A parameter perturbation method for controlling Chua's circuit: experimental setup

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Abstract. *Chaos control has been an expanding research area in the last decade, therefore a great amount of techniques have been developed for controlling or suppressing chaos. Some of the most popular techniques are the so-called parametric perturbation methods [4]. These techniques use an accessible system parameter to be changed dynamically to stabilize one of the system's unstable periodic orbits (UPO's). In this paper we report a variation of the OGY approach [13] and its application to control chaos in a Chua's circuit. This variation consists in using a Lorenz Map (LM) instead of a Poincaré Map to describe the system's dynamics in the neighborhood of an UPO. The proposed method allows us to avoid ill-conditioned mean squares procedures and to have a better implementation of the OGY control. Numerical and experimental results applied to the Chua's circuit are shown.*

Keywords. Chaos Control, Chua´s Circuit, Parameter Perturbation, Periodic Orbits Stabilization.

1. Introduction

In the last two decades there has been big efforts in the field of nonlinear dynamics, specially dealing with chaos control. This is due to the consideration that chaotic oscillations are a peculiar kind of irregular and unpredictable behaviors commonly considered as undesirable, and that is why in most cases this behavior is often regulated or avoided as much as possible. Although there have been a lot of efforts from researchers dealing with chaos control, there is no unified chaos control theory. Unlike the classic control problems where stabilization has a clear meaning, the stabilization of chaos can be understood in many different ways. Some authors define the control target as the stabilization of equilibrium points; the elimination of multiple attraction basins, and in some cases the stabilization of unstable periodic orbits among others. This takes us to different control approaches like: redesign control methods, parametric perturbation methods, external force methods, and engineering control methods [4].

Some parametric perturbation methods are derived from the OGY method [13]. This method is based in the statement that a chaotic attractor is a closure of an infinite number of Unstable Periodic Orbits (UPO's), and that using small parametric perturbations to an accessible parameter the system will stabilize to one of those UPO's. The method consists in forcing the system trajectory onto the stable manifold of a fixed point in a map that represents the system dynamics in a lower dimension. The proposed method uses a Lorenz Map (LM) [9] instead of the Poincaré Map originally proposed in [13]. The use of the LM avoids ill-conditioned identification problems in describing the local behavior of a selected UPO. One of the main features of this technique is that it can be implemented from time series without an *a priori* knowledge of the equations that describe the system behavior.

The paper is organized as follows, in the next section some convenient dynamical system definitions are given, section 3 deals with the parametric perturbation method used. In section 4, results of the application of the control method to the Chua´s circuit are shown in simulation, section 5 deals with the experimental implementation on the circuit and the results of this experiment are given. Finally in section 6 some concluding remarks are given.

[1] This work was done while the author was at the Research Center in Universidad La Salle, Mexico City.

2. Chaotic Systems

Let us consider a system described by:

$$\dot{x} = f(x, t, \beta) \qquad (1)$$

where $x \in \Re^n$ is the state, $f : \Re^n \to \Re^n$ is a smooth vector field, and β denotes the system parameters. The solution of (1) is some vector function $x = x(t)$ that describes the trajectories in the state space constructed with its coordinates. Depending on the parameter values, some systems may display different steady states, ranging from equilibrium points to chaotic attractors.

Definition 1 *(Chaotic Attractor) [15]. Consider a C^r ($r \geq 1$) autonomous vector field on $\Re^n$, defining a system like (1). Denote the flow generated by (1) as $\phi(t,x)$, and assume that $A \subset \Re^n$ is a compact set, invariant under $\phi(t,x)$. Then A is said to be chaotic if it has the following properties:*

i. Sensitive dependence on initial conditions. There exists $\varepsilon > 0$ such that, for any $x \in A$ and any neighborhood U of x, there exist $y \in U$ and $t > 0$ such that $|\phi(t,x) - \phi(t,y)| > \varepsilon$.

ii. Topological transitivity. For any two open sets U, $V \subset A$, there exists $t \in \Re$ such that $\phi(t,U) \cap V \neq \emptyset$. □

Definition 2. (Correlation Dimension). *Let A be a bounded subset of $\Re^n$ and C(r) a function proportional to the probability that two arbitrary points on the orbit in state space are closer together than r (for constructing C(r) see for example [14]). Then, the correlation dimension is defined by [8]:*

$$D_c(A) = \lim_{r \to 0} \frac{\log C(r)}{\log(r)} \qquad (2)$$

□

Typically, this quantity is not an integer number for a chaotic attractor *A*. When this situation occurs it is said that *A* is a *fractal* set.

3. Control of Chaotic Systems by Parameter Perturbation Methods

In 1990, Ott, Grebogi and Yorke [13] developed a technique for controlling nonlinear systems by stabilizing an UPO embedded in a chaotic attractor, with time dependent parametric perturbations. Since the first appearance of the method there have been many other approaches on parametric perturbation techniques [2], [3], [5], [12].

A chaotic attractor can be thought as the closure of an infinite number of unstable periodic orbits (UPO) [13], the OGY method takes this main idea and establishes that a little variation of an accessible system parameter is enough to stabilize the orbit.

The system trajectory is forced to cross close the stable manifold of a fixed point in a discrete map representing the system dynamics in a lower dimension, i.e. forces the trajectory to track a stable orbit in the original phase space.

We distinguish two approaches for the implementation of this control method. One of them is applied when the fractal dimension of the attractor is close to the integer and the other in the reverse case. If the fractal dimension is not close to the integer it is possible to use the classical approach. This approach consists mainly in reconstructing the attractor and identifying the local dynamics in a neighborhood of an unstable fixed point in the Poincaré Map. The second approach considers systems in which the attractor dimension is close to an integer. The main tool in this approach is the Lorenz Map (LM). This LM is obtained from the local maxima of the measured variable and delaying this very signal to form a vector of delayed coordinates similar to the one formed for the reconstruction of the attractor [11], [1], [7]. The Lorenz Map (LM) allows us to determine an UPO and locate easily the fixed point about which we want to control the chaotic system

According to this scheme it is only needed to identify a periodic unstable orbit of the attractor (a point in the LM), locally characterize its dynamics and determine the change of the attractor to external stimulus to find a suitable control law in order to stabilize the associated UPO.

One of the advantages of this method is that it is possible to implement it without any prior knowledge about the system equations. This feature is the one that makes this kind of methods so popular for the experimental control of chaotic systems. In what follows a detailed description of the proposed method is given.

3.1 Identification of unstable periodic motion

The LM is obtained from sampling consecutive local maxima of the measured variable, as shown in Fig. 1. Let us denote this sampled signal as $\{z\}_{n=1}^{N}$. The explicit nature of the dynamics of the LM is unknown; therefore, a reconstruction of this map is required in order to characterize the local dynamics around the selected fixed point. Likewise for attractor reconstruction [1], we apply the delay operator to this sampled signal to form a vector of *m* delayed coordinates. Now let us define a new vector $\xi \in \Re^m$ to denote the discrete time evolution of consecutive local maxima. For *m*=2, this vector is related to ζ as

$$\xi = \begin{bmatrix} \xi_n^{(1)} \\ \xi_n^{(2)} \end{bmatrix} = \begin{bmatrix} z(n) \\ z(n-1) \end{bmatrix} \qquad (3)$$

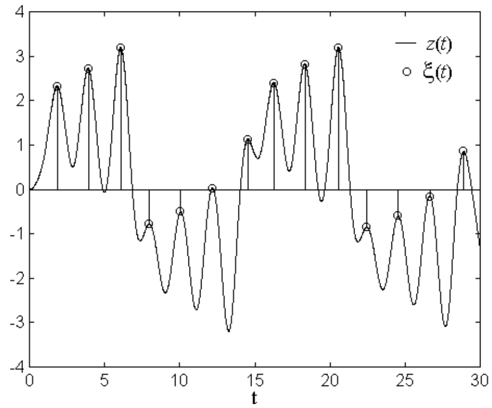

Fig. 1 Time series used for the Lorenz Map

Then, although explicitly unknown, the Lorenz Map associated to a specific value of a parameter β can be expressed by

$$\xi_n = P(\xi_{n-1}, \beta) \tag{4}$$

with P a nonlinear function. This map describes the dynamics of the (maximal) sampled values of the original measured signal and it has an m-dimensional fixed point ξ_F (related to a periodic orbit in the original space), the location of this point depends also on the parameter β, that is $\xi_F = \xi_F(\beta)$. Due to the nature of the unstable periodic orbit embedded in the chaotic attractor, the fixed point ξ_F is expected to be a saddle.

3.2 Determination of the attractor change to external stimulus

The change in the location of the fixed point due to a parameter perturbation can be estimated with a shift vector s:

$$s \equiv \frac{\partial}{\partial \beta} \xi_F(\beta)\bigg|_{\beta=\beta_n} \approx \frac{\xi_F(\beta_{n+1}) - \xi_F(\beta_n)}{\beta_{n+1} - \beta_n} \tag{5}$$

3.3 Characterization of the attractor form

We may represent the map (4) in a neighborhood of the fixed point using a square 2-dimensional matrix

$$\xi_{n+1} \approx \xi_F + M(\xi_n - \xi_F) \tag{6}$$

Matrix $\boldsymbol{M}$ is easily obtained from a mean square identification method and it is characterized by its stable λ_s and unstable λ_u eigenvalues and eigenvectors: e_s and e_u. The eigenvectors are normalized $e_u^T e_u = 1$, $e_s^T e_s = 1$. Now consider the dual base vectors $\{f_u, f_s\}$ in terms of vectors $\{e_u, e_s\}$ demanding $f_s^T e_s = 1 = f_u^T e_u$ and $f_u^T e_s = 0 = f_s^T e_u$, thus $\begin{bmatrix} f_u^T & f_s^T \end{bmatrix}^T = \begin{bmatrix} e_u & e_s \end{bmatrix}^{-1}$.

To achieve the control it is demanded that the next iteration falls near the stable direction. This leads to the next condition

$$f_u^T\left(\xi_{n+1} - \xi_F(\beta_n)\right) = 0 \tag{7}$$

Clearly, this condition means that the vector displacement from the fixed point to the next iteration do not have component along f_u^T.

3.4 The control law

After a few manipulations, equation (6) can be rewritten as:

$$\begin{aligned} f_u^T\left(\xi_{n+1} - \xi_F(\beta_n) - (\beta_{n+1} - \beta_n)s\right) \approx \\ \lambda_u f_u^T M\left(\xi_n - \xi_F(\beta_n) - (\beta_{n+1} - \beta_n)s\right) \end{aligned} \tag{8}$$

Using equation (7) and defining $\delta\beta_n \equiv \beta_{n+1} - \beta_n$ and $\delta\xi_n \equiv \xi_{n+1} - \xi_F(\beta_n)$:

$$0 - \delta\beta_n f_u^T s \approx \lambda_u f_u^T \delta\xi_n - \delta\beta_n \lambda_u f_u^T s \tag{9}$$

rewriting:

$$\delta\beta_n \approx \frac{\lambda_u}{(\lambda_u - 1)} \frac{f_u^T \delta\xi_n}{f_u^T s} \tag{10}$$

This is the so-called OGY formula [13].

4. Application to Chua´s Circuit

Chua's circuit is a simple nonlinear oscillator circuit, which exhibits a variety of behaviors including bifurcations and chaos [10]. The circuit contains three linear energy-storage elements, a linear resistor, and a single nonlinear resistor N_R as shown in Fig. 2. For analysis and simulation purposes, we have used previously proposed [10] dimensionless equations to describe the behavior of this system:

$$\begin{aligned} \dot{x} &= \alpha\left[-x + y - f(x)\right] \\ \dot{y} &= x - y + z \\ \dot{z} &= -\beta y \end{aligned} \tag{11}$$

Where $\alpha = C_1/C_2 > 0$ and $\beta = C_2/(LG^2) > 0$ are parameters and $f(x)$ can be expressed as

$$f(x) = m_0'x + \tfrac{1}{2}(m_1' - m_0')\left[|x + B_P| - |x - B_P|\right] \tag{12}$$

where $m_0' = m_0/G > 0$ and $m_1' = m_1/G < 0$. For having consistent results with the circuit, the chosen values for the parameters were $\alpha = 9$, $\beta = 14\ 2/7$, $m_0' = -5/7$, $m_1' = -8/7$. For these parameter values, the well-known Chua's double scroll attractor is formed.

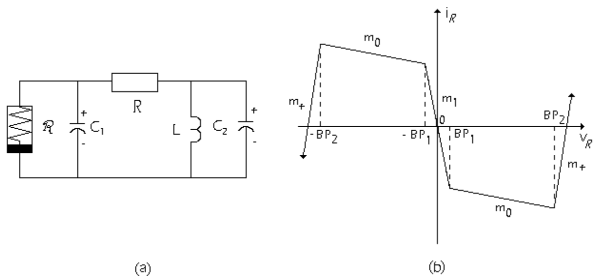

Fig. 2 (a) Chua's circuit, (b) Nonlinear resistor's *v-i* characteristic.

We have chosen β as the parameter to be perturbed, and we consider the measured variable to be *z* in simulation results.

4.1 Embedded unstable periodic motion and fixed points

In order to find a fixed point corresponding to an embedded periodic motion, we have built the LM obtaining a time series from the local maxima as seen in Fig. 1 and delayed this time series to find suitable coordinates for the LM shown in Fig. 3.

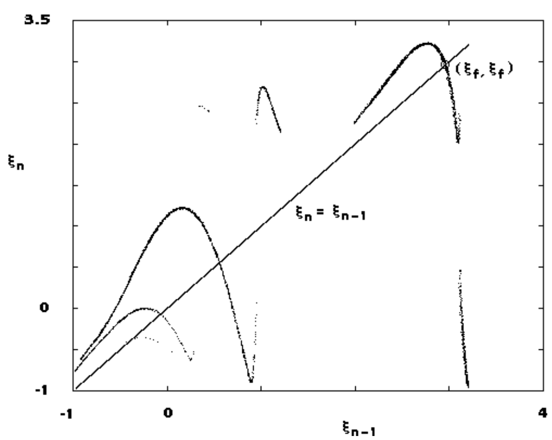

Fig. 3 Lorenz Map *P* and the line $\xi_n = \xi_{n-1}$ used for obtaining the unstable fixed point. (ξ_F, ξ_F)

The nominal parameter value $\beta=\beta^*$ was considered to be 14 2/7. Numerically, we obtained the intersection point by calculating the mass center of the points within a certain neighborhood nearby. This intersection with coordinates $(\xi_F, \xi_F) = (2.9652, 2.9652)$ is the unstable fixed point that we were looking for.

4.2 Displacement characterization in the fixed point due to external stimulus

The parameter value β was changed between periodic windows in the bifurcation diagram around the nominal parameter value β*. This was done in order to find a relationship between the parameter value and the unstable fixed point ξ_F. This was done in the range from 13.1 to 15.5 and the change of the location of the fixed point was observed to be proportional. The obtained points for each parameter value β were approximated by using a least mean squares algorithm to a straight line of the form $\xi_F = m\beta + b$, where $m = 0.0988$ and $b = 1.5496$. Therefore shift vector *s* becomes $s = [0.0988, 0.0988]$.

4.3 Local dynamics identification

In order to identify the local dynamics, we use a parameterization of equation (6). It is needed to choose a sequence of points from the $\xi(n)$ time series (at least 4 points) in a fixed point's neighborhood. These successive maxima must lie on the same direction, but in opposite sides in an alternative way. Using a least mean squares procedure we may calculate the *M* matrix elements and its eigenvalues, eigenvectors and dual basis vectors, which turns out to be:

$$M = \begin{bmatrix} -0.0162 & 0.9950 \\ 1.8894 & -1.6874 \end{bmatrix} \qquad (13)$$

thus λ_u=-2.4575 and $f_u = [0.9613 \quad 0.3918]$. With these values it is now possible to calculate the feedback vector gain in the OGY formula which turns out to be:

$$\delta\beta_n \approx [5.1109 \quad 2.0830]\delta\xi_n \qquad (14)$$

this control law was implemented directly to the system.

4.5 Numerical results

Fig. 4 shows the controlled variable $z(t)$ from Chua's equations. Parameter β is perturbed within the values $\beta_{min} < \beta^* < \beta_{max}$ which corresponds to a fixed point variation $\xi_{min} < \xi^* < \xi_{max}$.

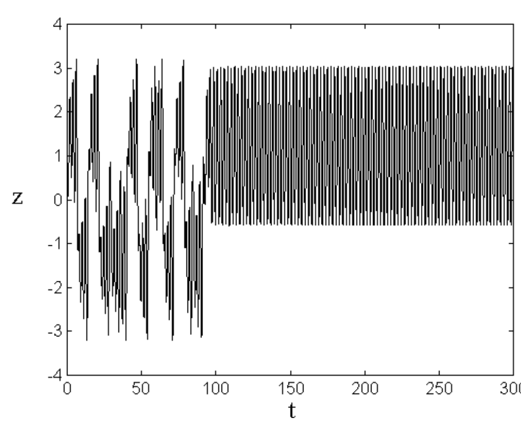

Fig. 4 Controlled $z(t)$ variable

The control law is activated when a couple of consecutive maxima $\xi(n)$ fall inside the above interval and it is shown in Fig. 5. Finally Fig. 6 shows the stabilized period one orbit.

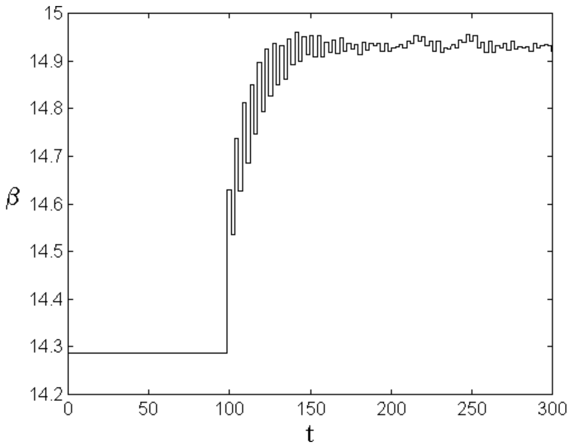

Fig. 5 Control law (parameter β variation)

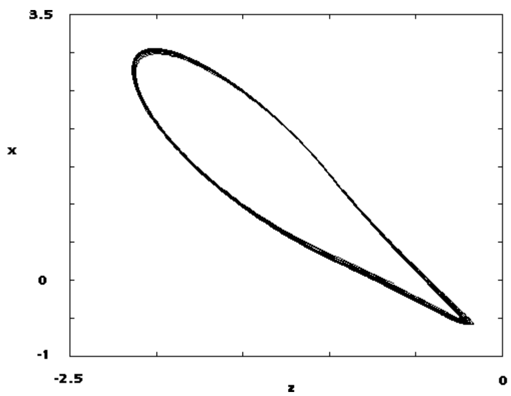

Fig. 6 Controlled UPO after applying the control law

5. Experimental setup

We have used the I/O DS1104 controller board from dSPACE to implement the control action directly to the electronic circuit. This system supports real time control and appropriate software is provided to communicate with Simulink from Mathworks. We have used this real time system to obtain all the measurements from the Chua's circuit variables and to implement the control law. In what follows a detailed description of the controller implementation is given.

5.1 Implementation of the control law

Since the Chua's circuit is an autonomous system, in order to implement the controller, an electronic input terminal should be added. This terminal allows us to change an available parameter, in this case the parameter to be varied is the linear resistance in Fig. 1. In Fig. 10 is shown how this control input was added to electronically change the resistance, which changes the circuit behavior.

The diode between the source and gate within a *p*-channel FET transistor was used as a voltage-driven resistance. When a voltage is applied to the gate, the high impedance of this diode changes proportionally with a change in the applied voltage. By incrementing this voltage, the circuit displays all its well-known behaviors until the double scroll appears. A capacitor (C2 in Fig. 7) was also added in order to isolate this resistance.

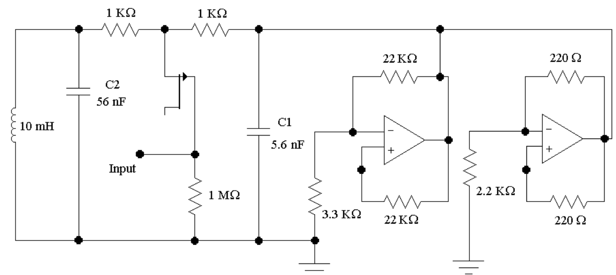

Fig. 7 Chua's circuit with control input

5.2 Results

According to the method described above, an OGY-like controller was applied to the modified Chua's circuit (Fig. 7). We have used the same process described in section 4 to implement the method electronically. First an appropriate range of variation for the available parameter, within the chaotic regime, was found experimentally. As mentioned before, the available parameter is voltage driven, thus this range for the experiment was found to be from 1.2 V to 2.2 V.

Time series were obtained from the circuit using an 84.6 KHz sampling frequency. This data was used to construct the Lorenz experimental map shown in Fig. 8 from where it can be seen 2 fixed points which are expected to be saddles, each of them corresponding to a scroll of the attractor. Thus, it is possible to select an UPO in the neighborhood of any of both scrolls. The next step is to find the variation in the position of the fixed point when the available parameter is changed. In order to obtain this relationship, several LM are obtained for different values of the parameter. We have selected arbitrarily the fixed point in the upper side of the LM to be stabilized and we have used a least squares method to determine the value of $s = [0.0058 \quad 0.0058]^T$ for the shift vector.

Now we may proceed to characterize the local dynamics around the selected fixed point. Again we repeat the procedure described in section 4 and we have found 4 consecutive points in the neighborhood of the fixed point in the LM. Using a standard least square procedure we have found:

$$M = \begin{bmatrix} 0 & 1 \\ 3.2095 & -2.5165 \end{bmatrix} \tag{15}$$

Eigenvalues λ=[0.6721 -3.4212] denote a saddle point behavior of the fixed point. Obtaining f_u=[1.1053 0.3115]^T we were able to find the feedback control law designed for the system:

$$\delta\beta_n \approx \begin{bmatrix} 22.27 & 6.27 \end{bmatrix} \delta\xi_n \qquad (16)$$

for the experimental setup. This control law was directly applied to the Chua's circuit via the controller board DS1104®. The measured signals of the controlled system are shown in Fig. 9.

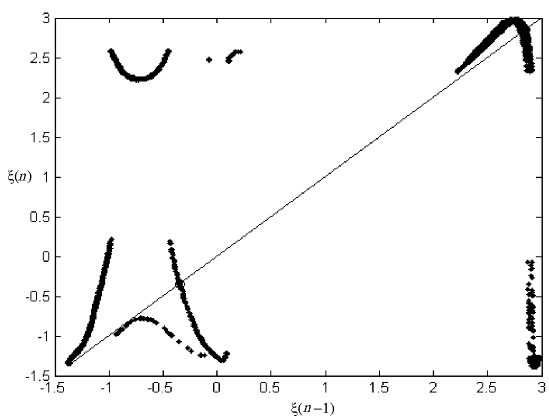

Fig. 8 Lorenz Map showing fixed points

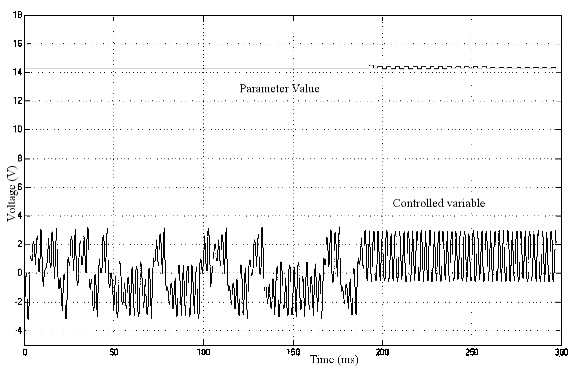

Fig. 9 Experimentally controlled system.

6. Concluding remarks

Control of chaotic systems is one of the most interesting research fields within the study of nonlinear dynamical systems and represents a great challenge for all fields. It can be seen from its actual development that its future will give us not only satisfactory answers to current problems, but also will provide a lot of new ideas. The OGY algorithm is probably the most popular method for controlling chaotic systems due to the possibility of implementing it without knowing the system's dynamical equations, it is just enough to use an experimental time series. This does not intend to mean that it is an easy task, it is composed by two principal stages: one developed offline (properly the controller design) and the application of the controller. It is possible to use any system parameter that is available for applying the parametric perturbations generated by the controller. For experimental controller's implementation, the noise and the unavoidable signal processing errors may cause that the controller does not work in such an effective way. In other words, before settling down to the desired periodic solution the trajectory exhibits some chaotic transients whose length depends on the moment in which the controller was turned on. Future research should lead to cope with these features.

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