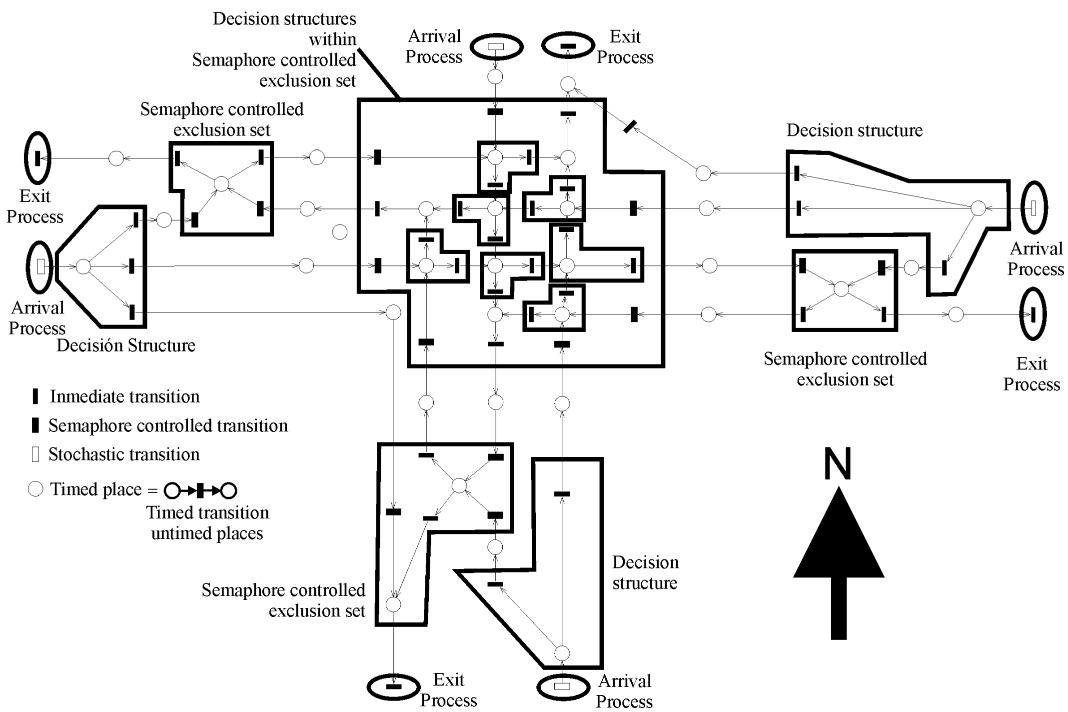

Figure 2: Simplified Petri Net Model (logical model) of the intersection considered