

# Haptic Guidance for Training Motor Skills

V. Parra-Vega, Omar A. Dominguez-Ramírez, and J. A. Méndez-Iglesias  
 Mechatronics Division, CINVESTAV, AP 14-740, México, DF, México

**Abstract**—Based on a novel formulation of the computation of the contact force of haptic interfaces, a new paradigm for haptic guidance is proposed. Guided kinesthetic feedback is provided to improve and effectively train the user with a sliding PID force robot control. Lyapunov stability theory is used to prove asymptotic stability of the closed loop system. Nonlinear Differential Algebraic set of Equations (DAE) arises from the nonlinear Euler Lagrange dynamics of the robotic haptic device and second order linear virtual environment. The DAE is solved to provide the constrained Lagrangian to establish stable interaction on deformable virtual environments, while the decentralized controller ensures passivity, in contrast to the widely used penalty-based method. The system introduces a training path using potential fields, which can be tuned according to the handicap score of the user, to gradually improve the motor skills of the user. Experimental data validate the proposed scheme, and results are discussed and analyzed for a complex calligraphy task.

**Index Terms**—Haptic Interfaces, Deformable Virtual Environments

## I. INTRODUCTION

Training with visual information is good for trajectory shape, however, the temporal attributes of this trajectory are better perceived with haptics. Haptic interfaces allows to feel unreal (virtual) objects, objects created into a virtual environment of a computer graphic environment. This is possible with kinesthetic stimulus at the contact point. So, the basic issue on haptic interfaces is the stimulus of the virtual, contact forces to the human operator, and the correlation of this contact forces to the visual images, real or virtual, perceived by the human operator. Certainly, the quality of kinesthetic feedback does not come only from the contact force, but visual cues are less important in the perception of virtual objects. In this paper, we explore a haptic scheme that allows better perception of temporal and spatial attributes of kinesthetic interaction to virtual environments for haptic training.

The application areas of haptic interfaces varies from typical virtual training stations to entertainment and

recently, manipulation of molecular docking of drugs and molecules. However, a new direction of haptic interfaces is on guided virtual teleoperators, wherein the haptic interface is now guided by another real teleoperator. This scheme mimics the role of a teacher (master teleoperator) holding virtually the hand of a student (the slave is holding the haptic interface). This is, the teacher guides and constraint the motion of the student to train him/her. Since the system is dynamical, thus a main issue is to yield dynamic-based contact force, in contrast to static-based contact forces. The high bandwidth of the tactile force-pressure physiological sensor of the operator requires high precision haptic interface to stimulate correctly the mechanoreceptors of the operator, while low bandwidth haptic rendering is required for visual stimuli. Also, the haptic rendering graphics should convey deformation of the virtual world accordingly to achieve bilateral force-pressure stimulus.

Realistic interaction to virtual environment through a haptic device is established as long as physics-based immersion is obtained, that is realistic contact force and realistic haptic rendering. It is evident that Hooke's Law-based contact force, or penalty-based method, will render limited kinesthetic feedback since a static mapping is used. This is not a realistic contact force.

In [13], we show that the Lagrangian-based control scheme allows to convey richer stimulus in comparison to hugely popular penalty-based method. High-end kinesthetic coupling that arises using the constrained Lagrangian method yields a more realistic contact force as a function on the dynamical properties of the whole system.

In [14] we propose a haptic guiding and exploration scheme to train the student using Lagrangian contact force. In [12] it was shown how to reproduce objects attributes such as shape, texture and roughness to allow a more realist compliant contacts and recreate real sensations of remote exploration.

### A. The problem

How can we guarantee that the closed-loop system of a manipulator immersed in a guided virtual environment yields a static or dynamic virtual object? We show in

We acknowledge support from Conacyt grant 39727 Y and CUDI grant for Internet2 Applications. Email: (vparra,omar,jmendez)@mail.cinvestav.mx

this paper the design of a control input that guarantees that the closed-loop equation produces the equation of the object, the properties of the object. When the object is a training task, training of motor skills is obtained. When there is a path to be followed over, or away of, the object, then a haptic guided scheme appears.<sup>1</sup>

### B. The contribution

In this paper, we prove from the point of view of control how a Lagrangian-based guided haptic interface produces effectively dynamic kinesthetic stimulus, that is the closed-loop equation shows that the haptic display, or manipulator, is in contact to a given virtual object, and force feedback is established with simultaneous control of position and velocity along the surface of this object, either static or dynamic. In this way, it is very easy to reproduce objects attributes such as shape, texture and roughness to allow a more realist contact force compliant to the real sensations of remote exploration. The PHANToM haptic interface is programmed to obtain real time experimental results to validate the proposed scheme.

### C. Organization

First, we introduces in Section II the robot and object dynamics, then the controller is proposed in Section III, with the stability analysis is presented. The guidance scheme is proposed in IV, and discussion on training motors skills are given in VI, with experimental data discussed in VIII.

## II. ROBOT AND OBJECT DYNAMICS

### A. Robot dynamics

Consider a mechanism of articulate links, with  $n$  revolute joints described in generalized joint coordinates  $(q^T, \dot{q}^T)^T \in R^{2n}$ . Physically, the robot is never in touch with a physical object, thus robot dynamics are the usual dynamics in free motion describe properly the haptic device, as follows

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau \quad (1)$$

<sup>1</sup>Haptic training involves two robotic systems, one robot, the master, guides remotely the other robot, the haptic system, in turn the haptic system is immersed into a virtual environment in a haptic guidance scheme. In the remote station, the remote robot can be a passive linkage robotic arm exploring the real object in contact; this remote arm is equipped with angular position sensors and force sensor to measure angular displacement and real contact forces with the object. At the local station, a haptic display is required to generate the force contact coming from the real contact at the remote station. In this paper, we consider that two Phantoms are involved in each side. In the master side, desired trajectories are generated on line, so without loss of generality we discuss only one haptic device.

where  $M(q) \in \mathbb{R}^{3 \times 3}$  denotes a symmetric positive definite inertial matrix,  $C(q, \dot{q}) \in \mathbb{R}^{3 \times 3}$  is a Coriolis and centripetal forces matrix,  $g(q) \in \mathbb{R}^3$  models the gravity forces,  $B \in \mathbb{R}^{3 \times 3}$  denotes the viscous coefficient, and  $\tau \in \mathbb{R}^3$  stands for the torque input

### B. Virtual object dynamics

1) *Static virtual object*: Usually a static, motionless, object is modelled in terms of the universal or the fixed inertial frame. It is easy then to express the object as an implicit equation of robot generalized coordinates as follows

$$\varphi(q) = 0 \quad (2)$$

Using the inverse kinematic, equation (2) can be expressed in term of operational coordinates  $X$  by  $q = f^{-1}(X)$  as  $\varphi_x(X) = 0$ . When the haptic device contacts to the object (2), then a contact force equivalent to the restitution force  $K\Delta X$  is transmitted to haptic interface. Intrinsicly, this contact force model assumes deformation with plastic index 1. For stiff objects, which is usually the case,  $K$  can be very large and provokes high frequencies as well as numerical instabilities, besides that colliding with stiff object produces high impact collision forces.

2) *Dynamic virtual object*: A second order system, mass-spring-damper system, is considered as follows

$$m\ddot{\varphi}(q) + b\dot{\varphi}(q) + k\varphi(q) = 0 \quad (3)$$

where  $m$  is the mass,  $b$  is damper and  $k$  is spring. It is considered that the lumped model (3) is a pointwise model. Note that the lumped model (3) can be casted easily as a FEM model.

## III. CONTROL DESIGN

A haptic interface moves around intermittently between non-contact and contact regime. Non-contact regime is when the gimbal (end-effector of the haptic device) is moving without any constraint, without touching anything in the virtual environment, while contact regime arises when the gimbal virtually touches the virtual object. Thus, physically, the control of the robot arm (5) switches between a gravity and friction free controller (free motion) to/from constrained controller (constrained motion).

### A. Free motion control

When moving freely in the virtual environment, in this regime the user must not feel any opposing force. Thus

the controller must only compensate for nonconservative forces as follows

$$\tau = g(q) + B_0\dot{q} \quad (4)$$

The closed loop system is then

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} = 0 \quad (5)$$

whose equilibrium is the whole reachable workspace  $R^n$ .

### B. Constrained lagrangian motion control

1) *Static object*: When the gimbal touches the virtual object, actuator  $\tau$  should produce a joint torque at each joint equivalent to produce the contact forces  $\lambda$  generated in the gimbal. That is,  $\tau$  should establish stable interaction, while providing  $\lambda$ . To this end, consider the following

$$\tau = \frac{\|J_\varphi J_\varphi^T\|}{J_\varphi M(q)^{-1} J_\varphi^T} \lambda \quad (6)$$

$$\lambda = -m\dot{J}_\varphi\dot{q} + mJ_\varphi M(q)^{-1}(C(q, \dot{q})q + g(q)) \quad (7)$$

when contact to a undeformable object (2). The closed loop equation between (6)-(7) into (2) is, after some algebraic manipulations,

$$\ddot{\varphi}(q) = 0 \quad (8)$$

which, for consistency to DAE equations,  $\varphi(q) = 0$  when a rigid arm is in contact to a rigid object, then  $\dot{\varphi}(q) = 0$  and  $\ddot{\varphi}(q) = 0$ , that is equation (8) becomes

$$\varphi(q) = 0 \quad (9)$$

which is nothing but the virtual object (2). This is, mathematically we prove that controller (6)-(7) yields contact to the virtual static object (8) with a contact force  $\lambda$  computed dynamically.

2) *Dynamic object*: Now, when there is virtual contact to a undeformable object, the controller is

$$\tau = \frac{\|J_\varphi J_\varphi^T\|}{mJ_\varphi M(q)^{-1} J_\varphi^T} \lambda \quad (10)$$

$$\lambda = -b\dot{\varphi}(q) - k\varphi(q) - m\dot{J}_\varphi\dot{q} + mJ_\varphi M(q)^{-1}(C(q, \dot{q})q + G(q)) \quad (11)$$

The closed loop equation between (5) and (10)-(11) is, after some algebraic manipulation,

$$m\ddot{\varphi}(q) + b\dot{\varphi}(q) + k\varphi(q) = 0 \quad (12)$$

which is nothing but the dynamics of virtual object (3). Similar to previous subsection, we prove mathematically that controller (10)-(11) yields stable contact to the virtual dynamical object (12) with a contact force  $\lambda$ .

**Comments 1.** The controllers (6)-(7) and (10)-(11) guarantee that the closed loop yields the object geometry, that is the gimbal, end effector of the haptic device is in contact to the object, without exerting any force on the object.

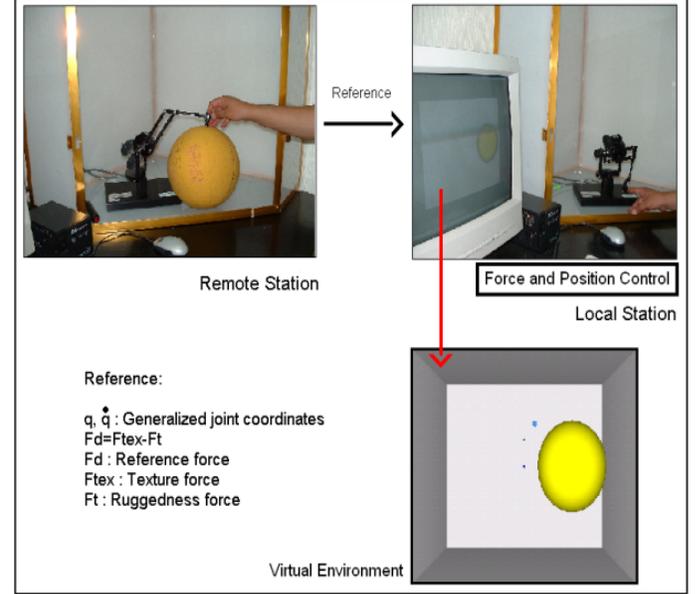


Fig. 1. Remote exploration of a real object through a haptic interface with haptic guidance controller

## IV. GUIDANCE CONTROL

So far, we have proposed three motion controller for three operational regimes. These controllers create a free floating haptic device in interaction. Notice that there are not desired trajectories, the human operator carries out the position control, while  $\tau$  implements a force control loop in the orthogonal direction of the subspace spanned by  $\dot{q}$ . This allows decoupling of position and force control, namely decoupling between motor control and the human control.

Now in order to achieve guidance or training motor skills of a human user holding the haptic device, desired trajectories need to be introduced in the previous controllers, and by tuning feedback gains, the allowable training error can be controlled. In this way, as training is improved, the allowable training error can be reduced. Also, during the initial stage it is expected that the trainee delivers the task with larger errors, so in order to help during the initial training phase, it is proposed potential fields to further control and improve the task.

Haptic guidance schemes are employed in tasks of remote training. The haptic device defines, in the master station, the position references (free motion) or position and forces (constrained motion) that will be reproduced

in the remote device (remote station). The switching algorithm over time is as follows, for  $\varepsilon \ll 1$  a threshold

- Phase a: Without interaction  $\varphi(q) > \varepsilon \rightarrow$  Free Motion Control
- Phase b: Collision detection  $-\varepsilon \leq \varphi(q) \leq \varepsilon \rightarrow$  Constrained Motion Control
- Phase c: Stable interaction with deformation  $\varphi(q) < -\varepsilon \rightarrow$  Constrained Motion Control

**Remark 1.** *It can be seen that applying the constrained Lagrangian method, in contrast to the penalty-based method, involves low or null frequencies over the virtual object. This allows a stable interaction without trembling for deformable objects.*

#### A. Guidance for free motion regime

A nonlinear PID control [7] is proposal for haptic guidance task in free movement. This control compensate the nonlinear dynamics in continuous mechanical plants with tracking capability. The nonlinear PID controller given by

$$\tau = \frac{\|J_\varphi J_\varphi^T\|}{J_\varphi M(q)^{-1} J_\varphi^T} \lambda \quad (13)$$

$$\lambda = -m \dot{J}_\varphi \dot{q} + m J_\varphi M(q)^{-1} (C(q, \dot{q})q + g(q) + \tau_c)$$

$$\begin{aligned} \tau_c = & -k_p \Delta\varphi - k_v \Delta\dot{\varphi} + k_d S_d - \\ & k_i \int_{t_0}^t \text{sgn}(\Delta\dot{\varphi} + \alpha \Delta\varphi - S_d) d\varsigma \end{aligned} \quad (14)$$

where  $S_d = (\Delta\dot{\varphi}(t_0) + \alpha \Delta\varphi(t_0)) \exp^{-k(t-t_0)}$ , for  $k_p, k_v, k, \alpha > 0$ , are positive feedback gains of appropriate dimensions. Tracking errors are defined as  $\Delta\varphi = \varphi - \varphi_d, \Delta\dot{\varphi} = \dot{\varphi} - \dot{\varphi}_d$  for position and velocity, respectively. Desired values are the position  $\varphi_d$  and velocity  $\dot{\varphi}_d$  generated by the master or teacher. This controller guarantees exponential tracking with using the model, similar to [7]. For dynamic object is similar, the proposal is similar along the lines of subsection III-B.2 and this section, and is therefore omitted.

#### B. Guidance for constrained motion regime

For this case, the control  $\tau_c$  of (12) is based on the control law [11] to ensure simultaneous tracking of force and position trajectories

$$\tau_c = -\tau_p - \tau_f - \tau_d \quad (15)$$

where

$$\begin{aligned} \tau_p &= -K_p(t) \Delta q - K_v(t) \Delta \dot{q} - K_{ip}(t) I_p \\ \tau_p &= K_F(t) \Delta F + K_\lambda(t) \Delta \lambda - K_{iF}(t) I_f \\ \tau_d &= K_g \mathcal{N} + \zeta(t) \end{aligned} \quad (16)$$

and  $K_p(t), K_v(t), K_{ip}(t), K_{iF}(t), K_F(t), K_\lambda(t), \dot{I}_F, \dot{I}_p, K_g(t), \mathcal{N}, \zeta(t)$  are time varying feedback gains that depends on matrix  $Q(q) = I - J_\varphi^T(q) \left( J_\varphi(q) J_\varphi^T(q) \right)^{-1}$  stands for the orthogonal projection of the normal of a matrix  $J_\varphi \in \mathbb{R}^{1 \times 3}$ , and on  $J_\varphi^T$ . Gains  $\alpha, \beta$  are positive constants,  $\Delta F = \int (\lambda_{sd} - \lambda_s) dt$ ,  $K_d = K_d^T \in \mathbb{R}^{3 \times 3}$  are positive gains. Desired contact force  $\lambda_{sd}$  is the commanded force by the master or teacher, while  $\lambda_s$  is the contact force being generated in the virtual environment by the trainee. This controller guarantees fast simultaneous tracking of position and force trajectories defined by the master operator. See [11]. For dynamic object is similar, the proposal is similar along the lines of subsection IV-B and is therefore omitted.

## V. PERCEPTION OF OBJECT PROPERTIES

Training is better realized under a realistic physics-based environment. On one hand, lagrangian based contact force allows more realistic interaction. On the other hand, even more realistic virtual contact can be obtained if object properties could be felt by the user, such as texture, roughness, and the shape of the object. Perception of object properties might be carried out with the real or virtual object, in any case, haptic rendering (visual images) must be consistent to what the user feels.

Parameterization of roughness, shape and texture properties arises in normal and tangential subspaces at the contact point. This in turn can be reproduced by the actuators of the real haptic device operational contact forces  $[f_x, f_y, f_z]^T$  along each unitary axis  $i, j, k$ . In this case, the master is physically in contact to the object, and moves in its surface to feel  $[f_x, f_y, f_z]$ , and with it, the roughness, shape and texture through force sensor. Now, how to reproduce object properties with only joint torques? In this section, along [12], we discuss an approach that synthesizes texture, roughness, and shape from  $f_x, f_y$ , and  $f_z$  measurements. In this way, better and more realistic kinesthetic coupling is established to be able to better train motor skills.

#### A. Roughness Perception

The sliding friction between two different materials with contact area defined by  $A$ , is equal to the load  $W$  divided by the flow stress  $P_m$  of the weaker of the two solids in contact. At this region of contact, the solid form a number of junctions as if they were welded together. Friction  $F$  represents the force required to shear these junctions apart. Mathematically, the theory is expressed as  $A = \frac{W}{P_m}, F = As, \mu = \frac{F}{W} = \frac{s}{P_m}$ , where  $s$  is the shear stress. Thus, the coefficient of friction  $\mu \ll 1$  may be represented by the ratio of shear stress to flow

stress of the material, and becomes its intrinsic property. Roughness arises as function of the sliding motion over the surface of the object, thus roughness is function of the tangential friction  $f_T$ . Since  $f_T$  arises at the tangential plane at the contact point, it is then function of joint velocity  $\dot{q}$ , in terms of  $\dot{X} = J\dot{q}$  as follows

$$f_T = \mu \sqrt{(f_x^2 + f_z^2)} \dot{X} \quad (17)$$

where,  $\vec{f}_x, \vec{f}_z \in S_t$ .  $\dot{X} = J\dot{q}$ , with  $J$  as the Phantom jacobian matrix. The torques based on the tangential friction force is defined by the equation,

$$\begin{aligned} \tau_c &= J^T f_T \\ &= \mu \sqrt{(f_x^2 + f_z^2)} J^T J \dot{q} \end{aligned} \quad (18)$$

In this way, we can model the roughness by simply assigning roughness properties through  $f_x, f_z$ .

### B. Texture Perception

The perception of surface texture is a specific design issue in force feedback interfaces. Manipulation of everyday objects, the perception of surface texture is fundamental to accurately identify contact points and apply the correct internal contact force. In a virtual environment also, haptic texture information can both increase the sense of realism of an object as well as convey information about what the object is and where it is. Phantom haptic device conveys texture by actuating kinesthetic forces on the users fingers. In this work we model the texture property as a periodic function

$$\tau_c = Amp (\sin(2\pi ft) + 1) \quad (19)$$

where  $Amp$  stands for half of the maximum value of texture torque,  $f$  stands for the frequency in hertz and  $t$  is the time in seconds.

### C. Shape Perception

Shape is perceived by the normal contact force of an object. Thus, equation (7) directly provides this perception in absence of roughness and texture.

## VI. TRAINING MOTOR SKILLS

For training tasks, it is required to generate on line the desired trajectory simply by substituting  $\varphi_d(t)$  as the desired temporal and spatial tasks. Controlling the feedback gains, it is possible to control the error of the task. For an excellent review of the subject, see the unique paper and reference in it.

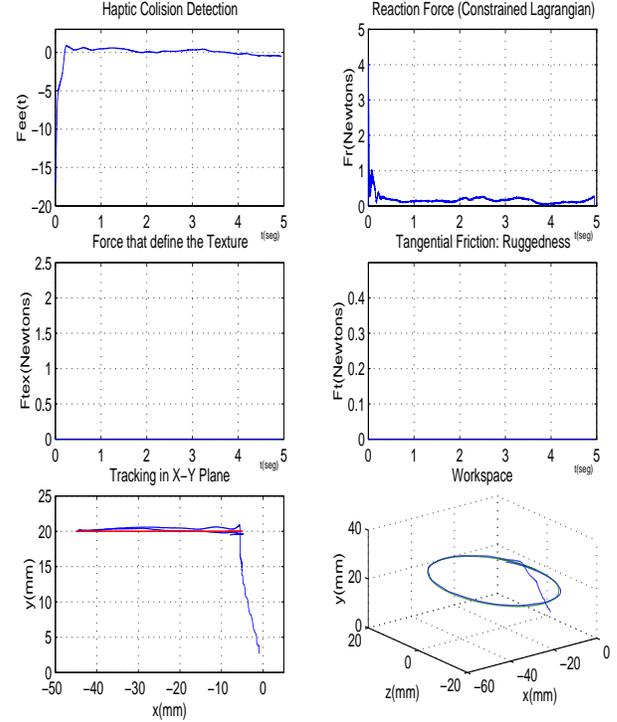


Fig. 2. Haptic control, with friction.

## VII. EXPERIMENTS

The conditions of the experiments are defined in a parallel plane  $S_t$  to the plane XZ, the human operator of the remote station develops a circular trajectory on the plane  $S_t$  with texture and ruggedness (in way emulated by means of references to the controller of force and position in the local station). The constraint surface is  $\varphi(q) = l_2 - l_2 \cos(q_3) + l_1 \sin(q_2) - y_0$ , where  $l_1 = l_2 = 139.7 \text{ mm}$ . The parametric equations that define the trajectory are  $x = h + r \cos(wt)$ ,  $y = y_0$ ,  $z = k + r \sin(wt)$ . These equations correspond to a circumference in the plane  $S_t$ , with center in  $C(h, y_0, k)$  and radius  $r$ . The Jacobian of PHANTOM is given by

$$J = \begin{bmatrix} l_1 c_1 c_2 + l_2 s_3 c_1 & -l_1 s_1 s_2 & l_2 s_1 c_3 \\ 0 & l_1 c_2 & l_2 s_3 \\ -(l_1 s_1 c_2 + l_2 s_1 s_3) & -l_1 s_2 c_1 & l_2 c_1 c_3 \end{bmatrix} \quad (20)$$

where  $c_* = \cos(*)$  and  $s_* = \sin(*)$ . The time in all the experiments is of  $t = 5$  seconds

### A. Experimental Setup

In all the experiments, the following parameters were used,  $h = -25.0 \text{ mm}$ ,  $y_0 = 20.0 \text{ mm}$ ,  $k = 0.0 \text{ mm}$ ,  $w = 2\pi/5$ . Texture properties are  $f_x = R \sin(2\pi f R t) \left[ \frac{J_{\varphi 11}}{\|J_{\varphi} J_{\varphi}^T\|} \right]$ ,  $f_z = R \sin(2\pi f R t) \left[ \frac{J_{\varphi 13}}{\|J_{\varphi} J_{\varphi}^T\|} \right]$ . In the table I presents the parameters used in each one of the experiments,

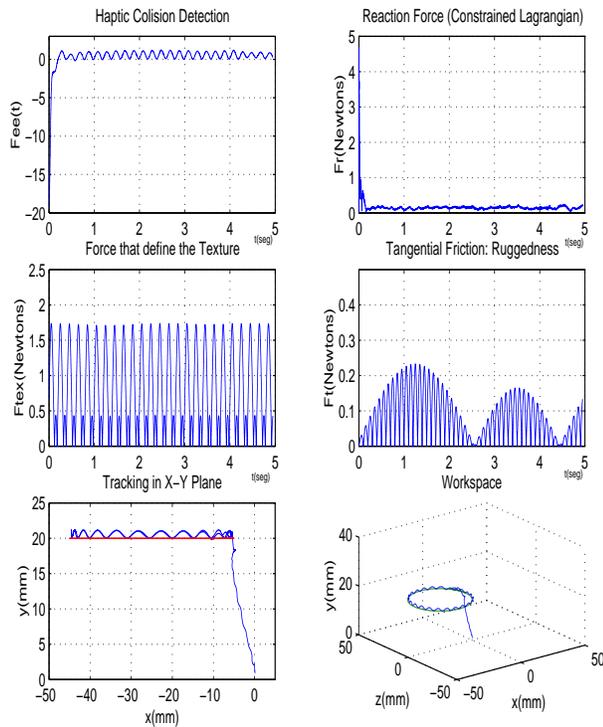


Fig. 3. Object properties.

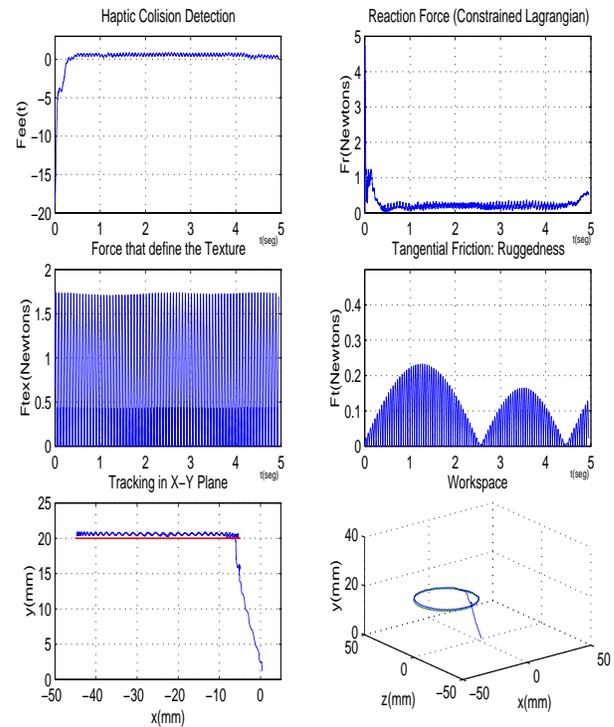


Fig. 4. Remote guidance with roughness properties.

Experiment	Amp	f	$\mu$	R	$f_R$
1	0	0	0	0	0
2	150	0.5	0.015	50	0.5
3	150	1.5	0.015	50	1.5
4 - deformation	150	0.5	0.015	50	0.5

TABLE I  
PARAMETERS USED IN EXPERIMENTS

B. Experiments

The figures that describe the experiments 1, 2, 3 and 4 are defined in Fig. 2, Fig. 3, Fig. 4 and Fig. 5.

VIII. DISCUSSIONS

A scheme to train motor skills of a trainee has been proposed. It is argued that a realistic, physics-based, haptic interface provides a better understanding of the attributes of the motor tasks. A general framework based on constrained robot dynamics renders a Lagrangian-based contact force controllers within a systematic way to produce contact to static and dynamic objects, and its attributes of shape, roughness and texture. The system is stable for free, collision and constrained motion by using a novel decentralized class of force-position robot control. This result has been supported theoretically, and experimental evidence suggest a successful training is easily achieved when desired trajectories are supervised

on line. The computational cost is quite low, and simple, yet formal stability arguments guarantee the stability of the closed-loop system convenient for remote virtual training.

REFERENCES

- [1] G.C. Burdea, "Force and Touch Feedback for Virtual Reality", John Wiley and Sons, Inc., Electrical and Computer Engineering Department, The State University of New Jersey, 1996.
- [2] T. H. Massie, J. K. Salisbury. "The PHANTOM Haptic Interface: A Device for Probing Virtual Objects", Proceedings of the ASME Winter Annual Meeting, Symposium on Haptic Interfaces for Virtual Environment and Teleoperator Systems, Chicago, IL, Nov. 1994.
- [3] Salisbury, J.K. and Mandayam A. Srinivasan. "Phantom-Based Haptic Interaction with Virtual Objects", IEEE Computer Graphics and Applications; Volume 17, Number 5, pp. 6-10; September-October 1997.
- [4] SensAble Technologies, Inc., "PHANTOM PREMIUM 1.0/1.5A, 3D Touch Components: Hardware Installation and Technical Manual", Revision 6.5, 18 August, 2000.
- [5] M. Moreyra, B. Hannaford, "A Practical Measure of Dynamic Response of Haptic Devices", International Conference on Robotics and Automation, Proceedings of the 1998 IEEE, pp. 369-374, Leuven Belgium, 1998.
- [6] T. Yoshikawa, A. Nagura, "Development and Control of Touch and Force Display Device for Haptic Interface", IFAC, pp. 127-132, 2000.
- [7] V. Parra-Vega, S. Arimoto "Nonlinear PID Control with Sliding Modes for Tracking of Robot Manipulator", Conference on Control Applications. Proceedings of the 2001 IEEE International.

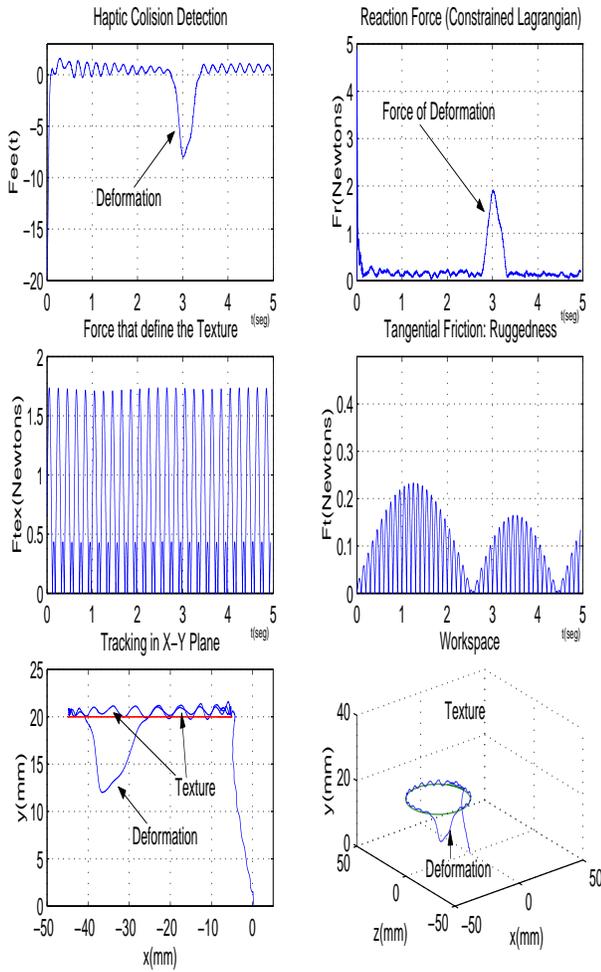


Fig. 5. Training path with deformation.

[8] C. Preusche, G. Hirzinger, "Scaling Issues for Teleoperation", The Fifth Phantom user's Group Workshop, Massachusetts Institute of Technology, Cambridge, MA, pp. 39-42, 2000.

[9] C.R. Carignan, K.R. Cleary, "Closed-Loop Force Control for Haptic Simulation of Virtual Environments", The Electronic Journal of Haptics Research, vol. 1, No. 2, 2000.

[10] S. Arimoto, "Control Theory of Non-Linear Mechanical Systems", Oxford Science Publications, 1996.

[11] V. Parra-Vega, A. Castillo-Tapia, "Perfect Tracking of Force and Position for Constrained Robot Manipulators with Orthogonal Nonlinear PID Control", International Symposium on Robotics and Automation 03, Toluca, México 2003.

[12] O. A. Dominguez-Ramirez, V. Parra-Vega, Texture, Roughness, and Shape Haptic, Perception of Deformable Virtual Objects with Constrained Lagrangian Formulation, IEEE-RSJ International Conference on Intelligent Robots and Systems, October 27 - 31, 2003 Bally's Las Vegas Hotel, Las Vegas, NV, USA.

[13] O. A. Dominguez-Ramirez, V. Parra-Vega, Constrained Lagrangian-Based Force-Position Control for Haptic Guidance, EuroHaptics 2003, Media Lab Europe, Ireland, 2003

[14] O. A. Dominguez-Ramirez, V. Parra-Vega, Haptic Remote Guided Exploration of Deformable Objects, Special Sessions on Telerobotics, Haptics, and Human Robot Collaboration of the 2003 ASME International Mechanical Engineering Congress and Exposition, November 16-21, 2003, Washington, USA.

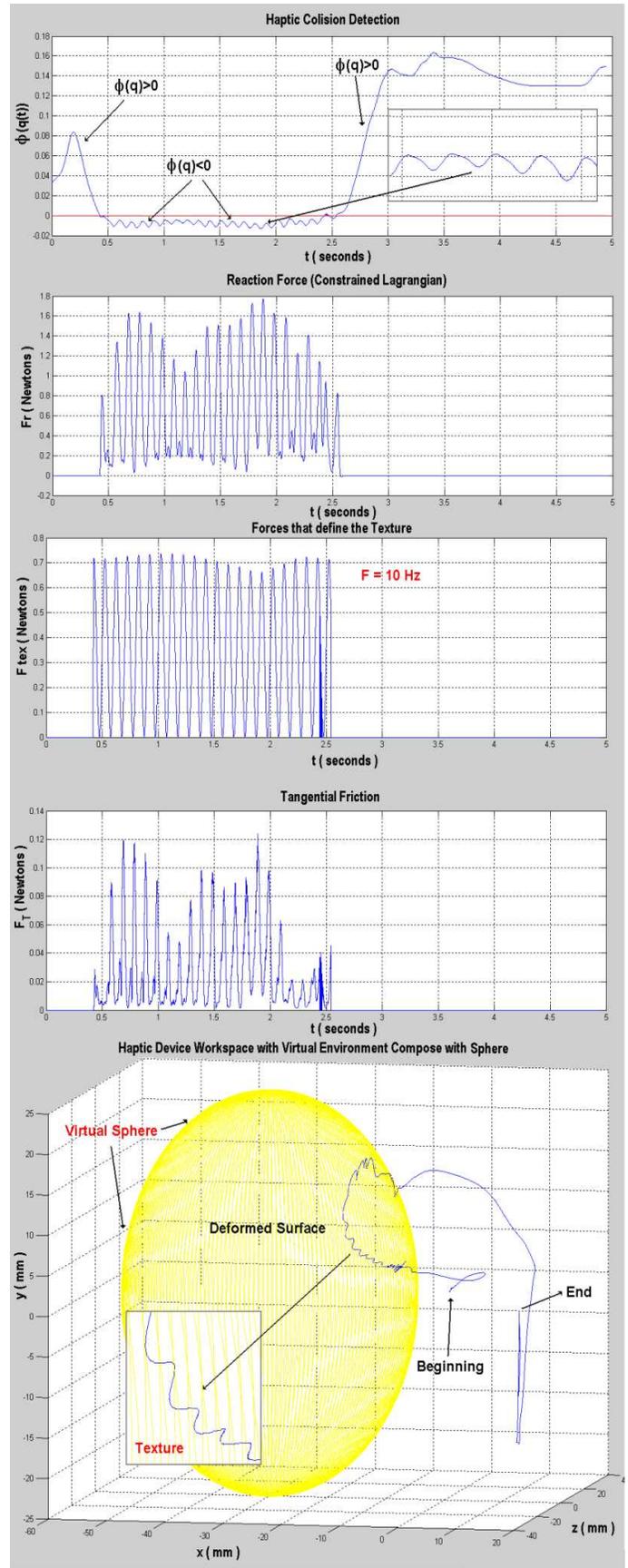


Fig. 6. Virtual sphere with texture define with a  $F_{text} = 0.7 \sin(40\pi t) + 0.7 N$