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# VARIABLE STRUCTURE CONTROL OF SYNCHRONOUS GENERATOR WITH SINGULARLY PERTURBED ANALYSIS

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Abstract: The synchronous generators have a natural different time scale dynamics. That is why for modeling and control design in such systems the methods of singular perturbations are widely used. In this paper the possibilities of sliding mode control design for synchronous generators are analyzed. With this aim the concept of singular perturbation is revised in order to use it for relay control system with a ***discontinuous right hand side.*** The obtained results are used for variable structure control of synchronous generator.

Keywords: sliding mode control, singular perturbations, nonlinear systems

## 1. INTRODUCTION

Simplifications of plant models is a classical tool for electric power systems control design, and the most typical way is the singular perturbation approach (see Sauer and Kokotovic 1998, Sauer and Pai 1998, Kokotovic *et al.* 1986, Krause 1986). From the other hand, a fruitful and relatively simple approach, especially when dealing with nonlinear plants subjected to perturbations, is based on Variable Structure Control technique with sliding mode (Utkin 1992). However the applying discontinuous (relay) control to a plant model with the singular perturbation leads to some problems. Classical methods of singular perturbation (see Vasil'eva *et al.* 1995, and Kokotovic *et al*. 1986) are based on the spectrum separation and consequently these approaches need the smoothness of the models and control law. That is why the classical methods of singular perturbations are not valid for Singularly Perturbed Relay Control Systems (SPRCS).

The decomposition methods for SPRCS were developed by Heck 1991, Su 1999, Fridman 2002a,b, Innocenti *et al.* 2003. The present paper discusses the advantages and possibilities of sliding mode control design for *nonlinear* SPRCS describing the synchronous generator dynamics. For the synchronous generators it is naturally to use a Two Step Control Design (TSCD) procedure:

I Eliminate the stator dynamics via singular perturbation methods and derive the reduced ($6^{th}$ order) model describing the mechanical and rotor fluxes dynamics.

II Design a sliding mode excitation control law using block control technique (Loukianov 1998).

So the order of the original SPRCS is reduced in two steps: first the elimination of the fast dynamics and then the reduction of the slow dynamics via sliding mode.

To justify TSCD procedure, first it is proved that the fast dynamics do not affect the entrance point into the sliding domain and the sliding mode equation of the slow dynamics outside of a boundary layer. Then conditions of the uniform asymptotic stability for the original SPRCS, are found. The obtained results are used to design a sliding mode control law for angular speed and voltage.

This paper is organized as follows. Section 2 introduces the basic equations of the synchronous generator. In Section 3 the concepts of singularly perturbed models with relay control are justified. In section 4 the singular perturbation approach is applied to design a synchronous generator controller. Simulation results are shown in Section 5.

## 2. SYNCHRONOUS GENERATOR MODELS

### 2.1 Basic Equations

The mathematical models for the synchronous generator are based on the mechanical and electric equilibrium equations The mechanical equilibrium equations for a synchronous generator are given by

$$\frac{d\delta}{dt} = \omega - \omega_b \tag{1}$$

$$\frac{d\omega}{dt} = \frac{\omega_b}{2H}\left(T_m - T_e\right) \tag{2}$$

where $\delta$ is the power angle (rad.), $\omega$ is the angular velocity (rad./sec.), $\omega_b$ is the synchronous angular velocity (rad./sec.), $H$ is the inertia constant (sec.), $T_m$ is the mechanical torque (p.u.), and $T_e$ is the electromechanical torque (p.u.). On the other hand, the electric equilibrium equations affected by the Park transformations (Park 1929), are expressed as

$$V = R\,i + \omega\,G\varphi + \frac{d\varphi}{d\bar{t}} \quad (3)$$

$$\varphi = L\,i \quad (4)$$

where $\bar{t} = \omega_b t$ , $\omega_b$ is the base angular velocity, $t$ is the time in seconds, $\bar{t}$ is the time in p.u., $i = [i_d, i_q, i_f, i_g, i_{kd}, i_{kq}]^T$ , $V = [V_d, V_q, V_f, 0, 0, 0]^T$ , $\varphi = [\varphi_d, \varphi_q, \varphi_f, \varphi_g, \varphi_{kd}, \varphi_{kq}]^T$

$$R = \begin{bmatrix} -r_s & & & & & \\ & -r_s & & 0 & & \\ & & r_f & & & \\ & & & r_g & & \\ & 0 & & & r_{kd} & \\ & & & & & r_{kq} \end{bmatrix} \qquad G = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} -L_d & 0 & L_{md} & 0 & L_{md} & 0 \\ 0 & -L_q & 0 & L_{mq} & 0 & L_{mq} \\ -L_{md} & 0 & L_f & 0 & L_{md} & 0 \\ 0 & -L_{mq} & 0 & L_g & 0 & L_{mq} \\ -L_{md} & 0 & L_{md} & 0 & L_{kd} & 0 \\ 0 & -L_{mq} & 0 & L_{mq} & 0 & L_{kq} \end{bmatrix}.$$

$V$ means voltage, $i$ means current, $\varphi$ means flux linkage, $r$ means resistance, $L$ means inductance, and the subscripts means: $s$ stator, $d$ direct axis circuit, $q$ quadrature axis circuit, $f$ field excitation circuit, $g$ quadrature field circuit, $kd$ direct axis damper, $kq$ quadrature axis damper, $md$ direct magnetizing, $mq$ quadrature magnetizing.

The equation for the electromechanical torque in terms of the currents and fluxes, is governed by

$$T_e = \varphi_d i_q - \varphi_q i_d \quad (5)$$

### 2.2 Time Scale Modeling

To simplify the model we will transform the system to the singularly perturbed form. With this aim we found a parasite parameter multiplying the stator dynamics. From (1)-(6) we obtain the stator singularly perturbed form

$$\frac{1}{\omega_b}\dot{\varphi}_d = \frac{\omega}{\omega_b}\varphi_q + r_s\,i_d + V_d \quad (6)$$

$$\frac{1}{\omega_b}\dot{\varphi}_q = -\frac{\omega}{\omega_b}\varphi_d + r_s\,i_q + V_q \quad (7)$$

### 2.3 Complete Model

From (1) to (7), we obtain the following model of synchronous generator of the 8th order:

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} F_1(\mathbf{x}_1, \mathbf{x}_2, \mathbf{z}, T_m) \\ F_2(\mathbf{x}_1, \mathbf{x}_2, \mathbf{z}) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} V_f \quad (8)$$

$$\mu\,\dot{\mathbf{z}} = F_3(\mathbf{x}_1, \mathbf{x}_2, \mathbf{z}) \quad (9)$$

where $\mathbf{x_1} = (x_1, x_2, x_3)^T$ , $\mathbf{x_2} = (x_4, x_5, x_6)^T$ , $\mathbf{z} = (z_1, z_2)^T$ , $x_1 = \delta$ , $x_2 = \omega$ , $x_3 = \varphi_f$ , $x_4 = \varphi_g$ , $x_5 = \varphi_{kd}$ , $x_6 = \varphi_{kq}$ $z_1 = \varphi_d$ , $z_2 = \varphi_q$ , $\mu = \frac{1}{\omega_b}$

$$F_1 = \begin{bmatrix} x_2 - \omega_s \\ d_m T_m - (a_{21}x_3 z_2 + a_{22}x_4 z_1 + a_{23}x_5 z_2 + a_{24}x_6 z_1 + a_{25}z_1 z_2) \\ a_{31}x_3 + a_{32}x_5 + a_{33}z_1 \end{bmatrix}$$

$$, F_2 = \begin{bmatrix} b_{11}x_4 + b_{12}x_6 + b_{13}z_2 \\ b_{21}x_3 + b_{22}x_5 + b_{23}z_1 \\ b_{31}x_4 + b_{32}x_6 + b_{33}z_2 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ b_3 \end{bmatrix} \text{ and } B_2 = 0 ,$$

$$F_3 = \begin{bmatrix} c_{11}x_2x_4 + c_{12}x_2x_6 + c_{13}x_2z_2 + c_{14}z_1 + c_{15}\sin x_1 \\ c_{21}x_2x_3 + c_{22}x_2x_5 + c_{23}x_2z_1 + c_{24}z_2 + c_{25}\cos x_1 \end{bmatrix}.$$

The coefficients of (8)-(9) depend on the parameters.

## 3. SINGULARLY PERTURBED APPROACH

### 3.1 Singularly Perturbed Model

In this paper we are dealing with the singularly perturbed model having the form:

$$\frac{dx}{dt} = f(x, z, \mu, u), \qquad x(0) = x_0 \quad (10)$$

$$\mu\frac{dz}{dt} = g(x, z, \mu), \qquad z(0) = z_0 \quad (11)$$

where $x \in R^n$ , $z \in R^m$ , $u \in R$ , $\mu \in R$ ; $f$ and $g$ are smooth functions of their argument and linear on $z$ and $u$, $\mu > 0$ is a small parameter, and $u$ is

$$|u| \le u_0 \text{ with } u_0 > 0 . \quad (12)$$

### 3.2 Control Design Procedure

The sliding mode control design procedure for original system (10), (11) consists of two steps.

*Step 1.* Setting $\mu = 0$ makes instantaneous the fast dynamics (11)

$$0 = g(x, z, 0). \quad (13)$$

Consider a smooth isolated solution of equation (13) for $z$

$$\bar{z} = h(x) \quad (14)$$

where $\bar{z}$ presents the quasi-steady state. Substituting (14) in (10) we obtain the reduced order model (ROM)

$$\frac{d\bar{x}}{dt} = f(\bar{x}, h(\bar{x}), 0, u) \quad (15)$$

where $\bar{x}(t)$ defines the solution of (15) for a fixed control $u(\bar{x})$ . Taking into account the specific feature of (8) and (9), we assume that (14) is linear with respect to $z$ and $u$, and solution (14) exists. Consequently, the ROM (15) is linear on $u$.

*Step2.* Design a nonlinear sliding surface $s(\bar{x}) = 0$ , $s \in R$ for the system (15) , such that the solution of the equation

$$\frac{ds}{dt} = \bar{G}f(\bar{x}, h(\bar{x}), 0, u_{eq}) = 0 , \ \bar{G} = \left\{\frac{ds}{d\bar{x}}\right\}$$

with respect to the equivalent control, $u_{eq}(\bar{x})$ (Utkin 1992), does exist, and the sliding mode equation (SME)

$$\frac{d\bar{x}}{dt} = f(\bar{x}, h(\bar{x}), 0, u_{eq}(\bar{x})) \quad (16)$$

$$s(\bar{x}) = 0 \quad (17)$$

has the desired properties. Second, taking into account (12), it is selected a discontinuous control

$$u(\bar{x}) = \begin{cases} u^+(\bar{x}) & if \quad s(\bar{x}) > 0, \quad \left|u^+(\bar{x})\right| \le u_0 , \\ u^-(\bar{x}) & if \quad s(\bar{x}) < 0, \quad \left|u^-(\bar{x})\right| \le u_0 \end{cases} \qquad (18)$$

that makes the sliding surface (24) to be attractive.

Note that (8), (9) is a particular case of (10), (11), when the functions $f$ and $g$ depend on $z$ and $u$ linearly and consequently exists a solution in Fillipov sense at least for a small $t$ (see for example Utkin 1992). Moreover, from (17) one of the vector $\bar{x}$ components can be expressed as a function of other ( $n-1$ ) components.

To justify the proposed control design (TSCD) procedure (see steps 1 and 2), first we will analyze the behavior of the original SPRCS when the state vector crosses the switching surface, and then the entrance into the sliding domain (subsection 3.3). Finally, the stability condition for SPRCS will be derived (subsection 3.4).

3.3 Analysis of SPRCS Solutions Crossing Sliding Surface

In this subsection we will study the behavior of the original SPRCS (10), (11) and (18) out from sliding mode domain. If a solution of the SPRCS is not crossing the discontinuity surface (17) it can be analyzed by classical method of singular perturbations (see Vasil'eva *et al.* 1995, and Kokotovic *et al.* 1986). From the other hand, the specific feature of SPRCS described the behavior of synchronous machines is that the equations of slow variables depend on the relay control (18), and consequently after a finite number of switches the trajectory of original SPRCS will enter into the sliding mode domain. In this subsection we will show that in the case of finite switches we can use the reduced order model to describe the slow motions in the SPRCS.

Denote the domains of definition for variables $z$ and $x$ as $Z$ and $X$. The discontinuity surface $s(x)=0$ divides the domains $X$ into the parts defined as $X^-$ for $s<0$ and $X^+$ for $s>0$, respectively; and define the system structure as

$$f^+(x,z,\mu) = f\left(x,z,\mu,u^+(x)\right) \quad \text{for } s \ge 0$$

$$f^-(x,z,\mu) = f\left(x,z,\mu,u^-(x)\right) \quad \text{for } s \le 0, \quad \text{with}$$

$$f^+ \in C^2\left[\bar{X}^+ \times [0,\mu_0]\right],\ f^- \in C^2\left[\bar{X}^- \times [0,\mu_0]\right].$$

*3.3.1. System in the domain* $s<0$.

Denote

$$\frac{ds^-}{dt}(x,z,\mu) = Gf^-(x,z,\mu), \quad \frac{ds^+}{dt}(x,z,\mu) = Gf^+(x,z,\mu).$$

Suppose that $x_0 \in X^-$, $z_0 \in Z$. It is natural to assume that for the original system (10), (11) and (18) the following conditions of the Tikhonov theorem (see, for example, Vasil'eva *et al.* 1995) hold:

[a1] The function $\bar{z} = h(\bar{x})$ is an isolated solution of $0 = g(x,z,0)$ for all $x \in X$.

[a2] The Cauchy problem for slow dynamics

$$\frac{d\bar{x}}{dt} = f^-\left(\bar{x}^-, h\left(\bar{x}^-\right), 0\right), \quad \bar{x}^-(0) = x_0 \qquad (19)$$

has a unique solution $\bar{x}^-(t)$ on $[0,\bar{t}_s]$, where $\bar{t}_s$ is the switching point i.e. the smallest root of the equation $s\left(\bar{x}^-(\bar{t}_s)\right) = 0$.

[a3] The equilibrium point $\Pi z = 0$ of the system

$$\frac{d(\Pi z)}{d\tau} = g\left(\bar{x}, \Pi z + h(\bar{x}), 0\right)$$

where $\Pi z = z - h(\bar{x})$, $\tau = t/\mu$, is asymptotically stable, moreover for all $x \in X$

$$\operatorname{Re}\ \operatorname{Spec}\ \frac{\partial g}{\partial \Pi z}\left(\bar{x}, h(\bar{x}), 0\right) < -\alpha < 0, \ \alpha > 0.$$

[a4] The trajectory of the reduced system (19) crosses the switching surface $s(x)=0$, without tangential touch, i.e.

$$\frac{d\bar{s}^-}{dt} = \bar{G}f^-\left(\bar{x}^-(\bar{t}_s), h\left(\bar{x}^-(\bar{t}_s)\right), 0\right) > 0.$$

***Remark*** . The function $g$ depends on $z$ linearly. This imply that every initial value of (11) belongs to the domain of attraction for the equilibrium point $\Pi z = 0$. Now from Vasil'eva theorem (Vasil'eva *et al.* 1995) and implicit function theorem it follows that for sufficiently small $\mu$ there exists a time moment $t = t_s(\mu)$ such that $s(x(t_s(\mu),\mu)) = 0$ and moreover, for all $\mu \in [0,\mu_0]$

$$\frac{ds^-}{dt} = Gf^-\left(x(t_s(\mu),\mu), z(t_s(\mu),\mu), \mu\right) > 0, \quad G = \left\{\frac{ds}{dx}\right\}$$

Now we have to consider two alternative variants for SPRCS solution behavior:

- a solution of original SPRCS will enter into the domain $X^+ \times Z$;
- a solution of original SPRCS will enter into the sliding domain.

*3.3.2 Entrance into the domain s>0*

From the condition [a4] and the boundary layer method (Vasil'eva *et al.* 1995), it follows that a solution of the original SPRCS will reach the switching surface $s(x(t_s(\mu),\mu)) = 0$ at the switching point

$$\left(x(t_s(\mu),\mu), z(t_s(\mu),\mu)\right) = \left(\bar{x}(\bar{t}_s) + O(\mu), h\left(\bar{x}(\bar{t}_s)\right) + O(\mu)\right).$$

This means that the point $\left(x(t_s(\mu),\mu), z(t_s(\mu),\mu)\right)$ does not belong to the sliding mode domain, and the solution of (10), (11) and (18) will enter into the domain $X^+ \times Z$. We can consider the coordinate of the switching point ( $x(t_s(\mu),\mu), z(t_s(\mu),\mu)$ ) as the initial condition for SPRCS into the domain $X^+ \times Z$ and suppose that for the original SPRCS, in the domain $X^+ \times Z$ the following conditions are satisfied:

[b1] $$\frac{d\bar{s}^+}{dt} = \bar{G}f^+\left(\bar{x}^-(\bar{t}_s), h\left(\bar{x}^-(\bar{t}_s)\right), 0\right) > 0$$

then from the Tikhonov theorem it follows that for sufficiently small $\mu$

$$\frac{d\,s^+}{dt} = Gf^+\left( x(t_s(\mu),\mu), z(t_s(\mu),\mu), \mu\right) > 0$$

[b2] Suppose that the Cauchy problem

$$\frac{d\bar{x}^+}{dt} = f^+\left(\bar{x}^+, h\left(\bar{x}^+\right), 0\right), \quad \bar{x}^+(\bar{t}_s) = \bar{x}^-(\bar{t}_s)$$

has a unique solution on $[\bar{t}_s, T]$.

The following lemma is true (Fridman 2002b):

***Lemma 1.*** *Suppose that the original SPRCS (10), (11) and (18) satisfies the conditions [a1]-[a4] and [b1]-[b2]. Then there exists small $\mu_0 > 0$ such that for all $\mu \in [0,\mu_0]$ there is a unique solution $(x(t,\mu), z(t,\mu))$ of Cauchy problem (10) and (11) on $[0,T]$, and*

$$\lim_{\mu\to 0} x(t,\mu) = \bar{x}(t) = \begin{cases} \bar{x}^-(t) & for\ \ t \in [0,\bar{t}_s] \\ \bar{x}^+(t) & for\ \ t \in [\bar{t}_s, T] \end{cases},$$

$$\lim_{\mu\to 0} z(t,\mu) = h(\bar{x}(t)) \quad \text{for} \quad t \in (0,T].$$

***Remark 1***. Also, we can prove that it is possible to use the slow motions equations to analyze the system (10)-(11) and (18) in the case when its solution leave the domain $X^+ \times Z$ and enter into the domain $X^- \times Z$.

*3.3.3 Transition into sliding domain*

The behavior of the original SPRCS (10), (11) and (18) into the sliding domain, is described. Denote as

$$S_0 = \left\{ x : \frac{ds^-}{dt}(\bar{x}, h(\bar{x}), 0) > 0, \frac{ds^+}{dt}(\bar{x}, h(\bar{x}), 0) < 0 \right\},$$

$$S_\mu = \left\{ (x,z,\mu) : \frac{ds^-}{dt}(x,z,\mu) > 0, \frac{ds^+}{dt}(x,z,\mu) < 0 \right\}$$

the sliding domains for the systems (15) and (10)-(11) respectively. Suppose that the control allows the sliding mode existence conditions (Utkin 1992):

[c1] $$\frac{d\,s^-}{dt}\left(\bar{x}^-(\bar{t}_s), h\left(\bar{x}^-(\bar{t}_s)\right), 0\right) > 0,$$

$$\frac{d\,s^+}{dt}\left(\bar{x}^-(\bar{t}_s), h\left(\bar{x}^-(\bar{t}_s)\right), 0\right) < 0$$

Now from the Tikhonov theorem it follows that for sufficiently small $\mu$

$$\frac{ds^-}{dt} = Gf^-\left(x(t_s(\mu)), z(t_s(\mu)), \mu\right) > 0,$$

$$\frac{ds^+}{dt} = Gf^+\left(x(t_s(\mu)), z(t_s(\mu)), \mu\right) < 0$$

which means that, a solution of the original system (10), (11) and (18) enter into the sliding domain $S_\mu$ without tangential touch. Therefore, we can consider $(x(t_s(\mu),\mu), z(t_s(\mu),\mu))$ as the initial condition for SPRCS into the sliding domain $S_\mu$. Hence, a solution of the Cauchy problem (10), (11) with (18) into $S_\mu$ is described by the following system (Utkin 1992):

$$\frac{dx^*}{dt} = f\left(x^*, z^*, \mu, u_{eq}(x^*, z^*, \mu)\right), \quad \mu\frac{dz^*}{dt} = g\left(x^*, z^*, \mu\right) \qquad (20)$$

$$x^*(t_s(\mu),\mu) = x(t_s(\mu),\mu), \quad z^*(t_s(\mu),\mu) = z(t_s(\mu),\mu), \quad s(x^*) = 0.$$

where $t \in [t_0(\mu), T]$, $x^* \in R^{n-1}$, $z^* \in R^m$, $u \in R$, $\mu \in [0,\mu_0]$, and $u_{eq}(x^*, z^*, \mu)$ is the equivalent control calculated as a solution of

$$\frac{ds}{dt} = Gf\left(x^*, z^*, \mu, u_{eq}\right) = 0, \quad s(x^*) = 0. \qquad (21)$$

Similar to the above case (subsection 3.3.2) we suppose that for the system (20)-(21) the following conditions of the Tikhonov theorem hold:

[c2] The reduced (by $\mu = 0$) sliding mode equation

$$\frac{d\bar{x}^*}{dt} = f\left(\bar{x}^*, h(\bar{x}^*), 0, \bar{u}_{eq}(\bar{x}^*)\right), \quad \bar{x}^*(\bar{t}_s) = x_0^*$$

with $\bar{u}_{eq}(\bar{x}^*) = u_{eq}(\bar{x}^*, h(\bar{x}^*), 0)$ has a unique solution $\bar{x}^*(t)$ on $[\bar{t}_s, T]$, and $\bar{x}^*(t) \in S_0$ for all $t \in [\bar{t}_s, T]$.

The following lemma is true (Fridman 2002b):

***Lemma 2***. *Suppose that the original SPRCS (10), (11) and (18) satisfies the conditions [a1]-[a4] and [c1]-[c2]. Then there exists a small $\mu_0 > 0$ such that for all $\mu \in [0,\mu_0]$ there is a unique solution $(x(t,\mu), z(t,\mu))$ of (10), (11) and (18) on $[0,T]$ and*

1) $\lim_{\mu\to 0} u_{eq}\left(x(t,\mu), z(t,\mu), \mu\right) = \bar{u}_{eq}\left(\bar{x}^*(t)\right)$ *for* $t \in [\bar{t}_s, T]$,

2) $\lim_{\mu\to 0} x(t,\mu) = \bar{x}(t) = \begin{cases} \bar{x}^-(t) & for\ \ t \in [0,\bar{t}_s] \\ \bar{x}^*(t) & for\ \ t \in [\bar{t}_s, T] \end{cases}$,

3) $\lim_{\mu\to 0} z(t,\mu) = h(\bar{x}(t))$ *for* $t \in (0,T]$.

***Remark 2***. If a solution of (10), (11) and (18) will leave the sliding modes it will not affect the zero approximation of the dynamics equations, because the slow integral manifold is continuous (Fridman 2002b).

3.4. Stability Analysis

Consider the case when the original SPRCS has an equilibrium into $S_\mu$. Solving (28) for $u_{eq}^*(x(t,\mu), z(t,\mu))$ and substituting it in (10 ), we obtain the *smooth* algebraic - differential singularly perturbed system (20) which describes the sliding mode dynamics. From the equation (17), taking into account that $G \neq 0$ one can express one coordinate or $x$ as a function of other $(n-1)$ coordinates. Then a sliding mode dynamics is governed by the singularly perturbed $(n+m-1)th$ order system:

$$\frac{dx^\otimes}{dt} = f^\otimes\left(x^\otimes, z^\otimes, \mu\right), \quad \mu\frac{dz^\otimes}{dt} = g^\otimes\left(x^\otimes, z^\otimes, \mu\right) \qquad (22)$$

where the vector $x^\otimes \in R^{n-1}$ consists of the $n-1$ independent coordinates of $x$, $z^\otimes = z$, $g^\otimes$ and $f^\otimes \in R^{n-1}$ are the values of $g$ and the corresponding component of $f$

computed at $u = u_{eq}\left(x^{\otimes}, z^{\otimes}, \mu\right)$. For the case of synchronous machine the equation $g^{\otimes}\left(\bar{x}^{\otimes}, h^{\otimes}\left(\bar{x}^{\otimes}\right), 0\right) = 0$ has a unique solution $\bar{z}^{\otimes} = h^{\otimes}\left(\bar{x}^{\otimes}\right)$, consequently the slow dynamics in (22) are described by the system

$$\frac{dx^{\otimes}}{dt} = \bar{f}^{\otimes}\left(\bar{x}^{\otimes}\right) = f^{\otimes}\left(\bar{x}^{\otimes}, h^{\otimes}\left(\bar{x}^{\otimes}\right), 0\right), 0 = g^{\otimes}\left(\bar{x}^{\otimes}, h^{\otimes}\left(\bar{x}^{\otimes}\right), 0\right) \quad (23)$$

Let us denote $x_{eq}{}^{\otimes}$ as an equilibrium point of (23). Then from Klimushchev – Krasovskii theorem (Klimushchev and Krasovskii 1962) it follows that the equilibrium point of system (22) is uniformly asymptotically stable for $\mu \in [0, \mu_0]$, if the matrices in (23):

[*ss*] $\frac{\partial \bar{f}^{\otimes}}{\partial x^{\otimes}}\left(x_{eq}{}^{\otimes}, h^{\otimes}\left(x_{eq}{}^{\otimes}\right), 0\right)$ and

[*sf*] $\frac{\partial g^{\otimes}}{\partial z^{\otimes}}\left(x_{eq}{}^{\otimes}, h^{\otimes}\left(x_{eq}{}^{\otimes}\right), 0\right)$, are Hurwitz.

Therefore we can conclude that in order to verify correctness of the proposed TSCP it is enough to check the conditions presented in the subsections 3.2 - 3.3.

## 4. CONTROL OF GENERATOR

In this section, we will derive a reduced model and a discontinuous control law for the generator.

### 4.1 Reduced Model of Synchronous Machine

The fast dynamics (8)-(9) can be neglected by making $\mu = 0$. The following reduced ($6^{th}$ order) model is obtained:

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} \bar{F}_1(\mathbf{x_1}, \mathbf{x_2}, T_m, h_1(\mathbf{x_1}, \mathbf{x_2}), h_2(\mathbf{x_1}, \mathbf{x_2})) \\ \bar{F}_2(\mathbf{x_1}, \mathbf{x_2}, h_1(\mathbf{x_1}, \mathbf{x_2}), h_2(\mathbf{x_1}, \mathbf{x_2})) \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} V_f \quad (24)$$

where

$$\bar{F}_1 = \begin{bmatrix} x_2 - \omega_s \\ d_m T_m - \left[(a_{22}x_4 + a_{24}x_6)\cdot h_1(\cdot) + (a_{21}x_3 + a_{23}x_5)\cdot h_2(\cdot) + a_{25}h_1(\cdot)h_2(\cdot)\right] \\ a_{41}x_3 + a_{42}x_4 + a_{43}x_5 + a_{44}x_6 + a_{45}\sin x_1 + a_{46}\cos x_1 \end{bmatrix}$$

$$, \bar{F}_2 = \begin{bmatrix} b_{11}x_4 + b_{12}x_6 + b_{13}h_2(\mathbf{x_1}, \mathbf{x_2}) \\ b_{21}x_3 + b_{22}x_5 + b_{23}h_1(\mathbf{x_1}, \mathbf{x_2}) \\ b_{31}x_4 + b_{32}x_6 + b_{33}h_2(\mathbf{x_1}, \mathbf{x_2}) \end{bmatrix}, \ B_1 = \begin{bmatrix} 0 \\ 0 \\ b_3 \end{bmatrix}.$$

The coefficients of (24) depend on the plant parameters. The condition [a2] for (24) is satisfied.

### 4.2 Angular Speed Control

The first subsystem of (24) has the Nonlinear Block Controllable form consisting of three blocks. Therefore we use the block control technique (Loukianov 1998). To satisfy rotor angle stability we define the control error as

$$\varsigma_2 = x_2 - \omega_b \quad (25)$$

The derivative of (25) along the trajectories of (24) is

$$\dot{\varsigma}_2 = f_2(\mathbf{x_1}, \mathbf{x_2}, T_m) + b_2(\mathbf{x_1}, \mathbf{x_2})x_3 \quad (26)$$

where

$f_2 = d_m T_m - \left(a_{22}x_4 h_1(\cdot) + a_{23}x_5 h_2(\cdot) + a_{24}x_6 h_1(\cdot) + a_{25}h_1(\cdot)h_2(\cdot)\right)$, $b_2 = a_{21}h_2(\cdot)$, and $b_2(t)$ is a positive function of the time. To introduce a new desired behavior we put

$$x_3 = -b_2(\mathbf{x_1}, \mathbf{x_2})^{-1}\left[f_2(\mathbf{x_1}, \mathbf{x_2}, T_m) + k_0\varsigma_2 - s_\omega\right], \ k_0 > 0 \quad (27)$$

Then using (27) the switching surface can be defined as

$$s_\omega(x) = b_2(\mathbf{x_1}, \mathbf{x_2})x_3 + f_2(\mathbf{x_1}, \mathbf{x_2}, T_m) + k_0(x_2 - \omega_B) = 0 \quad (28)$$

### 4.3 Stability Analysis.

*4.3.1 Sliding mode stability.*

Design the relay control law as follows

$$V_f = -u_0 sign(s_\omega), \quad u_0 > 0. \quad (29)$$

Then we can see that under

$u_0 \geq \left|V_{feq}(\mathbf{x_1}, \mathbf{x_2}, T_m)\right|$, $V_{feq} = b_s^{-1}(\mathbf{x_1}, \mathbf{x_2}) f_s(\mathbf{x_1}, \mathbf{x_2}, T_m)$, the conditions [a4], [b1] or [c1] are satisfied. That means the state vector convergence to (28) in a finite time $t_s$, and after this time a sliding mode motion occurs.

*4.3.2 Sliding dynamics stability*

Once the sliding mode motion is achieved, this motion is governed by the reduced order ($5^{th}$ order) system:

$$\dot{x}_1 = \varsigma_2\,; \dot{\varsigma}_2 = -k_0\varsigma_2 \quad (30)$$

$$\dot{\mathbf{x}}_2 = \bar{F}_2(\mathbf{x_1}, \mathbf{x_2}, h_1(\mathbf{x_1}, \mathbf{x_2}), h_2(\mathbf{x_1}, \mathbf{x_2})) \quad (31)$$

where the linear subsystem (30) describe the mechanical dynamics, with eigenvalue $-k_0$, while (31) represents the rotor flux dynamics. The subsystem (30) is stable, that is $\lim_{t\to\infty} \varsigma_2(t) = 0$ and $\mathbf{x}_2$ is referred to as the *zero dynamics*.

*4.3.3 Fast dynamics stability*

The slow motion manifold (23) does not depend on *u*

$$\Pi z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} - \begin{bmatrix} h_1(\mathbf{x_1}, \mathbf{x_2}) \\ h_2(\mathbf{x_1}, \mathbf{x_2}) \end{bmatrix}. \quad (32)$$

The derivative fast dynamics is governed by

$$\mu\frac{d(\Pi z)}{dt} = A_z(\mathbf{x_1})\Pi z + \mu\begin{bmatrix} \frac{\partial h_1}{\partial \mathbf{x_1}} & \frac{\partial h_1}{\partial \mathbf{x_2}} \\ \frac{\partial h_2}{\partial \mathbf{x_1}} & \frac{\partial h_2}{\partial \mathbf{x_2}} \end{bmatrix}\begin{bmatrix} F_1(\mathbf{x_1}, \mathbf{x_2}, \Pi z + h(\mathbf{x_1}, \mathbf{x_2})) + B_1 V_f \\ F_2(\mathbf{x_1}, \mathbf{x_2}, \Pi z + h(\mathbf{x_1}, \mathbf{x_2})) \end{bmatrix} \quad (33)$$

where $A_z(\mathbf{x_1}) = A_R$, and the control takes value $u_0$ or $-u_0$ if $s_\omega \neq 0$, and equal to $V_{feq}(\mathbf{x_1}, \mathbf{x_2}, T_m)$ on the surface $s_\omega = 0$. Setting $\mu = 0$ freezes the variable $\mathbf{x_1}$ and $\mathbf{x_2}$ at $t = 0$ and reduce (33) to the system

$$\frac{d(\Pi z)}{d\tau} = A_z(\mathbf{x_{1_0}})\Pi z, \ \mathbf{x_{1_0}} = \mathbf{x_1}(0).$$

The matrix $A_z(\mathbf{x_{1_0}})$ is Hurwitz, hence $\Pi z = 0$ and assumptions [a2] and [*sf*] in this case hold.

## 5. SIMULATION RESULTS

The performance of the proposed control algorithm was tested on the complete eight order model of synchronous generator connected to an infinite bus, Fig.1.

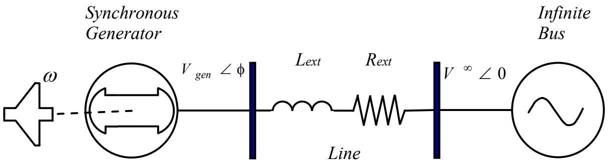


Fig. 1. Single machine with infinite bus.

The parameters of the synchronous machine and external network in p.u. are (Kundur 1994):

$T'_{do}=8.0$ sec. $T'_{qo}=1.0$sec. $T''_{do}=0.03$sec. $T''_{qo}=0.07$sec.

$L_d=1.81$, $L'_d=0.3$, $L''_d=0.23$ , $L_q=1.76$, $L'_q=0.6$ , $L_{ext}=0.1$, $R_{ext}=0.001$. From this we obtain the parameters of model (15)-(16), and (30). The controller gains was adjusted to $k_0=10$. The eigenvalues of (42) was calculated as, $\lambda_4=-38.77$ , $\lambda_5=-0.5024$ and $\lambda_6=-27.04$ . Figures 2-4 depict results under a three-phase short circuit (150 ms. long) simulated at the transformer terminals.

These Figures reveal some important aspects:

1 State variables hastily reach a steady state condition after small and large disturbances, exhibiting the stability of the closed-loop system.

2 The terminal voltage recovers their steady state value after the short circuit

## 6 CONCLUSIONS

In this paper the possibility of apply a sliding mode control algorithm for nonlinear SPRCS describing the synchronous generator model dynamics is analyzed. For SPRCS describing the behavior of synchronous machines this means that the slow equations depend on relay control. For such system the following two steps control design (TSCD) is proposed: firstly, the natural two scale properties of synchronous generator are used to obtain the reduced order model, and then a sliding mode control algorithm ensuring the desired behavior of the generator is designed. The effectiveness of proposed algorithm is illustrated on simulations with the parameters of a real generator.

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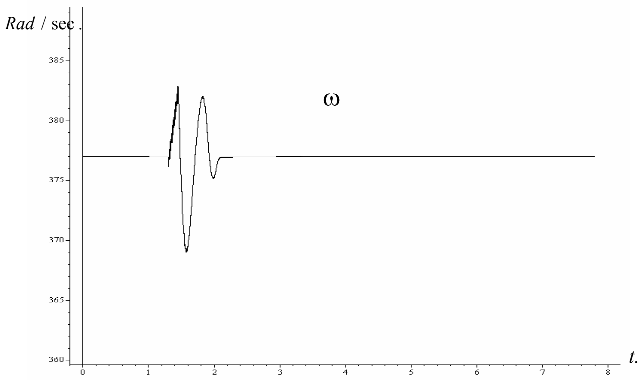


Fig. 2. Rotor angular velocity affected by a 0.15 sec. short circuit.

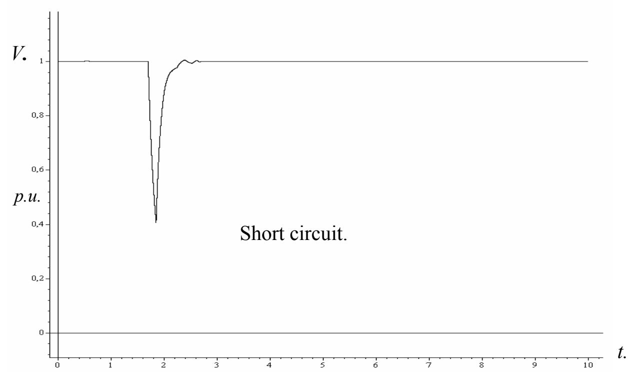


Fig. 3. Generator voltage affected by a 0.15 sec. short circuit.

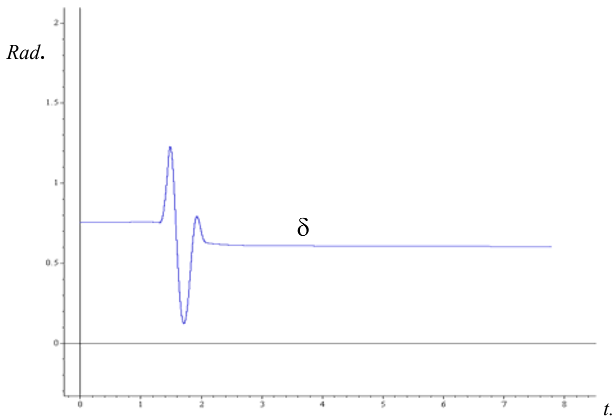


Fig. 4. Power angle affected by a 0.15 sec. short circuit.