# Control of differential mobile robots: a synchronization approach

**A. Rodriguez-Angeles**
Mexican Petroleum Institute
Program of Applied Mathematics and Computation
Eje Central Lazaro Cardenas 152
Mexico D.F., 07730, Mexico
Tel: (+52 55) 3003 7235, Fax: (+52 55) 3003 6277
E-mail: arangele@imp.mx

## Abstract

In this paper a synchronization controller for differential mobile robots is proposed. The synchronization goal is to control the position of each wheel to a desired trajectory and at the same time the differential (or synchronization) error between the coordinates of the two wheels. In this way it is ensured that the mobile robot follows the desired path trajectory. The controller is based on a computer torque architecture and a set of observers to estimate the angular velocity of the wheels. So that the controller only requires angular position measurements of the wheels.

## 1 Introduction

Nowadays the developments on technology and requirements on efficiency and quality in production processes have originated more complex and integrated systems. These integrated systems are a synergy of many different disciplines such as mechanics, electronics, control, etc.. The final goal of this synergy is to improve the performance, and in many cases to give rise to more flexible and robust systems.

In manufacturing processes, automotive applications and tele-operated systems there is a high requirement on flexibility and manoeuvrability of the involved systems. In most of these processes the use of mechanical systems, particularly robot manipulators and mobile systems, are widely spread, and their variety in uses is practically endless, e.g. assembling, transporting, painting, welding, grasping.

When transporting big and/or heavy objects, the use of a single mobile system is not enough. In such situations a set or team of two or more mobile robots is considered. The mobile robots in such a system usually work in a synchronous manner in order to execute the task (Siméon *et al.* 2002), (Sugar and Kumar 2002). The synchronization scheme gives manoeuvrability and dexterity that can no be achieved by a single system. In practice there are two basic synchronization schemes, coordinate schemes and cooperative (mutual) schemes. In coordinate system there exist a leader system and the other systems are supposed to coordinate with respect to the dominant one. In cooperative systems all system appears with the same hierarchy, such that the behavior of the system is the result of interactions between all the individual system.

The synchronization phenomenon was perhaps first reported by Huygens, (Huygens 1673), who observed that a pair of pendulum clocks hanging from a light weight beam oscillated with the same frequency. Synchronized sound in nearby organ tubes was reported by Rayleigh in 1877 (Rayleigh 1945), who observed similar effects for two electrically or mechanically connected tuning forks. In the last century synchronization received a lot of attention in the Russian scientific community since it was observed in balanced and unbalanced rotors and vibro-exciters (Blekhman 1988), (Blekhman *et al.* 1995). In mechanical systems there are several works about synchronization (Nijmeijer and Rodriguez-Angeles 2003), (Rodriguez-Angeles 2002) and robot synchronization in particular (Rodriguez-Angeles *et al.* 2002), (Liu *et al.* 1997).

In astronomy, synchronization theory is used to explain the motion of celestial bodies, such as orbits and planetary resonances, (Blekhman 1988). In biology, biochemistry and medicine many systems can be modelled as oscillatory or vibratory systems and those systems show a tendency towards synchronous behavior. Among evidences of synchronous behavior in the natural world, one can consider the chorusing of crickets, synchronous flash light in group of fire-flies.

Synchronization control provides a unique set of advantages and opportunities to solve the problem of control of differential mobile robot. The challenging problem behind control of differential mobile robots in a plane is that such systems possess three degrees of motion freedom while they are controlled by two external inputs, under non-holonomic constraints. Therefore such systems in the Cartesian space cannot be stabilized via differentiable, or even continuous, pure-state feedback (Brockett 1983). In contrast, if the controller is to directly stabilize the wheel coordinates to the desired state, then continuous feedback control law exists, since the wheel dynamics satisfies the passivity between input torques and output velocities. However, the convergence of wheel coordinates cannot guarantee the convergence of the robot configuration in Cartesian space, due to non-holonomic constraints in the mapping.

The synchronization control can be applied to solve the above problem. Since the heading angle of a differential robot is greatly affected by the differential (or synchronization) error between the coordinates of the two wheels, regulation of the wheel coordinates in a synchronous manner plays a key role in improvement of the heading angle control. Thus the synchronization control goal for differential robots can be stated as to control two wheel coordinates and one synchronization error between them by mean of two control inputs (inputs in the driving wheels).

In a similar manner to the work presented in (Sun *et al.* 2002) the goal is to controlled the coordinates of the wheels in the mobile robot, rather than the robot configuration, in a synchronous manner. The synchronization goal is to regulate the differential position error of two driving wheels to zero and thus regulate the heading angle of the mobile robot, so that the robot keeps in the desired path trajectory .

In (Sun *et al.* 2002) a tracking stabilization controller for the differential mobile robot using synchronization errors has been proposed. The controller is based on a computed torque architecture and sliding modes, it has on-line adaptation of the parameters of the robot, and measurements of the angular position and velocity of the wheels are assumed.

In this paper a synchronization controller for the differential mobile robot is proposed. The controller is an extension of the mutual synchronization controller proposed in (Rodriguez-Angeles 2002) and (Nijmeijer and Rodriguez-Angeles 2003). The proposed controller is based on a computed torque architecture and uses estimated values for the angular velocity of the wheels. So that only angular positions measurements are required.

The paper is organized as follows. The kinematic and dynamic model of the differential mobile robot is introduced in Section 2. In Section 3 the synchronization controller is proposed, while a simulation study is presented in Section 4. The paper closes with some conclusions in Section 5.

# 2 Model of the differential mobile robot

Consider a differential mobile robot with two driving wheels mounted on the same axis and a front passive wheel, see Figure 1. The configuration of a mobile robot in the Cartesian space is given by

$$q(t) = [\ x(t) \quad y(t) \quad \theta(t)\ ] \tag{1}$$

where $q$ denotes the generalized Cartesian coordinates, $x(t)$ and $y(t)$ are the coordinates of the mass center of the mobile robot, and $\theta(t)$ denotes the heading angle. The kinematic model of the mobile robot in the Cartesian coordinates $q$ is given by

$$\dot{x} = u\cos\theta, \qquad \dot{y} = u\sin\theta, \qquad \dot{\theta} = \omega \tag{2}$$

where no-slip of the wheels is assumed, and the forward velocity $u$ and the angular velocity $\omega$ are considered as inputs.

The no-slip condition on the wheels imposes the non-holonomic constraint

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0 \tag{3}$$

In the Cartesian space the kinematic model of the differential robot is given by (2). Nevertheless the synchronization controller here proposed is based on the wheel angle space.

Consider $\phi_1(t)$ and $\phi_2(t)$ as the angular displacements of the left and the right driving wheels respectively, and denote the linear velocity of each driving wheel by $v_i(t)$. Then the angular velocity of the mobile robot, $\dot{\theta}(t)$, is given by

$$\dot{\theta}(t) = \frac{v_2(t) - v_1(t)}{2R} = \frac{r_w}{2R}(\dot{\phi}_2(t) - \dot{\phi}_1(t)) \tag{4}$$

where $2R$ denotes the distance between the two driving wheels, and $r_w$ is the radius of the driving wheels, which are assumed to be identical. From (4) the heading angle of the mobile robot $\theta$ is given by

$$\theta(t) = \int_0^t \dot{\theta}(w)dw = \theta_c + \frac{r_w}{2R}(\phi_2(t) - \phi_1(t)) \tag{5}$$

where $\theta_c$ is a constant of integration, which depends on the initial conditions on the wheels of the mobile robot.

The dynamics of the angle displacement $\phi_i(t)$, $i = 1, 2$ of the wheels is given by

$$H_i(\phi_i)\ddot{\phi}_i + C_i(\phi_i, \dot{\phi}_i)\dot{\phi}_i + F_i(\phi_i, \dot{\phi}_i) = \tau_i \tag{6}$$

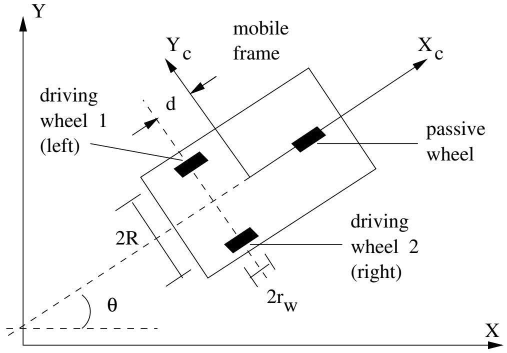


Figure 1: A differential mobile robot.

where $H_i(\phi_i)$ and $C_i(\phi_i, \dot{\phi}_i)$ denote the inertia and the nonlinear terms related to centrifugal forces respectively, $F_i(\phi_i, \dot{\phi}_i)$ denotes the force due to friction, and $\tau_i$ denotes the input torque.

# 3 Synchronization controller

Consider a desired path trajectory in the Cartesian space given by $q_d(t) = [\ x_{cd}(t) \quad y_{cd}(t) \quad \theta_d(t)\ ]$. It follows that for each trajectory $q_d(t)$ there exist a unique desired angle displacement of the wheels $i = 1, 2$ denoted by $\phi_{di}(t)$.

Since the mapping

$$q_d(t) \in \mathbb{R}^3 \rightarrow \begin{bmatrix} \phi_{d1}(t) \\ \phi_{d2}(t) \end{bmatrix} \in \mathbb{R}^2 \tag{7}$$

is unique, then $\phi_{di}(t)$ can be uniquely determined from $q_d(t)$ in the path planning. Note that the convergence $\begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix} \rightarrow \begin{bmatrix} \phi_{d1}(t) \\ \phi_{d2}(t) \end{bmatrix}$ does not necessarily lead to $q(t) \rightarrow q_d(t)$, because the mapping $\begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix} \rightarrow q(t)$ is not unique.

The above problem can be solved by considering synchronization control of the two driving wheels. The synchronization goal is to regulate the differential position error of the two driving wheels to zero and hence, regulate the heading angle $\theta(t)$, given by (5), so that the robot maintains in the desired trajectory path.

Figure 2 shows the two driving wheels of the robot in a desired curve path at instantaneous time $t$. The desired angular displacement of the wheels are denoted by $\phi_{d1}(t)$ and $\phi_{d2}(t)$.

The angular displacement error of each wheel is defined as

$$e_i(t) = \phi_i - \phi_{di}(t) \tag{8}$$

such that the linear displacement error of the wheels is $r_w e_i(t)$.

Note that $R_{ca}(t)$ and $R_{cd}(t)$ in Figure 2 denote the curvature radii of the path in which the center of the robot lies, when the robot is in its actual and desired positions, respectively.

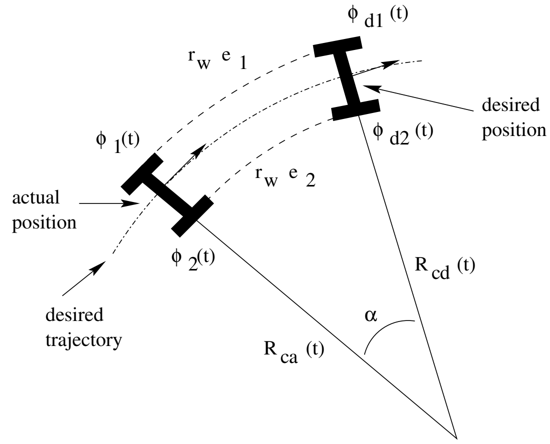


Figure 2: Two driving wheels in the desired trajectory.

Some assumptions are imposed on the desired path trajectory.

**Assumption 1** *The desired trajectory considered throughout this paper is smooth and continuous, and the curvature radius does not vary significantly so that*

- *The difference of curvature radii $R_{ca}(t)$ and $R_{cd}(t)$ is small enough to be neglected, i.e. $R_{ca}(t) \approx R_{cd}(t) = R_c(t)$.*
- *The curvature radius $R_c(t)$ is such that $R_c(t) > R$, and is twice differentiable.*

For a circular path, Assumption 1 holds since the curvature radius is constant. When the curvature radius varies along the path, Assumption 1 holds if $e_1(t)$ and $e_2(t)$ are small enough. The linear path is treated as a special case where the curvature radius is infinitely large.

From Figure 2 when the robot is in the desired trajectory path, it must satisfy the following kinematic relationship

$$\frac{r_w e_1(t)}{R_c(t) \pm R} = \frac{r_w e_2(t)}{R_c(t) \mp R} = \alpha \tag{9}$$

Under the condition that the initial error of the robot configuration is zero, keeping equation (9) valid ensures that the robot maintains in the desired path. To define the synchronization error that guarantees that the mobile robot maintains in the desired path, let rewrite (9) as

$$c_1(t)e_1(t) + c_2(t)e_2(t) = 0 \tag{10}$$

define the synchronization error $\epsilon$ of the two driving wheels as

$$\epsilon(t) = c_1(t)e_1(t) + c_2(t)e_2(t) \quad (11)$$

where $c_i(t)$ denotes the cross-coupling parameter which are defined by

$$c_1(t) = \frac{R_c(t)}{R_c(t) \pm R}, \qquad c_2(t) = -\frac{R_c(t)}{R_c(t) \mp R} \quad (12)$$

with $R_c(t)$ the curvature radius of the desired path trajectory $q_d$, and $2R$ the distance between the driving wheels. In the special case of straight line, $R_c = \infty$, then $c_1 = 1$ and $c_2 = -1$

Achieving the synchronization error $\epsilon(t) \to 0$ is equivalent to maintain the kinematic relationship (9), thus ensuring that the mobile robot maintains in the desired path. Therefore, ensuring that the synchronization error $\epsilon(t)$ converge to zero can be considered as an additional goal in addition to ensure that the tracking errors $e_i(t) \to 0$.

According to the work presented in (Sun *et al.* 2002) it follows that the control objective in the Cartesian space, i.e.

$$\Delta q(t) = \begin{bmatrix} x_c - x_{dc} & y_c - y_{dc} & \theta - \theta_d \end{bmatrix} \to 0 \quad (13)$$

which is widely used in mobile robots in the literature, is equivalent to the objective

$$\begin{bmatrix} e_1(t) & e_2(t) & \epsilon(t) \end{bmatrix} \to 0 \quad (14)$$

The Cartesian goal (13) is equivalent to the objective (14) under the condition that the initial error of the robot configuration is zero. The use of the objective given by (14) avoids involvement of non-holonomic constraint in the control design, and hence makes it possible to design a continuous feedback controller to achieve tracking of a desired path.

**Remark 2** *When the curvature radius varies along the trajectory path, a calculation error exists in the computation of $c_i(t)$ by (12). This is because the difference between $R_{ca}(t)$ and $R_{cd}(t)$ is neglected in Assumption 1. This calculation error affects the motion performance of the robot more obviously as the curvature radius changes significantly along the path. Since small errors $e_1(t)$ and $e_2(t)$ reduce this calculation error, the proposed synchronization strategy must ensured good and fast convergence of $e_i(t) \to 0$.*

In (Sun *et al.* 2002) a tracking stabilization controller for the differential mobile robot using the synchronization error (11) has been proposed. The controller is based on a computed torque architecture and sliding modes, it has on-line adaptation of the parameters of the robot, and measurements of the angular velocity of the wheels are assumed.

In this section a synchronization controller for the differential mobile robot is proposed. The controller is an extension of the mutual synchronization controller proposed in (Rodriguez-Angeles 2002) and (Nijmeijer and Rodriguez-Angeles 2003).

For the design of the proposed synchronization controller for the differential mobile robot the following assumptions are introduced.

**Assumption 3** *Only angular positions of the driving wheels $\phi_1, \phi_2$ are measured. No angular velocities, neither angular accelerations are measured.*

**Assumption 4** *Friction effects in the driving wheels are neglected, so*

$$F_i(\phi_i, \dot{\phi}_i) = 0, \qquad i = 1, 2 \quad (15)$$

**Assumption 5** *The initial tracking errors $e_i = \phi_i - \phi_{di}$ and the initial error in the heading angle $\theta - \theta_d$ are all equal to zero. It implies that the mobile robot is in the desired trajectory and with the proper orientation as initial condition.*

**Assumption 6** *The desired path trajectory $q_d(t)$ is smooth and continuous.*

**Remark 7** *Assumption 5 is required for stability analysis of the proposed controller. Nevertheless the synchronization controller can deal with initial errors different from zero as shown in section 4.*

Consider that the controller for the driving wheels $i = 1, 2$ is given by

$$\tau_i = H_i(\phi_i)\widehat{\ddot{\phi}}_{ri} + C_i(\phi_i, \widehat{\dot{\phi}}_i)\widehat{\dot{\phi}}_{ri} - K_{d,i}\widehat{\dot{s}}_i - K_{p,i}s_i \quad (16)$$

where $H_i(\phi_i)$, $C_i(\phi_i, \widehat{\dot{\phi}}_i)$ are defined as in (6), $K_{p,i}, K_{d,i}$ are positive gains, $s_i$ and $\widehat{\dot{s}}_i$ are synchronization errors, and $\widehat{\ddot{\phi}}_{ri}$, $\widehat{\dot{\phi}}_{ri}$ are nominal reference variables, which are based in the desired path trajectories $\phi_{di}$ and the synchronization goal given by (11).

The synchronization errors $s_i$ and $\widehat{\dot{s}}_i$ are defined by

$$s_i = \phi_i - \phi_{ri}, \qquad \widehat{\dot{s}}_i = \widehat{\dot{\phi}}_i - \widehat{\dot{\phi}}_{ri} \quad (17)$$

with $\phi_{ri}$, $\widehat{\dot{\phi}}_{ri}$, and $\widehat{\ddot{\phi}}_{ri}$ nominal references that are defined as

$$\phi_{ri} = \phi_{di} + \beta_i \int_0^t \epsilon(w)dw \quad (18)$$

$$\hat{\dot{\phi}}_{ri} = \dot{\phi}_{di} + \beta_i \epsilon \quad (19)$$

$$\hat{\ddot{\phi}}_{ri} = \ddot{\phi}_{di} + \beta_i \dot{\epsilon} \quad (20)$$

with the synchronization error $\epsilon(t)$ given by (11).

The first term on the right hand side of $\phi_{ri}$ gives the desired path trajectory $\phi_i$, thus is in charge of convergence between the angular position and its desired value. The second term is a feedback of the synchronization error, thus regulates the heading angle and induces the synchronous behavior between the driving wheels. Note that the definition of the synchronization error $s_i$ and the nominal reference $\phi_{ri}$ imply a trade off between the tracking errors $e_i = \phi_i - \phi_{di}$ and the synchronization error $\epsilon$. Therefore those errors can be penalized to favor either convergence of $e_i$ or small heading angle errors by better convergence of $\epsilon$.

The synchronization control (16) and the nominal references (18 - 20) are based on estimated angular velocities $\hat{\dot{\phi}}_i$, which are obtained by the full state nonlinear observer, $i = 1, 2$

$$\begin{aligned} \frac{d}{dt}\hat{\phi}_i &= \hat{\dot{\phi}}_i + \mu_{i,1}\tilde{\phi}_i \\ \frac{d}{dt}\hat{\dot{\phi}}_i &= -H_i(\phi_i)^{-1}\left[C_i(\phi_i, \hat{\dot{\phi}}_i)\hat{\dot{\phi}}_i - \tau_i\right] + \\ &\quad \mu_{i,2}\tilde{\phi}_i \end{aligned} \quad (21)$$

where $\mu_{i,1}$, $\mu_{i,2}$ are positive gains, $\hat{\phi}_i, \hat{\dot{\phi}}_i$ represent estimated values for $\phi_i, \dot{\phi}_i$, and the estimation position error $\tilde{\phi}_i$ is defined by

$$\tilde{\phi}_i := \phi_i - \hat{\phi}_i \quad (22)$$

After substitution of the synchronization controller (16) and the nominal references (18 - 20) the nonlinear observer (21) can be written as

$$\begin{aligned} \frac{d}{dt}\hat{\phi}_i &= \hat{\dot{\phi}}_i + \mu_{i,1}\tilde{\phi}_i \\ \frac{d}{dt}\hat{\dot{\phi}}_i &= \hat{\ddot{\phi}}_{ri} - H_i(\phi_i)^{-1}\Big[C_i(\phi_i, \hat{\dot{\phi}}_i)\hat{\dot{s}}_i + \\ &\quad K_{d,i}\hat{\dot{s}}_i + K_{p,i}s_i\Big] + \mu_{i,2}\tilde{\phi}_i \end{aligned} \quad (23)$$

The purpose of this paper is to present possible extensions of the general ideas about synchronization developed in (Rodriguez-Angeles 2002) and (Nijmeijer and Rodriguez-Angeles 2003). Therefore a stability analysis of the closed loop between the driving wheels modelled by (6) and the synchronization controller (16) is beyond the scope of the paper, the interested reader is referred to (Rodriguez-Angeles 2002) and (Nijmeijer and Rodriguez-Angeles 2003). As a manner of validation of the proposed extension a simulation study is presented. This simulation study does not intend to substitute the stability proof, but gives an idea that the proposed synchronization controller (16) yields tracking between the mobile robot and a desired path trajectory.

# 4 Simulation study

The dynamic model of the driving wheels, when friction effects are neglected and not centrifugal forces are considered, is given by the very simple equation

$$H_i\ddot{\phi}_i = \tau_i \quad (24)$$

The parameters of the differential mobile robot have been chosen arbitrarily and only for simulation purposes. The parameters of the robot are the radius of the wheel $r_w = 0.04$ [m], the separation between wheels $2R = 0.29$ [m], and the inertial values $H_1 = 0.50$ and $H_2 = 0.52$ [kg · m $^2$]. The gains in the synchronization controller (16) and the observer (23) have been chosen by trial and error, the values of the gains are the same for both driving wheels and are given by $K_p = 1000$, $K_d = 100$, $\mu_1 = 20$, $\mu_2 = 500$ and $\beta = 500$.

The differential mobile robot is commanded to move along a circular path with radius $R_c = 2.0$ [m]. The circular trajectory in terms of the desired angle positions is given by

$$\phi_{d1}(t) = \left(0.3571t^2 - 0.0119t^3\right)\frac{R_c + R}{R_c} \qquad \text{[rad]} \quad (25)$$

$$\phi_{d2}(t) = \left(0.3571t^2 - 0.0119t^3\right)\frac{R_c - R}{R_c} \qquad \text{[rad]} \quad (26)$$

As claimed in Remark 7 the synchronization controller can deal with initial errors different from zero, as to show this the initial position of the wheels are set as $\phi_1 = 0.2$ and $\phi_2 = 0.1$ [rad]. So the initial wheel errors $e_i = \phi_i - \phi_{di}$ are $e_1 = 0.2$ and $e_2 = 0.1$ [rad], see Figure 3.

For the circular trajectory the coupling parameters $c_i(t)$ in the synchronization goal (11) are constant and given by

$$c_1 = \frac{R_c}{R_c + R}, \qquad c_2 = -\frac{R_c}{R_c - R} \quad (27)$$

Figure 3 shows the angle position errors (tracking errors) $e_i = \phi_i - \phi_{di}$ and the synchronization error $\epsilon = c_1e_1 + c_2e_2$. The synchronization controller forces the synchronization error $\epsilon$ to zero, thus eliminating the error in the heading angle. Note that the tracking errors $e_1$ and $e_2$ and the synchronization error $\epsilon$ show fast convergence.

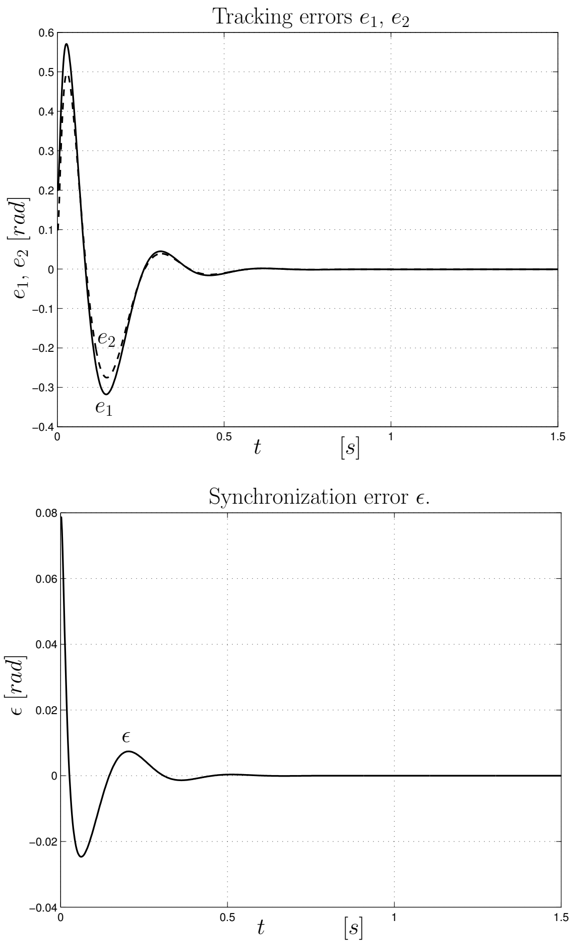


Figure 3: Tracking wheel errors $e_1$ (solid), $e_2$ (dashed), and synchronization error $\epsilon$.

# 5 Conclusions

A synchronization control strategy for tracking of differential mobile robots has been proposed. The synchronization strategy guarantees stabilization and tracking of the wheels of the mobile robots and of the differential error between the wheels. Such that the differential mobile robot follows a desired path trajectory if the initial value of the errors is zero, i.e. the robot starts on the desired path.

The synchronization controller regulates the coordinate of the wheel directly, thus it avoids the common problems yield by the non-holomonic constraints to which the mobile robot is subjected. The proposed controller is based on a computer torque strategy and uses estimates for the velocity of the wheels, that are obtained by model based observers.

# References


Blekhman, I. I. (1988). *Synchronization in Science and Technology*. ASME Press Translations. New York.

Blekhman, I. I., P. S. Landa and M. G. Rosenblum (1995). Synchronization and chaotization in interacting dynamical systems. *ASME Applied Mechanical Review* **48**, 733–752.

Brockett, R.W. (1983). Asymptotic stability and feedback stabilization. In: Differential Geometric Control Theory, Brockett,R.W., Millman, R.S., and Sussman, H.J., Eds.. pp. 181–191. Birkhäuser. Boston, MA, USA.

Huygens, C. (1673). *Horoloqium Oscilatorium*. Paris, France.

Liu, Y. H., S. Arimoto, V. Parra-Vega and K. Kitagaki (1997). Decentralized adaptive control of multiple manipulators in cooperations. *International Journal of Control* **67**, 649–673.

Rodriguez-Angeles, A. (2002). Synchronization of mechanical systems. PhD thesis. Eindhoven University of Technology, The Netherlands.

Rodriguez-Angeles, A., H. Nijmeijer and H. A. van Essen (2002). Coordination of rigid and flexible joint robot manipulators, In Advanced Dynamics and Control of Structures and Machines, H. Irschik and K. Schlander (Eds.). pp. –. Springer-Verlag. New York.

Nijmeijer, H. and A. Rodriguez-Angeles (2003). *Synchronization of mechanical systems*. World Scientific Publishing. Singapore.

Rayleigh, J. (1945). *Theory of Sound*. Dover. New York.

Siméon, T., S. Leroy and J.-P. Laumond (2002). Path coordination for multiple mobile robots: a resolution-complete algorithm. *IEEE Transactions on Robotics and Automation* **18**, 42–49.

Sugar, T.G. and V. Kumar (2002). Control of coopearting mobile manipulators. *IEEE Transactions on Robotics and Automation* **18**, 94–103.

Sun, D., H.N. Dong and S.K. Tso (2002). Tracking stabilization of differential mobile robots using adaptive synchronized control. In: *Proceedings of the 2002 IEEE International Conference on Robotics and Automation*. pp. 2638–2643.