A Globally Adaptive Internal Model Regulator for SISO Linear Systems.

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Abstract— **The problem of compensating noise and/or track some reference signal with n uknown frequencies in general linear SISO systems is treated in this work. We derive a frequency estimator that ensures closed loop robust regulation in some neighborhod of the nominal values of the system as well.**

I. Introduction.

In many industrial and defense applications noise and vibration are important problems. Some common class of those are the periodic and/or quasi-periodic noises, which include engine noise in turboprop aircraft [2] and automobiles [3], and ventilation noise in HVAC systems [10] and landing systems [17].

The problem of rejecting sinusoidal noise was addressed in [4], [6], [5] and recently in [7], using adaptive observers scheme developed in [9], and [15] for a class of nonlinear systems. In [4] a local solution for a single frequency and for stable SISO system and matching condition case is given, while in [7] a global solution is given for stable SISO system and matching condition case for one frequency. In [5], a locally exponentially stable adaptive control law is proposed for MIMO linear systems, based on Youla parameterization. Also, in [6] a supervisory control scheme is proposed for the case of k frequencies and SISO linear system, considering that the number of frequencies is known and that all the frequency values lie in a pre-defined closed and bounded set Ω. In [15], the same problem is analyzed for a minimum phase SISO nonlinear system using a high gain feedback technique combined with regulator theory. There, the values of the frequencies must belong to a some pre-specified bounded set Σ. If the values of the frequencies leave this set, the regulator gains must be changed in order to keep the stability property. Along the same lines, in this work an adaptive control scheme for the case of SISO linear system for which the number of the frequencies are known but not necessarily belonging to a finite set of frequencies Ω nor bounded set of frequencies Σ is proposed, relaxing as well the minimum phase and matching conditions.

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II. Problem Statement

Consider a linear system subject to perturbation and put it in a canonical controller form

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bu(t) + Dd(t) && (1) \\ e(t) &= c^T x(t) + \tilde{q}^T d(t) && (2) \end{aligned}$$

where $x \in \Re^n$ is the state, $u \in \Re^1$ is the input, $d \in \Re^{(k+1)}$ is a disturbance and/or reference signal, and $e \in \Re^1$ represents the tracking error between the plant output $c^T x(t)$ and a reference signal $-\tilde{q}^T d(t)$. The parameters of $A \in \Re^{n\times n}$, $(b,\, c) \in \Re^{n\times 1}$, $D \in \Re^{n\times(k+1)}$ and $\tilde{q} \in \Re^{(k+1)}$ may possibly vary in some neighborhood of the nominal values (A_0, c_0, b_0, D_0 and $\tilde{q}_0$). We consider a vector $d(t)$ consisting of a constant signal with unknown magnitude B_0 and k sinusoidal signals with unknown magnitudes B_1, .., B_k, frequencies α_i and phases φ_i for $i = 1..k$, namely

$$d(t) = \begin{bmatrix} B_0 & B_1 \sin(\alpha_1 t + \varphi_1) & \cdots & B_k \sin(\alpha_k t + \varphi_k) \end{bmatrix}^T \quad (3)$$

where $\alpha_i \neq \alpha_j$ if $i \neq j$. Assuming these signals to be generated by an external dynamical generator or exosystem ([13]), the system (1)-(3) can be rewritten as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bu(t) + Pw(t) && (4) \\ \dot{w}(t) &= Sw(t) && (5) \\ e(t) &= c^T x(t) + q^T w(t) \end{aligned}$$

where $w \in \Re^{(2k+1)}$, $P \in \Re^{n\times(2k+1)}$, and the matrix $S \in \Re^{(2k+1)\times(2k+1)}$ is

$$S = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & S_i & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & S_k \end{bmatrix}, \quad S_i = \begin{bmatrix} 0 & 1 \\ -\alpha_i^2 & 0 \end{bmatrix}.$$

The problem we face here is that of finding a dynamic error feedback control

$$\begin{aligned} \dot{\xi}(t) &= \Xi(\xi(t), e(t)) \\ u(t) &= h(\xi(t)) \end{aligned}$$

such that, for any initial conditions $x(0)$, $w(0)$, $\xi(0)$, the solution of the closed loop system

$$\begin{aligned}\dot{x}(t) &= Ax(t) + bh(\xi(t)) + Pw(t)\\ \dot{w}(t) &= Sw(t)\\ \dot{\xi}(t) &= \Xi(\xi(t), c^T x(t) + q^T w(t))\end{aligned}$$

satisfies that $\lim_{t\to\infty} e(t) = 0$.

In the case of known frequencies, it is well known that this problem can be solved by directly applying the regulation theory, ([11], [12], [13]). However in practice, usually not all the frequencies are known, so the regulator theory need to be modified in order to handle with this situation. This case will be treated in this work.

III. Regulation Theory Preview.

From the Robust Regulation Theory [11], [12], [13], [14], it is known that if the following assumptions hold:

A.1 The pair (A, b) is controllable and the pair (A, c) is observable.

A.2. The matrix S is neutrally stable, that is, all its eigenvalues are on the imaginary axis,

A.3. For every P and q there exists a solution $\Pi \in \Re^{n\times(2k+1)}$ and $\gamma \in \Re^{2k\times 1}$ to the Francis equations

$$\begin{aligned}\Pi S &= A\Pi + b\gamma^T + P &(6)\\ 0 &= c^T\Pi + q^T, &(7)\end{aligned}$$

then by the transformations $\tilde{x}(t) = x(t) - \Pi w(t)$ and

$$\begin{aligned}z_1(t) &= \gamma^T w(t)\\ \dot{z}_i(t) &= z_{i+1}(t) = \gamma^T S^i w(t), \quad i = 1, ..., 2k\\ \dot{z}_{2k+1}(t) &= -\left(\Pi_{i=1}^k \alpha_i^2\right) z_2(t) - .... - \left(\Sigma_{i=1}^k \alpha_i^2\right) z_{2k}(t),\end{aligned}$$

the interconnected system (4)-(5), can be transformed into

$$\begin{aligned}\dot{\tilde{x}}(t) &= A\,\tilde{x}(t) + bu(t) - bh^T z(t) &(8a)\\ \dot{z}(t) &= \Phi(\theta)z(t) &(8b)\end{aligned}$$

with the tracking error $e(t) = c^T\tilde{x}(t)$, where $\Phi(\theta) \in \Re^{(2k+1)\times(2k+1)}$ and $h \in \Re^{(2k+1)}$ are defined as

$$\Phi(\theta) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0\\ 0 & 0 & 1 & \cdots & 0 & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & -\theta_1 & 0 & \cdots & -\theta_k & 0\end{bmatrix},\ h = \begin{bmatrix}1\\0\\ \vdots\\0\end{bmatrix},$$

and the parameters $\theta = (\theta_1, ..., \theta_k)^T$ calculated using the Cayley-Hamilton Theorem, from the characteristic polynomial of the matrix S, wich is $s\left(s^{2k} + \left(\Sigma_{i=1}^k \alpha_i^2\right) s^{2k-2} + \cdots + \left(\Pi_{i=1}^k \alpha_i^2\right)\right) = 0$, with $\theta_1 = \Pi_{i=1}^k \alpha_i^2, ..., \theta_k = \Sigma_{i=1}^k \alpha_i^2$.

Finally, the control $u(t) = k^T\tilde{x}(t) + h^T z(t)$, where k calculated such that the matrix $(A+bk^T)$ is Hurwitz, stabilizes the system and guarantees also the perturbation rejection.

In the case when only the error $e(t)$ is measurable, the control action can be provided by a dynamical system

$$\begin{aligned}\dot{\xi}(t) &= F\xi(t) + ge(t)\\ u(t) &= \tilde{h}^T\xi(t)\end{aligned}$$

where the matrix F and vectors g and $\tilde{h}$ are chosen such that the matrix

$$\begin{bmatrix} A & b\tilde{h}^T\\ gc^T & F\end{bmatrix}$$

is Hurwitz. Then the solvability of the output regulation problem can be stated in terms of the existence of matrices Π and Σ that solve the linear equations

$$\begin{aligned}\Pi S &= A\Pi + b\tilde{h}^T\Sigma + P &(9)\\ \Sigma S &= F\Sigma &(10)\\ 0 &= c^T\Pi + q^T. &(11)\end{aligned}$$

If the frequencies are known then a robust controller can be constructed as an observer for the system (8a)-(8b), namely

$$\begin{aligned}\begin{bmatrix}\dot{\xi}_1(t)\\ \dot{\xi}_2(t)\end{bmatrix} &= \begin{bmatrix} A_0 + b_0k^T - g_1c_0^T & 0\\ -g_2c_0^T & \Phi(\theta)\end{bmatrix}\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix}\\ &\quad + \begin{bmatrix}g_1\\ g_2\end{bmatrix} e(t) &(12)\\ u(t) &= \begin{bmatrix} k^T & h^T\end{bmatrix}\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix} = \tilde{h}^T\xi(t),\end{aligned}$$

where k, g_1 and g_2 are chosen such that the matrices $\left[A_0 + b_0k^T\right]$ and $\begin{bmatrix} A_0 - g_1c^T & -b_0h^T\\ -g_2c_0^T & \Phi(\theta)\end{bmatrix}$ are Hurwitz. This controller is robust with respect to plant parameter variation ([12]). Again, in the case when the frequencies are unknown then the parameters $\theta_1, ..., \theta_k$ of matrix $\Phi(\theta)$ are also unknown and the controller (12) does not further guarantees the regulation properties. To solve this problem, we propose the use of an adaptive scheme, by first change the form of the observer to get some desired structure.

IV. The Change of Coordinates.

To derive the adaptive regulator, we need first to rewrite the system (8a)-(8b) and the controller (12) in a particular form. To this end, we use the following transformation:

$$\begin{bmatrix}\delta(t)\\ r(t)\end{bmatrix} = \begin{bmatrix} I & -\tilde{\Pi}(\theta)\tilde{\Phi}^{-1}(\theta)\\ 0 & \tilde{\Phi}^{-1}(\theta)\end{bmatrix}\begin{bmatrix}\tilde{x}(t)\\ z(t)\end{bmatrix}$$

where

$$\tilde{\Phi}(\theta) = \begin{bmatrix} m^T(\theta)\\ m^T(\theta)\Phi(\theta)\\ \vdots\\ m^T(\theta)\Phi^{2k}(\theta)\end{bmatrix}$$

and $m(\theta)$ and $\tilde{\Pi}(\theta)$ to be defined later. Under this transformation, and from the fact that $\tilde{\Phi}^{-1}(\theta)\Phi(\theta)\tilde{\Phi}(\theta) = \Phi(\theta)$ and $h^T\tilde{\Phi}(\theta) = m^T(\theta)$, the nominal system (8a)-(8b) can be represented as

$$\begin{aligned}\dot{\delta}(t) &= A_0\delta(t) + b_0u(t) + \Delta r(t) &(13a)\\ \dot{r}(t) &= \Phi(\theta)r(t) &(13b)\\ e(t) &= c_0^T\delta(t) + \mu^T r(t) &(13c)\end{aligned}$$

where

$$\begin{aligned}\Delta &= A_0\tilde{\Pi}(\theta) - \tilde{\Pi}(\theta)\Phi(\theta) - b_0 m^T(\theta) \quad (14)\\ \mu^T &= c_0^T\tilde{\Pi}(\theta) \quad (15)\end{aligned}$$

and $\mu = \left[\begin{array}{ccccc}\mu_1 & \mu_2 & \cdots & \mu_{2k} & \mu_{2k+1}\end{array}\right]^T$. Equations (14)-(15) are also in the form of Francis equations, and if assumptions $A.1$ through $A.3$ hold, the solutions $\tilde{\Pi}(\theta)$ and $m(\theta)$ exist for every Δ and μ, so the transformation is well defined.

From the input $u = k^T\tilde{x} + h^T z$, it follows that

$$\begin{aligned}u(t) &= k^T(\delta(t) + \tilde{\Pi}(\theta)r(t)) + h^T\tilde{\Phi}(\theta)r(t)\\ &= k^T\delta(t) + \left(k^T\tilde{\Pi}(\theta) + m^T(\theta)\right)r(t).\end{aligned}$$

In the same way, the controller (12) can be transformed to get a suitable form to be used for the adaptive version. Considering this, we have the following result:

Lemma 1: Assuming assumptions $A1$ *through* $A3$ *hold, the regulator*

$$\begin{aligned}\dot{\zeta}_1(t) &= (A_0 - \bar{g}_1 c_0^T)\zeta_1(t) + b_0 u(t) + \bar{g}_1 e(t)\\ \dot{\zeta}_2(t) &= -\bar{g}_2 c_0^T\ \zeta_1(t) + (\Phi(\theta) - \bar{g}_2\mu^T)\zeta_2(t) + \bar{g}_2 e(t)\end{aligned} \quad (16)$$

$$\begin{aligned}u(t) &= k^T\zeta_1(t) + \varphi^T(\theta)\zeta_2(t) \quad (17)\\ \varphi^T(\theta) &= (m^T(\theta) + k^T\tilde{\Pi}(\theta)) \quad (18)\end{aligned}$$

with the coefficients of the vector μ *chosen such that the characteristic equation*

$$\sigma^{2k} + \frac{\mu_{2k}}{\mu_{2k+1}}\sigma^{2k-1} + \cdots + \frac{\mu_3}{\mu_{2k+1}}\sigma^2 + \frac{\mu_2}{\mu_{2k+1}}\sigma + \frac{\mu_1}{\mu_{2k+1}} = 0 \quad (19)$$

is stable, $\bar{g}_1$ *such that the matrix* $(A_0 - \bar{g}_1 c_0^T)$ *is Hurwitz, and*

$$\begin{aligned}\bar{g}_2 &= \left[\begin{array}{cccccc}0 & \cdots & 0 & \vartheta_1 & \vartheta_2\end{array}\right]^T \quad (20)\\ \Delta &= \bar{g}_1\mu^T \quad (21)\end{aligned}$$

with $\vartheta_1 = \frac{1}{\mu_{2k+1}}$ *and* $\vartheta_2 > 0$, $\mu_{2k+1} > 0$, *guarantees robust regulation.*

Proof: We first see that if $A1$ through $A3$ hold then the equations (6), (7), (14) and (15) are satisfied. Now define the errors $\tilde{\zeta}_1(t) = \zeta_1(t) + \tilde{\Pi}(\theta)r(t)$ and $\tilde{\zeta}_2(t) = \zeta_2(t) - r(t)$, then it is straightforward to verify that the composite system is described by

$$\left(\begin{array}{ccc}\dot{\tilde{x}}(t) & \dot{\tilde{\zeta}}_1(t) & \dot{\tilde{\zeta}}_2(t)\end{array}\right)^T = F\left(\begin{array}{ccc}\tilde{x}(t) & \tilde{\zeta}_1(t) & \tilde{\zeta}_2(t)\end{array}\right)$$

where

$$F = \left(\begin{array}{ccc}A & bk^T & b\varphi^T(\theta)\\ \bar{g}_1 c^T & A_0 + b_0 k^T - \bar{g}_1 c_0^T & b_0\varphi^T(\theta)\\ \bar{g}_2 c^T & -\bar{g}_2 c_0^T & \Phi(\theta) - \bar{g}_2\mu^T\end{array}\right)$$

whose dynamical matrix for nominal values is similar to

$$\left(\begin{array}{ccc}A_0 + b_0 k^T & b_0 k^T & b_0(m^T(\theta) + k^T\tilde{\Pi}(\theta))\\ 0 & A_0 - \bar{g}_1 c_0^T & 0\\ 0 & -\bar{g}_2 c_0^T & \Phi(\theta) - \bar{g}_2\mu^T\end{array}\right).$$

Now, by assumption, $\left(A_0 + b_0 k^T\right)$ and $\left(A_0 - \bar{g}_1 c_0^T\right)$ are stable. To show that also $\left(\Phi(\theta) - \bar{g}_2\mu^T\right)$ is stable, it suffices to observe that the matrix

$$(\Phi(\theta) - \bar{g}_2\mu^T) = \left(\begin{array}{cc}\Phi_2 & 0\\ v^T(\theta) & -\vartheta_2\mu_{2k+1}\end{array}\right)$$

with

$$\begin{aligned}\Phi_2 &= \left(\begin{array}{cccc}0 & 1 & \cdots & 0\\ 0 & 0 & \ddots & \vdots\\ 0 & 0 & \cdots & 1\\ -\frac{\mu_1}{\mu_{2k+1}} & -\frac{\mu_2}{\mu_{2k+1}} & \cdots & -\frac{\mu_{2k}}{\mu_{2k+1}}\end{array}\right),\\ v^T(\theta) &= \left(\begin{array}{cccc}-\vartheta_2\mu_1 & -(\theta_1 + \vartheta_2\mu_2) & \cdots & -(\theta_k + \vartheta_2\mu_{2k})\end{array}\right)\end{aligned}$$

has eigenvalues with negative real part since the polynomial (19) is stable and $\vartheta_2\mu_{2k+1} > 0$. Thus, the system remains stable in some neighborhood of the nominal values.

The steady state is then computing as

$$\begin{aligned}\tilde{x} &= 0\\ \zeta_{1,\,ss}(t) &= -\tilde{\Pi}(\theta)r(t)\\ \zeta_{2,\,ss}(t) &= r(t)\\ u_{ss}(t) &= m^T(\theta)r(t)\end{aligned}$$

and since $e(t) = c^T\tilde{x}(t)$ this in turn implies that $e(t) \longrightarrow 0$.

Note that the exact steady state input $m^T(\theta)r(t)$ which guarantees zero output tracking is obtained. ■

Remark 2: The controller (16) presents the interesting feature that if the term $\varphi^T(\theta)\zeta_2(t)$ is disconnected from the input $u(t)$ and therefore from the dynamics of $\zeta_1(t)$, the closed loop system remains stable. In particular the eigenvalues of $\left(\Phi(\theta) - \bar{g}_2\mu^T\right)$ do not change for possible variations on the frequencies. This property will be exploited in the design of the adaptive estimator of these frequencies.

V. The adaptive regulator.

Following the previous discussion, we present in the following result an estimator which guarantees global stability of the closed loop system.

Theorem 3: The regulator with

$$\begin{aligned}\dot{\zeta}_1(t) &= (A_0 - \bar{g}_1 c_0^T)\zeta_1(t) + b_0 u(t) + \bar{g}_1 e(t)\\ \dot{\zeta}_2(t) &= -\bar{g}_2 c_0^T\ \zeta_1(t) + (\Phi(\hat{\theta}) - \bar{g}_2\mu^T)\zeta_2(t) + \bar{g}_2 e(t) \quad (22)\\ u(t) &= k^T\zeta_1(t) + \varphi^T(\hat{\theta})\zeta_2(t)\\ \dot{\hat{\theta}} &= -\Lambda\bar{\zeta}_2(e(t) - c_0^T\zeta_1(t) - \mu^T\zeta_2(t))\end{aligned}$$

$$\begin{aligned} \zeta_2 &= \begin{bmatrix} \zeta_{2,1} & \zeta_{2,2} & \cdots & \zeta_{2,2k} & \zeta_{2,2k+1} \end{bmatrix}^T \\ \bar{\zeta}_2 &= \begin{bmatrix} \zeta_{2,2} & \zeta_{2,4} & \cdots & \zeta_{2,2k-2} & \zeta_{2,2k} \end{bmatrix}^T \\ \Lambda &= diag(\varrho_1, ..., \varrho_n) > 0, \end{aligned}$$

where $(A_0 - \bar{g}_1 c_0^T)$ and $(A_0 + b_0 k^T)$ are *Hurwitz matrices*, the polynomial (19) is stable, and $\bar{g}_2$ given by (20), is such that the closed loop of the nominal system is stable, $\hat{\theta} \to \theta$ and $e(t)$ tends globally exponentially to zero.

Proof: First we observe that the closed loop system at the nominal values of the parameters (8a)-(22) is

$$\begin{aligned} \dot{\tilde{x}} &= A_0 \tilde{x} + b_0 k^T \zeta_1 + b_0 \varphi^T(\hat{\theta}) \zeta_2(t) - b_0 m(\theta) r \\ \dot{\zeta}_1 &= (A_0 + b_0 k^T - \bar{g}_1 c_0^T)\zeta_1 + b_0 \varphi^T(\hat{\theta})\zeta_2(t) + \bar{g}_1 c_0^T \tilde{x} \\ \dot{\zeta}_2 &= \Phi(\hat{\theta})\zeta_2 + \bar{g}_2 \left(c_0^T \tilde{x} - c_0^T \zeta_1 - \mu^T \zeta_2 \right) \\ \dot{\hat{\theta}} &= -\Lambda \bar{\zeta}_2 (c_0^T \tilde{x} - c_0^T \zeta_1 - \mu^T \zeta_2). \end{aligned}$$

Now, defining the errors $e_1 = \tilde{x} - \tilde{\Pi}(\theta) r - \zeta_1$, $e_2 = r - \zeta_2$ and $e_3 = \hat{\theta} - \theta$, it is not difficult to see that the error dynamics take the form

$$\begin{aligned} \dot{\bar{e}}_1 &= \bar{A}\ \bar{e}_1 \\ \dot{e}_{2,2k+1} &= -\bar{\beta}_1^T \bar{e}_1 - \beta_2 e_{2,2k+1} + \bar{\zeta}_2^T e_3 \\ \dot{e}_3 &= -\Lambda \bar{\zeta}_2 (\bar{q}_1^T \bar{e}_1 + \mu_{2k+1} e_{2,2n+1}) \end{aligned} \tag{23}$$

where

$$\begin{aligned} \bar{e}_1 &= \begin{bmatrix} e_1^T & e_{2,1} & e_{2,2} & \cdots & e_{2,2k} \end{bmatrix}^T \\ e_2 &= \begin{bmatrix} e_{2,1} & e_{2,2} & \cdots & e_{2,2k} & e_{2,2k+1} \end{bmatrix}^T \\ \bar{\beta}_1 &= \begin{bmatrix} \vartheta_2 c_0^T & \vartheta_2 \mu_1 & (\theta_1 + \vartheta_2 \mu_2) & \cdots & (\theta_k + \vartheta_2 \mu_{2k}) \end{bmatrix}^T \\ \beta_2 &= \vartheta_2 \mu_{2k+1} \\ \bar{A} &= \begin{bmatrix} A - \bar{g}_1 c & 0 \\ -\upsilon_2 c & \Phi_2 \end{bmatrix} \\ \upsilon_2 &= \begin{bmatrix} 0 & 0 & \cdots & 0 & \vartheta_1 \end{bmatrix}^T \in \Re^{2k} \qquad \vartheta_1 = \frac{1}{\mu_{2n+1}} \\ \bar{q}_1 &= \begin{bmatrix} c_0^T & \mu_1 & \mu_2 & \cdots & \mu_{2k-1} & \mu_{2k} \end{bmatrix}^T \in \Re^{n+2k}. \end{aligned}$$

and the dynamics of $\tilde{x}(t)$ in term of the error variables takes the form

$$\dot{\tilde{x}} = (A_0 + b_0 k^T)\tilde{x} - b_0 k^T e_1 - b_0 \varphi^T(\theta) e_2 + b_0 \tilde{\varphi}^T(e_3)\zeta_2(t) \tag{24}$$

with $\tilde{\varphi}^T(e_3) = \varphi^T(\hat{\theta}) - \varphi^T(\theta)$. and $\tilde{\varphi}^T(0) = 0$.

Now assume without loss of generality that $\bar{A}$ has distinct eigenvalues, then there exists a matrix T such that

$$\begin{aligned} \tilde{A} &= T^{-1} \bar{A} T = diag(-\lambda_1, ..., -\lambda_{n+2k}) \\ \bar{e}_1 &= T \tilde{e}_1 \\ \tilde{q}_1^T &= \bar{q}_1^T T, \quad \tilde{\beta}_1^T = \bar{\beta}_1^T T, \end{aligned}$$

and therefore there exists some diagonal matrix $P > 0$ such that

$$P\ \tilde{A} = -\kappa I$$

for every $\kappa > 0$. With this transformation the system becomes

$$\begin{aligned} \dot{\tilde{e}}_1 &= \tilde{A}\ \tilde{e}_1 \\ \dot{e}_{2,2k+1} &= -\tilde{\beta}_1^T \tilde{e}_1 - \beta_2 e_{2,2k+1} + \bar{\zeta}_2^T e_3 \\ \dot{e}_3 &= -\Lambda \bar{\zeta}_2 (\tilde{q}_1^T \tilde{e}_1 + \mu_{2k+1} e_{2,2k+1}). \end{aligned}$$

Let us now consider the Lyapunov candidate function

$$\begin{aligned} V(\tilde{e}_1, e_{2,2k+1}, e_3) &= \frac{1}{2} \tilde{e}_1^T P \tilde{e}_1 + \frac{\rho}{2} e_{2,2k+1}^2 \\ &\quad + \frac{\rho}{\mu_{2k+1}} \tilde{q}_1^T \tilde{e}_1 e_{2,2k+1} \\ &\quad + \frac{\rho}{2\mu_{2k+1}} e_3^T \Lambda^{-1} e_3 \end{aligned}$$

which is positive definite for some matrix P. The derivative of $V(e)$ is given by

$$\begin{aligned} \dot{V}(e) &= -\kappa \left\| \tilde{e}_1 \right\|^2 - \frac{\rho}{\mu_{2k+1}} \tilde{e}_1^T\ \tilde{q}_1 \tilde{\beta}_1^T\ \tilde{e}_1 - \rho \beta_2 e_{2,2k+1}^2 \\ &\quad + \rho \left(\frac{1}{\mu_{2k+1}} \tilde{q}_1^T \tilde{A} - \tilde{\beta}_1^T - \frac{1}{\mu_{2k+1}} \tilde{q}_1^T \beta_2 \right) \tilde{e}_1 e_{2,2k+1} \\ &\le \left(-\kappa + \frac{\rho}{\mu_{2k+1}} \left\| \tilde{q}_1 \tilde{\beta}_1^T \right\| \right) \left\| \tilde{e}_1 \right\|^2 \\ &\quad - \rho \beta_2 e_{2,2k+1}^2 + \rho \omega \tilde{e}_1 e_{2,2k+1}, \end{aligned}$$

then, for every vector $\omega = \left(\frac{1}{\mu_{2k+1}} \tilde{q}_1^T \tilde{A} - \tilde{\beta}_1^T - \frac{1}{\mu_{2k+1}} \tilde{q}_1^T \beta_2 \right)$, and given $\rho > 0$ there exists some $\kappa > 0$ such that $\dot{V}(e)$ is negative, then the invariant set is

$$\Omega = \{ (\tilde{e}_1, e_{2,2k}, e_3) \mid \tilde{e}_1 = 0, e_{2,2k} = 0 \}$$

and the error system is stable, moreover, because $\bar{\zeta}_2$ tends exponentially to its steady state which is periodic, then it satisfies the persistence excitation condition. Applying the Persistency of Excitation Lemma (see [8], [15], [18]), to system (23) we can conclude that ($\tilde{e}_1 = 0, e_{2,2n} = 0$, $e_3 = 0$) is a globally exponentially stable equilibrium point for (23) hence by the lemma (1) we have that $e(t) \to 0$. ■

Now, concerning the robustness issue, we show in the following result that, since the regulator is calculated to guarantee robustness with respect to parameter variation in a neighborhood of the nominal values, this property is maintained by the overall scheme regulator plus adaptive frequency estimator.

Corollary 4: The regulator (22) preserves the robust regulation property for some neighborhood of the nominal parameters (A_0, b_0, c_0).

Proof: From the theorem (3) the origin of the nominal system is exponentially stable. Then by a converse Lyapunov theorem there exists a Lyapunov function $V(\bar{z})$ such that

$$\begin{aligned} c_1 \left\| \bar{z} \right\|_2 &\le V(\bar{z}) \le c_2 \left\| \bar{z} \right\|_2 \\ \frac{\partial V(\bar{z})}{\partial \bar{z}} f(\bar{z}) &\le -c_3 \left\| \bar{z} \right\|_2 \\ \left\| \frac{\partial V(\bar{z})}{\partial \bar{z}} \right\|_2 &\le c_4 \left\| \bar{z} \right\|_2 \end{aligned}$$

where

$$\begin{aligned}\bar{z} &= \begin{bmatrix} \tilde{x}^T & e_1^T & e_2^T & e_3^T \end{bmatrix}^T \\ \dot{\bar{z}} &= f(\bar{z})\end{aligned}$$

is the nominal system. Now without loss of generality, we suppose the system in the form

$$G(s) = \frac{b_1\left(s^m + c_m s^{m-1} + \cdots + c_2 s + c_1\right)}{s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots + a_1 s + a_0} \quad \text{with } m < n$$

whose dynamics matrices are

$$\begin{aligned}A_0 &= \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 1 \\ -a_0 & -a_1 & \cdots & -a_{n-1} \end{bmatrix} \\ b_0 &= \begin{bmatrix} 0 & \cdots & 0 & b_1 \end{bmatrix}^T \\ c_0 &= \begin{bmatrix} c_1 & \cdots & c_m & 1 & 0 & \cdots & 0 \end{bmatrix}^T\end{aligned}$$

and with perturbation given by

$$\begin{aligned}\Delta A_0 &= \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\Delta a_0 & -\Delta a_1 & \cdots & -\Delta a_{n-1} \end{bmatrix} \\ \Delta b_0 &= \begin{bmatrix} 0 & \cdots & 0 & \Delta b_1 \end{bmatrix}^T \\ \Delta c_0 &= \begin{bmatrix} \Delta c_1 & \cdots & \Delta c_m & 0 & \cdots & 0 \end{bmatrix}^T\end{aligned}$$

The closed loop system is then

$$\begin{aligned}\dot{\tilde{x}} &= \left(A_0 + \Delta A_0\right)\tilde{x} - (b_0 + \Delta b_0)m(\theta)r \\ &\quad + (b_0 + \Delta b_0)k\zeta_1 + (b_0 + \Delta b_0)\varphi^T(\hat{\theta})\zeta_2 \\ \dot{\zeta}_1 &= (A_0 + b_0 k^T - \bar{g}_1 c_0^T)\zeta_1 + b_0\varphi^T(\hat{\theta})\zeta_2 \\ &\quad + \bar{g}_1\left(c_0^T + \Delta c_0^T\right)\tilde{x} \\ \dot{\zeta}_2 &= \Phi(\hat{\theta})\zeta_2 + \bar{g}_2\left(\left(c_0^T + \Delta c_0^T\right)\tilde{x} - c_0^T\zeta_1 - \mu^T\zeta_2\right) \\ \dot{\hat{\theta}} &= -\Lambda\,\bar{\zeta}_2(\left(c_0^T + \Delta c_0^T\right)\tilde{x} - c_0^T\zeta_1 - \mu^T\zeta_2).\end{aligned}$$

Defining $\beta = \frac{\Delta b_1}{b_1}$, $e_1 = \tilde{x} - (1+\beta)\tilde{\Pi}(\theta)r - (1+\beta)\zeta_1$, $e_2 = r - \zeta_2$, $e_3 = \hat{\theta} - \theta$, after some algebra we have

$$\begin{aligned}\dot{\tilde{x}} &= (A_0 + b_0 k)\tilde{x} - b_0 k e_1 - b_0\varphi^T(\theta)e_2 + b_0\tilde{\varphi}^T(e_3)\zeta_2 \\ &\quad + \left(\Delta A_0\tilde{x} + \beta b_0\varphi^T(\theta)e_2 + \beta b_0\tilde{\varphi}^T(e_3)\zeta_2\right) \\ \dot{e}_1 &= (A_0 - \bar{g}_1 c_0)e_1 + (\Delta A_0 - \beta\bar{g}_1(c_0 + \Delta c_0) - \bar{g}_1\Delta c_0)\tilde{x}\end{aligned}$$

with $\tilde{\varphi}^T(e_3) = \varphi^T(\theta) - \varphi^T(\hat{\theta})$ and $\tilde{\varphi}^T(0) = 0$. Then the dynamics of e_2 is

$$\begin{aligned}\dot{e}_{2,i} &= e_{2,i}, \qquad i = 1 \ldots (2k-1) \\ \dot{e}_{2,2k} &= -c_0^T e_1 - \frac{\mu_1}{\mu_{2k+1}}e_{2,1} - \cdots - \frac{\mu_{2k}}{\mu_{2k+1}}e_{2,2k} \\ &\quad -\frac{1}{\mu_{2k+1}}\left(\Delta c_0^T + \frac{\beta}{1+\beta}c_0^T\right)\tilde{x} + \frac{1}{\mu_{2k+1}}\left(\frac{\beta}{1+\beta}\right)c_0^T e_1 \\ \dot{e}_{2,2k+1} &= -\bar{\beta}_1^T\bar{e}_1 - \beta_2 e_{2,2k+1} + \bar{\zeta}_2^T e_3 \\ &\quad -\vartheta_2\left(\Delta c_0^T + \frac{\beta}{1+\beta}c_0^T\right)\tilde{x} + \vartheta_2\left(\frac{\beta}{1+\beta}\right)c_0^T e_1\end{aligned}$$

and finally

$$\begin{aligned}\dot{e}_3 &= -\Lambda\bar{\zeta}_2\left[\bar{q}_1^T\bar{e}_1 + \mu_{2k+1}e_{2,2k+1}\right] \\ &\quad -\Lambda\bar{\zeta}_2\left[\left(\Delta c_0^T + \frac{\beta}{1+\beta}c_0^T\right)\tilde{x} - \left(\frac{\beta}{1+\beta}\right)c_0^T e_1\right]\end{aligned}$$

from this we can see that the system can be written as $\dot{\bar{z}} = f(\bar{z}) + g(\bar{z})$ where $g(0) = 0$, and $g(\bar{z})$ is a vanishing perturbation term and satisfies that $\|g(\bar{z})\| < \gamma_0\|\bar{z}\|$ for some constant γ_0 and some region D around the origin. Then if this constant satisfies that $\gamma_0 < \frac{c_3}{c_4}$ the Lyapunov function satisfies that $\dot{V}(\bar{z}) < (-c_3 + c_4\gamma_0)\|\bar{z}\|^2 < 0$ and the system remains stable. ■

Remark 5: Note that in general the solution $\tilde{\Pi}(\theta)$ and $m(\theta)$ of equations (14)-(15) may depend on the determinant of this system, and therefore the term $\varphi^T(\hat{\theta}) = (m^T(\hat{\theta}) + k^T\tilde{\Pi}(\hat{\theta}))$ can be written in the form

$$\varphi^T(\theta) = (m^T(\theta) + k^T\tilde{\Pi}(\theta)) = \frac{\eta^T(\theta)}{\psi(\theta)}$$

where $\psi(\theta)$ is the determinant of system equations (14)-(15). However, even if the determinant $\psi(\theta)$ is in general well defined, when considering the estimate $\hat{\theta}$ of θ, it may happen that $\psi(\hat{\theta})$ crosses the zero value. In order to avoid this problem, that appears also in the estimator proposed in [7], we modify the last equation to get

$$\frac{\eta(\hat{\theta})}{\psi(\hat{\theta})} \approx \frac{\psi(\hat{\theta})\eta^T(\hat{\theta})}{\psi(\hat{\theta})^2 + \epsilon e^{-\kappa_1\psi(\hat{\theta})^2}} = \varphi_0^T(\hat{\theta}) \quad \text{for } \epsilon > 0,\ \kappa_1 > 0, \tag{25}$$

which never reaches the singular point. Note that the global exponential stability of error system is preserved. Another possibility is estimate, if possible, directly the term $\varphi^T(\theta)$. In the following, we present a special class of systems for which the later is possible, namely class of strictly positive real systems.

VI. Simulation.

Let us consider the non-minimum phase system

$$\begin{aligned}\dot{x}_1 &= x_2 + 10 * \sin(t) \\ \dot{x}_2 &= (1+\nu_1)x_1 - x_2 + (1+\nu_2)u + 10 * \cos(2t) \\ y &= -x_1 + x_2\end{aligned}$$

where we suppose that the frequencies $\alpha_1 = 1, \alpha_2 = 2$, are unknown and ν_1, ν_2 are unknown parameters. In this case we obtain, applying the theorem 3 that the regulator take the form (22) with the following parameters:

$$\begin{aligned}
\bar{g}_1 &= \begin{bmatrix} -8 \\ -5 \end{bmatrix}, \; k = \begin{bmatrix} -5 & -3 \end{bmatrix} \\
\mu^T &= \begin{bmatrix} 15 & 18.5 & 7.5 & 1 \end{bmatrix}, \quad \vartheta_2 = 1.5 \\
\psi(\hat{\theta}) &= \left(1. + \hat{\theta}_1 + \hat{\theta}_2\right) \\
\varphi_0(\hat{\theta}) &= \begin{bmatrix} 1095. - 2306.\ \hat{\theta}_1 + 1095.\ \hat{\theta}_2 + \hat{\theta}_1^2 + \hat{\theta}_1\hat{\theta}_2 \\ 2550.5 - 851.5\ \hat{\theta}_1 + 2550.5\ \hat{\theta}_2 \\ 3242.5 - 159.5\ \hat{\theta}_1 - 158.5\ \hat{\theta}_2 + \hat{\theta}_1\hat{\theta}_2 + \hat{\theta}_2^2 \\ 3386.5 - 15.5\ \hat{\theta}_1 - 15.5\ \hat{\theta}_2 \end{bmatrix} \\
\varphi(\hat{\theta}) &= \varphi_0(\hat{\theta})\frac{1}{\psi(\hat{\theta})};
\end{aligned}$$

then the polynomial $\psi(\hat{\theta})$, may take the zero value for some values of $\hat{\theta}$, in this case we utilize the signal (25). In Figure 1 the output signal is shown. A change of parameters $\nu_1 = \nu_2 = 0$ to $\nu_1 = \nu_2 = 0.03$ .is introduced at $t = 60\ s$. We observe that the output is regulated despite the variations on the parameters value. In Figure 2, a change of frequency from $\alpha_2 = 2$ to $\alpha_2 = 3$ is performed at $t = 60\ s$. Again, the performance of the estimator suggests the validity of the proposed scheme.

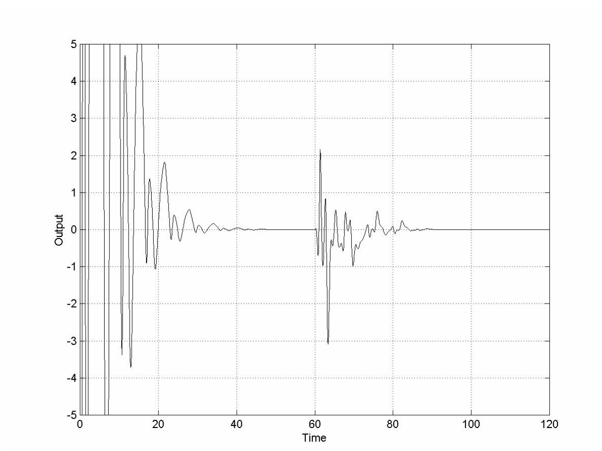
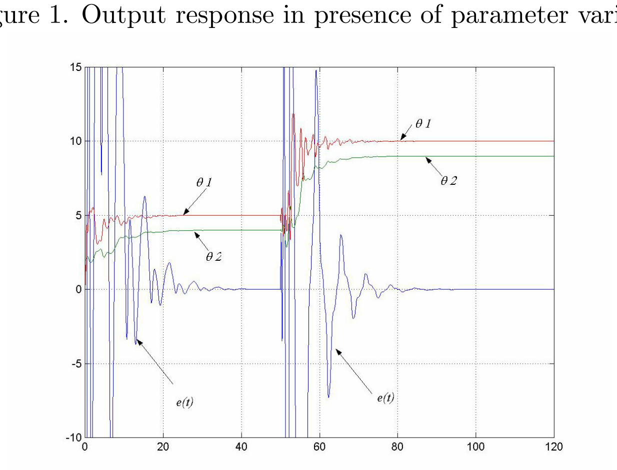


Figure 1. Output response in presence of parameter variation.



Fugure 2. Output response and Estimates.

VII. Conclusion.

In this paper, we present a new compensator with adaptive internal model, for minimum and non-minimum phase in linear SISO systems. It is shown that by assuring global convergence of the estimator preserves the autput regulation properties despite variations on the parameters of external signal generator. The property of robustness in presence of variations in a neighborhood of nominal values of the systems´s parameters is also guaranteed. Simulations result suggests the validity of the proposed scheme.

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