Global Estimation of Frequency in Multiple Signals Without Overparameterization.

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Abstract— **In this paper we study the global adaptive estimation of the frequencies in multiple sinusoidal signals without overparameterization.**

Keywords— **Estimation, Stability, Non linear systems, Signal Processing.**

I. Introduction

We are interesting in the estimation of the frequencies and states of multiple sinusoidal signals given by

$$\begin{aligned} Y(t) &= \begin{bmatrix} y_1 & y_2 & \cdots & y_m \end{bmatrix}^T \\ y_j &= B_j + \sum_{i=1}^{n} A_{i\ j} \sin(\alpha_i\ t + \varphi_{i\ j}) \qquad j = 1...m \end{aligned} \tag{1}$$

where the parameters B_j, A_{ij}, α_i, φ_{ij} are unknown. The goal is estimate the parameters only once. The estimation of the frequency, is a very important issue in control theory due the number of practical applications, see for example [1], [11] for disturbance rejection; [2] for control of sound and vibration, [3] for helicopter vibration, and [9] for landing system.

The problem of frequency estimation of the signal has been estudied with differents techniques both in the offline case, for example [10] and the online estimation [5]. For one signal and one frequency a globally stable estimator is proposed in [4], in this work the stability property of the filter depends of the amplitud of the mesured signal. Recently two estimators have been proposed for one signal and n frequencies case [6], [7]. In [6] the dimension of the estimator is $(5n-1)$ states, while in [7] the dimension of the estimator is $3n$. In both cases the estimators are globally stables.

In this context, if we have more than one signal (for example "m" signals) and all them are mesured, we can apply "m" times the estimator proposed in [6] or [7], but we estimate the parameters "m" times. Now the question is: How to construct an estimator that estimate the parameters only once?. First we remember the case of one signal and "n" frequencies.

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II. The single signal estimator.

The estimator proposed in [7] take the form:

$$\begin{aligned} \dot{x}_i &= \lambda_i x_{i+1} \quad i = 1...(2n-1) \\ \dot{x}_{2n} &= \lambda_{2n} x_{2n+1} + \zeta_1 (y - \hat{y}) \\ \dot{x}_{2n+1} &= -\left(\sum_{i=1}^{n} \frac{\sigma_i x_{2n+1+i}}{\Pi_{j=2i}^{2n} \lambda_j} x_{2i} \right) + \zeta_2 (y - \hat{y}) \\ \dot{x}_{2n+1+i} &= -\zeta_{i+2} x_{2i} (y - \hat{y}) \quad i = 1...n \\ \hat{y} &= \left(\sum_{i=1}^{2n} \frac{k_i}{\Pi_{j=i}^{2n} \lambda_j} \right) x_i + k_{2n+1} x_{2n+1} \end{aligned} \tag{2}$$

with constants

$$\begin{aligned} \zeta_1 &= \frac{\lambda_{2n}}{k_{2n+1}}, \quad \zeta_{i+2} > 0 \quad i = 0...n \\ \lambda_i &> 0 \quad i = 1...2n, \quad \sigma_i > 0 \quad i = 1...n \end{aligned}$$

and the polynomial

$$P(\xi) = \sum_{i=0}^{2n} \frac{k_{i+1}}{k_{2n+1}} \xi^i$$

chosen stable. This estimator guarantees that

$$\begin{aligned} \lim_{t \to \infty} (y - \hat{y}) &= 0, \\ x_1 &\to (w_1 = \eta_1), \\ x_i &\to \left(\frac{w_i}{\Pi_{j=1}^{i-1} \lambda_j} = \eta_i \right) \quad i = 2...2n+1, \\ x_{2n+1+i} &\to \left(\frac{a_{2i-2}}{\sigma_i} \right) \quad i = 1...n \end{aligned}$$

globally when $t \to \infty$, where

$$\begin{aligned} \dot{w}_i &= w_{i+1} \quad i = 1...2n \\ \dot{w}_{2n+1} &= -\theta_1 w_2 - \theta_2 w_4 - ... - \theta_n w_{2n} \\ y(t) &= \sum_{i=1}^{2n+1} c_i w_i \\ \theta_1 &= \left(\Pi_{i=1}^{n} \alpha_i^2 \right), \ \theta_2 = \left(\sum_{j=1}^{n} \frac{\Pi_{i=1}^{n} \alpha_i^2}{\alpha_j^2} \right), ..., \\ \theta_{n-1} &= \sum_{i=1}^{n-1} \left(\sum_{j=i+1}^{n} \alpha_i^2 \alpha_j^2 \right), \ \theta_n = \left(\sum_{i=1}^{n} \alpha_i^2 \right) \end{aligned}$$

The problem we face here is a little different, we are interested in construct a common parameter estimator for all signals, this is, avoid the repetition of these estimates for each signal.

III. The "m" signals estimator.

In this section we give the "m" signals estimator. We first observe that the signal (1) can be given by

$$\begin{aligned} \dot{w}_{(i,j)} &= w_{((i+1),j)} \quad i=1...2n \quad j=1...m \\ \dot{w}_{((2n+1),j)} &= -\theta_1 w_{(2,j)} - \theta_2 w_{(4,j)} - ... - \theta_n w_{(2n,j)} \\ y_j(t) &= \sum_{i=1}^{2n+1} c_{(i,j)} w_{(i,j)}(t) \end{aligned}$$

where the constants $c_{(i,j)}$ take here are chosen such that the observability of this system is preserved. A particular choice is

$$c_{(i,j)} = \frac{k_{(i,j)}}{\Pi_{q=1}^{2n}\lambda_q} \quad i=1...(2n+1), \quad j=1...m$$

Now we present the main result.

Theorem 1: Consider the signals (1), then the estimator of the form

$$\begin{aligned} \dot{x}_{(i,j)} &= \lambda_i x_{((i+1),j)}\ , \quad i=1..(2n-1)\ , \quad j=1..m \\ \dot{x}_{(2n,j)} &= \lambda_{2n} x_{[(2n+1),j]} + \zeta_{(1,j)}(y_j - \hat{y}_j) \qquad (3) \\ \dot{x}_{((2n+1),j)} &= -\left(\sum_{i=1}^{n} \frac{\sigma_i x_{(i,0)}}{\Pi_{q=2i}^{2n}\lambda_q} x_{(2i,j)}\right) + \zeta_{(2,j)}(y_j - \hat{y}_j) \\ \dot{x}_{(i,0)} &= -\gamma_i \left(\sum_{j=1}^{m} x_{(2i,j)}\,(y_j - \hat{y}_j)\right) \quad i=1...n \end{aligned}$$

$$\hat{y}_j = \left(\sum_{i=1}^{2n} \frac{k_{(i,j)}}{\Pi_{q=i}^{2n}\lambda_q} x_{(i,j)}\right) + k_{((2n+1),j)} x_{((2n+1),j)}$$

with constants

$$\begin{aligned} \zeta_{(1,j)} &= \frac{\lambda_{2n}}{k_{((2n+1),j)}}\ , \quad \zeta_{(2,j)} > 0 \quad j=1...m \\ \lambda_i &> 0 \quad i=1...2n, \quad \sigma_i > 0\ , \quad \gamma_i > 0 \quad i=1...n \end{aligned}$$

and

$$P_j(\xi) = \sum_{i=0}^{2n} \frac{k_{((i+1),j)}}{k_{((2n+1),j)}} \xi^i \quad j=1...m$$

stable polynomials, with distinct roots, is such that

$$\begin{aligned} \lim_{t\to\infty}(y_j - \hat{y}_j) &= 0,\ x_{(1,j)} \to w_{(1,j)}, \\ x_{(i,j)} &\to \left(\frac{w_{(i,j)}}{\Pi_{j=1}^{i-1}\lambda_j}\right)\ , \quad x_{(i,0)} \to \left(\frac{\theta_i}{\sigma_i}\right) \end{aligned}$$

exponentially if the signal

$$F_x(t) = \sum_{j=k_1}^{k_2} \rho_j \bar{x}_j(t) \qquad (4)$$

$$\bar{x}_j = \left[\begin{array}{ccccc} \lambda_1 x_{(2,\ j)} & \cdots & \Pi_{i=1}^{2i-1}\lambda_i x_{(2i,\ j)} & \cdots & \Pi_{i=1}^{2n-1}\lambda_i x_{(2n,\ j)} \end{array}\right]^T$$

contain all the "n" frequencies for some constants $1 \leq k_1 \leq k_2 \leq m$ and ρ_j.

Proof: We first considere the error system

$$\begin{aligned} e_{(1,j)} &= w_{(1,j)} - x_{(1,j)} \quad j=1...m,\ i=2...2n+1 \\ \dot{e}_{(i-1,j)} &= e_{(i,j)} = w_{(i,j)} - \left(\Pi_{q=1}^{i-1}\lambda_q\right) x_{(i,j)} \\ e_{(i,0)} &= \sigma_i x_{(i,0)} - a_{2i-2} \quad i=1...n \end{aligned}$$

then each signal $(y_j - \hat{y}_j)$ can be written in the form

$$\begin{aligned} (y_j - \hat{y}_j) &= \sum_{i=1}^{2n+1} \frac{k_{(i,j)} w_{(i,j)}}{\Pi_{q=1}^{2n}\lambda_q} - \sum_{i=1}^{2n} \frac{k_{(i,j)} x_{(i,j)}}{\Pi_{q=i}^{2n}\lambda_q} \\ &\quad - k_{((2n+1),j)} x_{((2n+1),j)} \\ &= \sum_{i=1}^{2n} \frac{k_{(i,j)}}{\Pi_{q=1}^{2n}\lambda_q}\left(w_{(i,j)} - \Pi_{q=1}^{i-1}\lambda_q x_{(i,j)}\right) \\ &\quad + \frac{k_{(2n+1,j)}}{\Pi_{q=1}^{2n}\lambda_q}\left(w_{(2n+1,j)} - \Pi_{q=1}^{2n}\lambda_q x_{((2n+1),j)}\right) \\ &= \frac{1}{\Pi_{q=1}^{2n}\lambda_q}\left(\sum_{i=1}^{2n+1} k_{(i,j)} e_{(i,j)}\right) \end{aligned}$$

Now, the system can takes the form

$$\begin{aligned} \dot{\bar{e}}_j &= \bar{A}_j\ \bar{e}_j \quad j=1...m \qquad (5) \\ \dot{e}_{((2n+1),j)} &= -\bar{\beta}_j^T \bar{e}_j - \beta_{(2,j)} e_{((2n+1),j)} + \bar{x}_j^T \bar{e}_0 \\ \bar{e}_0 &= -\Gamma \sum_{j=1}^{m} \bar{x}_j \left(\bar{k}_j^T \bar{e}_j + k_{((2n+1),\ j)} e_{((2n+1),\ j)}\right) \end{aligned}$$

where for $i=1...n$ and $j=1...m$ we have

$$\begin{aligned} \bar{A}_j &= \left[\begin{array}{cc} 0 & I \\ -\frac{k_{(1,j)}}{k_{((2n+1),\ j)}} & \bar{a}_j \end{array}\right] \\ \bar{a}_j &= \left[\begin{array}{ccc} -\frac{k_{(2,j)}}{k_{((2n+1),\ j)}} & \cdots & -\frac{k_{(2n,\ j)}}{k_{((2n+1),\ j)}} \end{array}\right] \\ \bar{e}_j &= \left[\begin{array}{ccccc} e_{(1,j)} & e_{(2,j)} & \cdots & e_{(2n-1,j)} & e_{(2n,j)} \end{array}\right] \\ \bar{\beta}_j^T &= \left[\begin{array}{ccc} \zeta_{(2,j)} k_{(1,j)} & \cdots & (\theta_n + \zeta_{(2,j)} k_{(2n,\ j)}) \end{array}\right] \\ \beta_{(2,j)} &= \zeta_{(2,j)} k_{((2n+1),j)} \\ \bar{x}_j &= \left[\begin{array}{ccc} \lambda_1 x_{(2,\ j)} & \cdots & \Pi_{q=1}^{2i-1}\lambda_q x_{(2i,\ j)} \end{array}\right]^T \\ \bar{e}_0 &= \left[\begin{array}{ccccc} e_{(1,0)} & e_{(2,0)} & \cdots & e_{(n-1,0)} & e_{(n,0)} \end{array}\right]^T \\ \Gamma &= diag\left(\begin{array}{ccc} \frac{\sigma_1\gamma_1}{\left(\Pi_{q=1}^{2n}\lambda_q\right)(\lambda_1)} & \cdots & \frac{\sigma_i\gamma_i}{\left(\Pi_{q=1}^{2n}\lambda_q\right)\left(\Pi_{q=1}^{2i-1}\lambda_q\right)} \end{array}\right) \\ \bar{k}_j &= \left[\begin{array}{cccc} k_{(1,j)} & k_{(2,j)} & \cdots & k_{(2n,j)} \end{array}\right]^T \end{aligned}$$

Now since the matrix $\bar{A}_j$ has distinct eigenvalues, because the polynomials $P_j(\xi)$ has distinct roots, then there exist T_j such that

$$\begin{aligned} \tilde{A}_j &= T_j^{-1}\bar{A}_j T_j = diag(-\mu_{(1,j)}, ..., -\mu_{(2n,j)}) \\ \bar{e}_j &= T_j \tilde{e}_j \\ \tilde{k}_j^T &= \bar{k}_j^T T_j, \quad \tilde{\beta}_j^T = \bar{\beta}_j^T T_j \end{aligned}$$

and the system take the form

$$\begin{aligned}\dot{\tilde{e}}_j &= \tilde{A}_j\ \tilde{e}_j \qquad j=1...m\\ \dot{e}_{((2n+1),j)} &= -\tilde{\beta}_j^T\tilde{e}_j-\beta_{(2,j)}e_{((2n+1),j)}+\bar{x}_j^T\bar{e}_0\\ \bar{e}_0 &= -\Gamma\sum_{j=1}^m\bar{x}_j\left(\tilde{k}_j^T\tilde{e}_j+k_{((2n+1),\ j)}e_{((2n+1),\ j)}\right)\end{aligned}$$

Take the j-th Lyapunov function like

$$V_j(\tilde{e}_j,e_{((2n+1),j)},\bar{e}_0)=$$

$$\begin{aligned}&\frac{1}{2}\tilde{e}_j^TP_j\tilde{e}_j+\frac{p_{((2n+1),j)}}{2}e^2_{((2n+1),j)}\\ &+\bar{P}^T_{(j,2n+1)}\tilde{e}_je_{((2n+1),j)}+\frac{1}{2m}\bar{e}_0^T\Gamma^{-1}\bar{e}_0\end{aligned}$$

where

$$P_j=\kappa\ diag(\frac{1}{\mu_{(1,j)}},...,\frac{1}{\mu_{(1,2n)}}) \tag{6}$$

wich is positive definite for some constant κ and where $\bar{P}_{(j,2n+1)}\in\Re^{2n\times 1}$ is a vector definite latter wich dos not depent of the constant κ. We take the sum of these functions to obtein

$$V_0(\tilde{e}_j,e_{((2n+1),j)},\bar{e}_0)_{j=1...m}=\sum_{j=1}^m V_j(\tilde{e}_j,e_{((2n+1),j)},\bar{e}_0)$$

$$\begin{aligned}=&\sum_{j=1}^m\left(\frac{1}{2}\tilde{e}_j^TP_j\tilde{e}_j+\frac{p_{((2n+1),j)}}{2}e^2_{(2n+1),j)}\right)\\ &+\sum_{j=1}^m\left(\bar{P}^T_{(j,2n+1)}\tilde{e}_je_{((2n+1),j)}+\frac{1}{2m}\bar{e}_0^T\Gamma^{-1}\bar{e}_0\right)\\ =&\sum_{j=1}^m\left(\frac{1}{2}\tilde{e}_j^TP_j\tilde{e}_j+\frac{p_{((2n+1),j)}}{2}e^2_{(2n+1),j)}\right)\\ &+\sum_{j=1}^m\left(\bar{P}^T_{(j,2n+1)}\tilde{e}_je_{((2n+1),j)}\right)+\frac{1}{2}\bar{e}_0^T\Gamma^{-1}\bar{e}_0\end{aligned}$$

wich is positive definite. The derivative is

$$\dot{V}_0=$$

$$\begin{aligned}&\sum_{j=1}^m\left(-\tilde{e}_j^TQ_j\tilde{e}_j\right)-\sum_{j=1}^m p_{((2n+1),j)}e_{((2n+1),j)}\tilde{\beta}_j^T\tilde{e}_j\\ &+\sum_{j=1}^m p_{((2n+1),j)}e_{((2n+1),j)}\left(-\beta_{(2,j)}e_{((2n+1),j)}+\bar{x}_j^T\bar{e}_0\right)\\ &+\sum_{j=1}^m\bar{P}^T_{(j,2n+1)}\tilde{e}_j\left(-\tilde{\beta}_j^T\tilde{e}_j-\beta_{(2,j)}e_{((2n+1),j)}+\bar{x}_j^T\bar{e}_0\right)\\ &+\sum_{j=1}^m\bar{P}^T_{(j,2n+1)}\tilde{A}_j\ \tilde{e}_je_{((2n+1),j)}\\ &-\bar{e}_0^T\left(\sum_{j=1}^m\bar{x}_j\left(\tilde{k}_j^T\tilde{e}_j+k_{((2n+1),\ j)}e_{((2n+1),\ j)}\right)\right)\end{aligned}$$

wich considering (6) and choosing

$$\begin{aligned}\bar{P}^T_{(j,2n+1)} &= \tilde{k}_j^T\\ p_{((2n+1),j)} &= k_{((2n+1),\ j)}\end{aligned}$$

we have

$$\dot{V}_0\leq$$

$$\begin{aligned}&\sum_{j=1}^m\left(\left(-\kappa+\left\|\tilde{k}_j\tilde{\beta}_j^T\right\|\right)\|\tilde{e}_j\|^2-k_{((2n+1),j)}\beta_{(2,j)}e^2_{((2n+1),j)}\right)\\ &+\sum_{j=1}^m\left(\tilde{k}_j^T\tilde{A}_j-\tilde{k}_j^T\beta_{(2,j)}-k_{((2n+1),j)}\tilde{\beta}_j^T\right)\tilde{e}_je_{((2n+1),j)}\end{aligned}$$

and there for is always negative if $k_{((2n+1),j)}>0$ and $\beta_{(2,j)}>0$, because the constant κ is freely chosen. Then the invariant set is $\Omega=\left\{\left(\tilde{e}_j,e_{((2n+1),j)}\right)\mid\tilde{e}_j=0,e_{((2n+1),j)}=0,j=1...m\right\}$, the equations for $e_{((2n+1),j)}$ are

$$\dot{e}_{((2n+1),j)}=\bar{x}_j^T\bar{e}_0=0,\quad j=1...m$$

and then

$$\left(\sum_{j=k_1}^{k_2}\rho_j\bar{x}_j^T\right)\bar{e}_0=0,\quad 1\leq k_1\leq k_2\leq m$$

Now if the signal $\left(\sum_{j=k_1}^{k_2}\rho_j\bar{x}_j^T\right)$ contein all the frequencies, applying the Persistency of Excitation Lemma (see [12], [13]), to system (5) we can conclude that ($\tilde{e}_1=0,e_{2,2n}=0$, $e_3=0$) is a globally exponentially stable equilibrium point for (5). ■

Remark 2: The condition for the signal (4) always is possible, because we can choose the polynomials $P_j(\xi)$ and the constants k_1,k_2,ρ_j arbitrarely.

IV. Simulation.

In this section we show some simulations results for $n=2$ and $m=3$. We consider the signals

$$\begin{aligned}y_1 &= 1-2\sin(2t+1)\\ y_2 &= 1+3\cos(\sqrt[2]{2}t)\\ y_3 &= -2+2\sin(2t+1)-3\cos(\sqrt[2]{2}t).\end{aligned}$$

Taking the values

$$\begin{aligned}k_{(5,j)} &= j,\ j=1..3,\\ \sigma_1 &= \sigma_2=1,\ \lambda_i=1,\ i=1..4,\\ \zeta_{(1,j)} &= \frac{1}{j},\ \zeta_{(2,j)}=1,\\ \gamma_1 &= 1700,\gamma_2=250\end{aligned}$$

and the polynomials $P_j(\xi)$ of the following form

$$\begin{aligned}P_1(\xi) &= (\xi+1)(\xi+1.5)(\xi+2)(\xi+2.5)\\ P_2(\xi) &= (\xi+1)(\xi+3)(\xi+4)(\xi+4.5)\\ P_3(\xi) &= (\xi+3)(\xi+5)(\xi+6)(\xi+6.5)\end{aligned}$$

the results are showed in the figures 1-4, we can see that the estimator work and estimate the parameters one time.

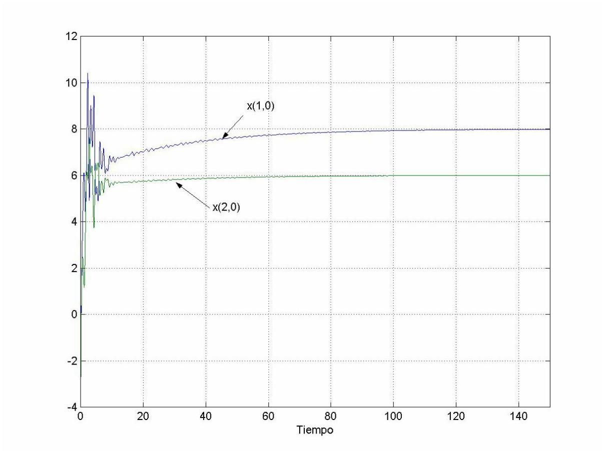

Figure 1. Estimates $x_{(1,0)}$, $x_{(2,0)}$.

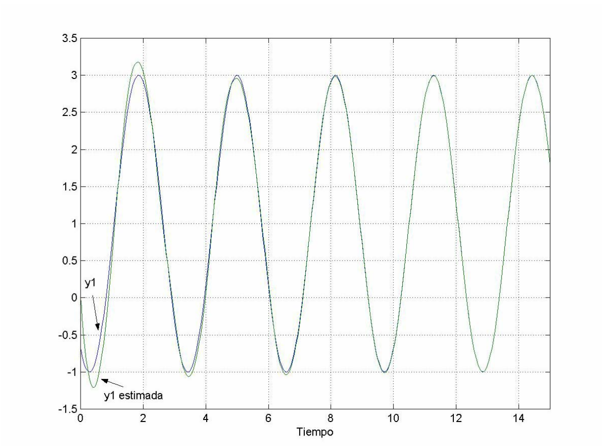

Figure 2. Signal y_1 and their estimate.

The signals y_2 and y_3 together with theirs estimates are showed in the follows figures.

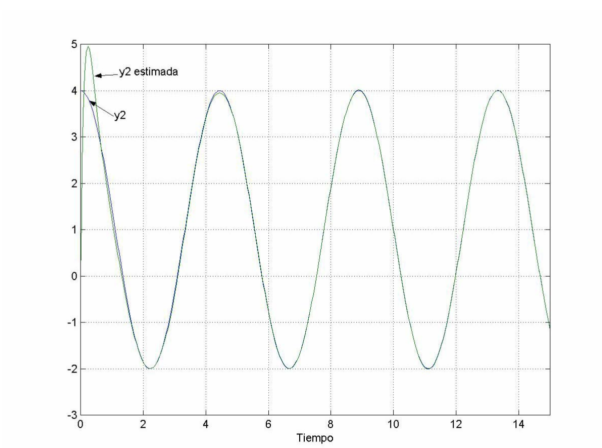

Figure 3. Signal y_2 and their estimate.

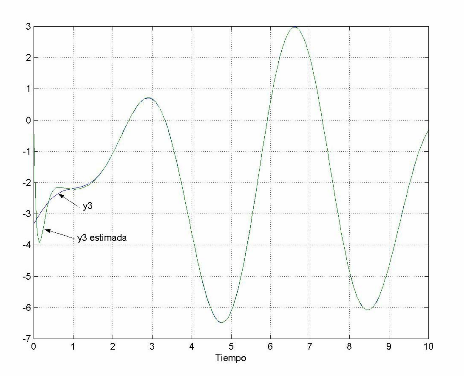

Figure 4. Signal y_3 and their estimate.

As it may be observed, the estimator exhibites a good convergence properties, so this resulto suggest the validity of the proposed estimator

V. Conclusion.

In this note the problem of global state and frequency simultaneous estimation for multiple signals without over-parameterization is addressed. We propose a new simple estimator with similar estructure to that given in [7]. This estimator whose dimension is $(2nm+n)$, provides a solution to this important problem in system and suignal theory. The estimator is globally convergent for all frequencies and initial conditions. The extensive performed simulations allows us to validate the proposed escheme.

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