Nonlinear $\mathcal{H}_\infty$-Position Regulation of Friction Mechanical Manipulators*

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Abstract

Nonlinear $\mathcal{H}_\infty$-controller synthesis is developed and applied to a regulation problem for mechanical systems with friction. Due to the nature of the approach, the resulting controller additionally yields the desired robustness properties against the discrepancy between the real friction and that described in the model. To facilitate exposition, the friction model chosen for treatment is confined to the Dahl model augmented with viscous friction. Performance issues of the nonlinear $\mathcal{H}_\infty$-regulation controller are illustrated in a simulation study made for a two degrees-of-freedom robot manipulator.

keywords: Nonlinear $\mathcal{H}_\infty$-control; Nonsmooth dynamic systems; Mechanical manipulator; Friction.

1. Introduction

Robust control of mechanical systems has attracted considerable research interest. However, in spite of the rich and varied literature on this topic (see, e.g., related surveys compiled by Sage *et al.* [9]), the following factors, relevant in practice, are far from apropriately handled in combination:

1. The system dynamics is nonlinear and, due to frictional effects, it is nonsmooth.

2. The motion of the system is affected by unknown disturbances. Incomplete and imperfect state measurements are only available.

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Thus, it is of interest to develop consistent control methods that are capable of handling all of the listed factors and thereby yielding good performance on real systems.

In the present paper the nonlinear $\mathcal{H}_\infty$-control approach is extended to nonsmooth systems and then applied to regulation control problems for mechanical manipulators with friction. Due to the nature of the approach, the resulting controller is additionally expected to yield the desired robustness properties against the discrepancy between the real friction and that described in the model.

The $\mathcal{H}_\infty$-controller design presented here is based on the game-theoretic approach from [1], and the $\mathcal{L}_2$-gain analysis [7], and [10], providing complete solutions for nonlinear autonomous systems. Our approach [8] extends their results to account for nonsmooth friction dynamic models (the Dahl model, the LuGre model etc.) which have been brought into play to accurately describe observed frictional effects (the stiction behavior, the Stribeck effect etc.). The nonsmooth analysis from Clarke [4] will also be instrumental in our design.

The paper is outlined as follows. The $\mathcal{H}_\infty$-control problem for a class of nonsmooth systems is studied in Section 2. Sufficient conditions for the existence of a global solution of the problem are given in terms of solvability of two Hamilton-Jacobi-Isaacs inequalities which arise in the state-feedback and, respectively, output-injection design.

In Section 3, the synthesis procedure is extended to a regulation control problem for mechanical systems with friction. To facilitate exposition, the friction model chosen for treatment is confined to the Dahl model augmented with viscous friction. This simplest dynamic model captures all the essential features of the general treatment thereby making the extension to other friction dynamic models straightforward. Since the position is assumed to be the only available measurement on the system the resulting nonlinear $\mathcal{H}_\infty$-controller necessarily includes velocity and friction compensators. Performance issues of the controller developed are illustrated in a simulation study made for a two degrees-of-freedom robot manipulator. Finally, Section 4 presents conclusions.

2. Nonlinear $\mathcal{H}_\infty$-Control Synthesis

2.1. Problem Statement

Consider a nonlinear system of the form

$$\begin{aligned} \dot{x} &= f_1(x) + f_2(x) + g_1(x)w + g_2(x)u \\ z &= h_1(x) + k_{12}(x)u \\ y &= h_2(x) + k_{21}(x)w \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state space vector, $u \in \mathbb{R}^m$ is the control input, $w \in \mathbb{R}^r$ is the unknown disturbance, $z \in \mathbb{R}^l$ is the unknown output to be controlled, $y \in \mathbb{R}^p$ is the only available measurement on the system. The following assumptions are made throughout.

A1) The functions $f_1(x)$, $f_2(x)$, $g_1(x)$, $g_2(x)$, $h_1(x)$, $h_2(x)$, $k_{12}(x)$, $k_{21}(x)$ are locally Lipschitz continuous in x.

A2) $f_1(0) = 0$, $f_2(0) = 0$, $h_1(0) = 0$, and $h_2(0) = 0$.

A3)

$$h_1^T(x)k_{12}(x) = 0,\ k_{12}^T(x)k_{12}(x) = I$$
$$k_{21}(x)g_1^T(x) = 0,\ k_{21}(x)k_{21}^T(x) = I.$$

These assumptions are made for technical reasons. Assumption A1) guarantees the well-posedness of the above dynamic system, while being enforced by integrable exogeneous inputs. Along with this, Assumption A1) admits nonsmooth nonlinearities. In the subsequent local analysis nonsmooth nonlinearities are only absorbed into the term $f_2(x)$ whereas the other terms, including $f_1(x)$, are smooth enough. Assumption A2) ensures that the origin is an equilibrium point of the undriven ($u = 0$) disturbance-free ($w = 0$) dynamic system (1). Assumption A3) is a simplifying assumption inherited from the standard $\mathcal{H}_\infty$-control problem.

A causal dynamic feedback compensator

$$u = \mathcal{K}(y), \tag{2}$$

with internal state $\xi \in \mathbb{R}^s$, is said to be a globally (locally) admissible controller if the closed-loop system (1), (2) is globally (uniformly) asymptotically stable when $w = 0$.

Given a real number $\gamma > 0$, it is said that system (1), (2) has $\mathcal{L}_2$-gain less than γ if the response z, resulting from w for initial state $x(0) = 0, \xi(0) = 0$, satisfies

$$\int_0^T \|z(t)\|^2\,dt < \gamma^2 \int_0^T \|w(t)\|^2\,dt \tag{3}$$

for all $T > 0$ and all piecewise continuous functions $w(t)$.

The nonsmooth $\mathcal{H}_\infty$-control problem is to find a globally admissible controller (2) such that $\mathcal{L}_2$ -gain of the closed-loop system (1), (2) is less than γ. In turn, a locally admissible controller (2) is said to be a local solution of the $\mathcal{H}_\infty$-control problem if there exists a neighborhood U of the equilibrium such that inequality (3) is satisfied for all $T > 0$ and all piecewise continuous functions $w(t)$ for which the state trajectory of the closed-loop system starting from the initial point $(x(0), \xi(0)) = (0, 0)$ remains in U for all $t \in [0, T]$.

2.2. Local State-Space Solution

Our local analysis additionally assumes the following.

A4) The functions $f_1(x)$, $g_1(x)$, $g_2(x)$, $h_1(x)$, $h_2(x)$, $k_{12}(x)$, $k_{21}(x)$ are twice continuously differentiable in x locally around the origin $x = 0$ whereas their first and second order state derivatives are piecewise continuous.

A5) The vector $\zeta = 0$ is a proximal supergradient of the components $f_{2i}(x)$, $i = 1, \dots, n$ of the function $f_2(x)$ at $x = 0$.

Recall that a vector $\zeta(\hat{x}) \in \mathbb{R}^n$ is a proximal supergradient of a scalar function $f(x)$ at $\hat{x} \in \mathbb{R}^{n+1}$ if there exists some $\sigma(\hat{x}) > 0$ such that

$$f(x) \le f(\hat{x}) + \zeta^T(\hat{x})(x - \hat{x}) + \sigma(\hat{x})\|x - \hat{x}\|^2 \tag{4}$$

for all x in some neighborhood $U(\hat{x})$ of $\hat{x}$. The set $\partial^P f(\hat{x})$ of proximal supergradients of f at $\hat{x}$ is referred to as the proximal superdifferential of f at $\hat{x}$.

So, the components $f_{2i}(x)$, $i = 1, \dots, n$ of the function $f_2(x)$ are such that

$$0 \in \partial^P f_{2i}(0) \tag{5}$$

or equivalently,

$$f_{2i}(x) \le \sigma\|x\|^2 \tag{6}$$

for some $\sigma > 0$, and all $x \in U(0)$.

Assumptions A1)-A5), coupled together, allow one to linearize the corresponding Hamilton-Jacobi-Isaacs inequalities thereby yielding a local solution of the nonsmooth $\mathcal{H}_\infty$-control problem.

The subsequent local analysis involves the linear $\mathcal{H}_\infty$-control problem for the system

$$\begin{aligned} \dot{x} &= A_1x + B_1w + B_2u \\ z &= C_1x + D_{12}u \\ y &= C_2x + D_{21}w \end{aligned} \tag{7}$$

where

$$A_1 = \frac{\partial f_1}{\partial x}(0),\ B_1 = g_1(0),\ B_2 = g_2(0),$$

$$C_1 = \frac{\partial h_1}{\partial x}(0),\ C_2 = \frac{\partial h_2}{\partial x}(0),\ D_{12} = k_{12}(0),\ D_{21} = k_{21}(0). \quad (8)$$

Such a problem is now well-understood if the linear system (7) is stabilizable and detectable from u and y, respectively. Under these assumptions, the following conditions are necessary and sufficient for a solution of the problem to exist (see, e.g., Doyle *et al.* [6]):

C1) there exists a bounded positive semidefinite symmetric solution of the equation

$$PA_1 + A_1^T P + C_1^T C_1 + P[\frac{1}{\gamma^2}B_1B_1^T - B_2B_2^T]P = 0 \quad (9)$$

such that the system

$$\dot{x} = [A_1 - (B_2B_2^T - \gamma^{-2}B_1B_1^T)P]x(t) \quad (10)$$

is exponentially stable;

C2) there exists a bounded positive semidefinite symmetric solution to the equation

$$AZ + ZA^T + B_1B_1^T + Z[\frac{1}{\gamma^2}PB_2B_2^TP - C_2^TC_2]Z = 0 \quad (11)$$

such that the system

$$\dot{x} = [A - Z(C_2^TC_2 - \gamma^{-2}PB_2B_2^TP)]x(t) \quad (12)$$

is exponentially stable and $A = A_1 + \frac{1}{\gamma^2}B_1B_1^TP$.

According to the bounded real lemma, conditions C1) and C2) ensure that there exists a positive constant ε_0 such that the system of the perturbed Riccati equations

$$P_\varepsilon A_1 + A_1^T P_\varepsilon + C_1^T C_1 + P_\varepsilon[\frac{1}{\gamma^2}B_1B_1^T - B_2B_2^T]P_\varepsilon + \varepsilon I = 0, \quad (13)$$

$$A_\varepsilon Z_\varepsilon + Z_\varepsilon A_\varepsilon^T + B_1B_1^T + Z_\varepsilon[\frac{1}{\gamma^2}P_\varepsilon B_2B_2^TP_\varepsilon - C_2^TC_2]Z_\varepsilon + \varepsilon I = 0 \quad (14)$$

has a unique bounded, positive definite symmetric solution $(P_\varepsilon, Z_\varepsilon)$ for each $\varepsilon \in (0, \varepsilon_0)$ where $A_\varepsilon = A_1 + \frac{1}{\gamma^2}B_1B_1^TP_\varepsilon$.

In what follows, equations (13) and (14) are utilized to derive a local solution of the nonlinear $\mathcal{H}_\infty$-control problem for (1).

Theorem 1. *Let conditions C1) and C2) be satisfied and let $(P_\varepsilon,\ Z_\varepsilon)$ be the bounded positive definite solution of (13), (14) under some $\varepsilon > 0$. Then hypotheses H1) and H2) hold locally around the equilibrium $(x, \xi) = (0, 0)$ with*

$$V(x) = x^TP_\varepsilon x, \quad (15)$$

$$W(x, \xi) = \gamma^2(x - \xi)^TZ_\varepsilon^{-1}(x - \xi), \quad (16)$$

$$F(x) = \frac{\varepsilon}{2}\|x\|^2, \quad (17)$$

$$G = Z_\varepsilon C_2^T, \quad (18)$$

$$Q(x, \xi) = \frac{\varepsilon}{2}\gamma^2\|Z_\varepsilon^{-1}\|^2\|x - \xi\|^2, \quad (19)$$

and the output feedback

$$\dot{\xi} = f_1(\xi) + f_2(\xi) + \left[\frac{1}{\gamma^2}g_1(\xi)g_1^T(\xi) - g_2(\xi)g_2^T(\xi)\right]P_\varepsilon\xi + Z_\varepsilon C_2^T[y - h_2(\xi)], \quad (20)$$

$$u = -g_2^T(\xi)P_\varepsilon\xi \quad (21)$$

is a local solution of the $\mathcal{H}_\infty$-control problem.

Proof of Theorem 1 in the smooth case can be found in [8]. The general proof is nearly the same and it is therefore ommited.

Thus, conditions C1) and C2) ensure the existence of a local solution of the nonlinear $\mathcal{H}_\infty$-control problem. Under an appropriate assumption these conditions become not simply sufficient but also necessary for such a solution to exist. Indeed, let there exist a local solution of the nonlinear $\mathcal{H}_\infty$-control problem and let its linear approximation exponentially stabilize the disturbance-free version of the linearized system (7). Then following Proposition 6 in [10] one can observe that the above assumptions guarantee solvability of the linear $\mathcal{H}_\infty$-control problem for the linearized system (7), thereby proving that (7) is stabilizable and detectable (from u and y, respectively) and hence (see Doyle *et al.* [6]) conditions C1) and C2) are satisfied.

3. $\mathcal{H}_\infty$-Position Control of Mechanical Manipulators with Friction

3.1. Problem Statement

Theoretical results of the previous section are now applied to a position control problem for friction mechanical manipulators, whose links are joined together with revolute joints. A mathematical model for such a manipulator is given by

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + F(\dot{q}) = \tau + w_1 \quad (22)$$

where $q \in \mathbb{R}^n$ is a position, $\tau \in \mathbb{R}^n$ is a control input, $w_1 \in \mathbb{R}^n$ is an external disturbance, $F(\dot{q})$, $G(q)$ $M(q)$, and $C(q, \dot{q})$ are matrix functions of appropriate dimensions. From the physical point of view, q is the vector of generalized coordinates, τ is the vector of external torques, $M(q)$ is the inertia matrix, symmetric and positive definite for all $q \in \mathbb{R}^n$, $C(q, \dot{q})\dot{q}$ is the vector of Coriolis and centrifugal torques, $G(q)$ is the vector of gravitational torques, the components $F_i(\dot{q}_i)$, $i = 1, \ldots, n$ of $F(\dot{q})$ are the friction forces acting independently in each joint and they are represented as a combination

$$F_i = f_{vi}\dot{q}_i + F_{di},\ i = 1, \ldots, n \quad (23)$$

of viscous friction $f_{vi}\dot{q}_i$ and the Dahl friction F_{di} governed by the following dynamic model (cf. that of Dahl [5]):

$$\dot{F}_{di} = \sigma_{1i}\dot{q}_i - \sigma_{1i}|\dot{q}_i|\frac{F_{di}}{F_{Ci}} + w_{2i} \quad (24)$$

where $\sigma_{0i} > 0$, $\sigma_{1i} > 0$, and $F_{Ci} > 0$ are the viscous friction coefficient, the stiffness, and the Coulomb friction level, respectively, corresponding to the i-th manipulator joint; w_{2i} is an external disturbance which is involved to account for inadequacies of the friction modeling. Clearly, the above component-wise relations can be rewritten in the vector form

$$F = f_v\dot{q} + F_d \quad (25)$$

$$\dot{F}_d = \sigma_1\dot{q} - \sigma_1 diag\{|\dot{q}_i|\}F_C^{-1}F_d + w_2 \quad (26)$$

where $F = col\{F_i\}$, $F_d = col\{F_{di}\}$, $q = col\{q_i\}$ $f_v = diag\{f_{vi}\}$, $\sigma_1 = diag\{\sigma_{1i}\}$, $F_C = diag\{F_{ci}\}$, $w_2 = col\{w_{2i}\}$, the notations *diag* and *col* are used to denote a diagonal matrix and a column vector, respectively.

Let $q_d = col\{q_{di}\}$ be the desired position. Then if there were no initial and external disturbances the desired position would be an equilibrium point of the closed loop system driven by $\tau = G(q_d)$. Our objective is to design a controller of the form

$$\tau = G(q_d) + u \quad (27)$$

that imposes on the disturbance-free manipulator motion desired stability properties around q_d while also locally attenuating the effect of the disturbances. Thus, the controller to be constructed consists of a gravity compensator and a disturbance attenuator u, internally stabilizing the closed-loop system around the desired position.

For certainty, we confine our investigation to the position regulation control problem where

1. the output to be controlled is given by

$$z = \rho\begin{bmatrix} 0 \\ q - q_d \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \quad (28)$$

with a positive weight coefficient ρ, and

2. the position measurements

$$y = q + w_0, \quad (29)$$

corrupted by the error vector $w_0(t) \in \mathbb{R}^n$, are only available.

The extension to a general case is straightforward.

The $\mathcal{H}_\infty$ position control problem for robot manipulators with friction can formally be stated as follows. Given a mechanical system (22)-(29), a desired position q_d, and a real number $\gamma > 0$, it is required to find (if any) a causal dynamic feedback controller (2) with internal state $\xi \in \mathbb{R}^s$ such that the undisturbed closed-loop system is uniformly asymptotically stable around the desired position and its $\mathcal{L}_2$-gain is locally less than γ, i.e., inequality (3) is satisfied for all $T > 0$ and all piecewise continuous functions $w = ((w_0, w_1, w_2)^T$ for which the state trajectory of the closed-loop system starting from the initial point $(q(0), \dot{q}(0), F_d(0), \xi(0)) = (q_d, 0, 0, 0)$ remains in some neighborhood of this point.

3.2. $\mathcal{H}_\infty$ Synthesis

To begin with, let us introduce the state deviation vector $x = (x_1, x_2, x_3)^T$ where $x_1 = q_d - q$ is the position deviation from the desired position q_d, $x_2 = \dot{q}$ is the velocity, and $x_3 = F_d$ is the Dahl friction. After that let us rewrite the state equations (22)-(29) in terms of the state vector x:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -M^{-1}(q_d - x_1)[C(q_d - x_1, x_2)x_2 \\ &\quad + G(q_d - x_1) - G(q_d) + f_v x_2 + x_3 - u - w_1] \\ \dot{x}_3 &= \sigma_1 x_2 - \sigma_1 diag\{|x_{2i}|\}F_C^{-1}x_3 - w_2 \end{aligned} \quad (30)$$

Clearly, the $\mathcal{H}_\infty$ regulation problem in question is nothing else than the earlied studied nonlinear $\mathcal{H}_\infty$ control problem for the nonsmooth system (1) specified as follows:

$$f_1(x) = \begin{bmatrix} x_2 \\ -M^{-1}(q_d - x_1)[C(q_d - x_1, x_2)x_2 - G(q_d - x_1)] \\ \sigma_1 x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -M^{-1}(q_d - x_1)[G(q_d) + f_v x_2 + x_3] \\ 0 \end{bmatrix},$$

$$f_2(x) = \begin{bmatrix} 0 \\ 0 \\ -\sigma_1 diag\{|x_{2i}|\}F_C^{-1}x_3 \end{bmatrix}, \quad (31)$$

$$\begin{aligned} g_1(x) &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & -M^{-1}(q_d - x_1) & 0 \\ 0 & 0 & -1 \end{bmatrix}, \\ g_2(x) &= \begin{bmatrix} 0 \\ -M^{-1}(q_d - x_1) \\ 0 \end{bmatrix}, \\ h_1(x) &= \rho\begin{bmatrix} 0 \\ x_1 \end{bmatrix}, \; h_2(x) = x_1, \\ k_{12}(x) &= \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \; k_{21}(x) = [\ 1\ 0\ 0\ ] \end{aligned} \quad (32)$$

where by virtue of the well-known inequality

$$2|a|b \le a^2 + b^2, \; a, b \in \mathbb{R}$$

the function $f_2(x)$ satisfies (6) for $\sigma = 0.5\max_i \sigma_{1i}F_{ci}^{-1}$ and all $x \in \mathbb{R}^n$. Now by applying Theorem 1 to system (1) thus specified, we derive a local solution of the $\mathcal{H}_\infty$ regulation control problem.

Theorem 2. *Let conditions C1) and C2) hold for the matrix functions* A, B_1, B_2, C_1, C_2*, governed by (8),(31), (32), and let* $(P_\varepsilon, Z_\varepsilon)$ *be the corresponding positive definite solution of (13), (14) under some* $\varepsilon > 0$*. Then the output feedback*

$$\dot{\xi} = f_1(\xi) + f_2(\xi) + [\frac{1}{\gamma^2} g_1(\xi) g_1^T(\xi) - g_2(\xi) g_2^T(\xi)] P_\varepsilon \xi$$

$$+ Z_\varepsilon C_2^T(t)[y(t) - h_2(\xi)], \quad (33)$$

$$u = -g_2^T(\xi) P_\varepsilon \xi \quad (34)$$

subject to (31), (32) is a local solution of the $\mathcal{H}_\infty$*-position regulation problem.*

Proof. Since by inspection Assumptions A1)-A5) hold for the system (30), which represents the manipulator equations (22)-(29) in terms of the state deviation with respect to the desired position q_d, the validity of Theorem 2 is concluded by applying Theorem 1. ∎

3.3. Simulation Results

The controller performance was studied by simulation made for a two-degrees-of-freedom robot manipulator moving in the vertical plane. The dynamics of the manipulator satisfies (22) and it is affected by viscous and Dahl frictions (23), (24) where the model parameters

$$M(q) = \begin{bmatrix} 8.77 + 1.02\cos(q_2) & 0.76 + 0.51\cos(q_2) \\ 0.76 + 0.51\cos(q_2) & 0.62 \end{bmatrix},$$

$$C(q, \dot{q}) = \begin{bmatrix} -0.51\sin(q_2)\dot{q}_2 & -0.51\sin(q_2)(\dot{q}_1 + \dot{q}_2) \\ 0.51\sin(q_2)\dot{q}_1 & 0 \end{bmatrix},$$

$$G(q) = \begin{bmatrix} 74.48\sin(q_1) + 6.174\sin(q_1 + q_2) \\ 6.174\sin(q_1 + q_2) \end{bmatrix},$$

$$f_v = \begin{bmatrix} 1 & 0 \\ 0 & 1.2 \end{bmatrix}, \sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, F_C = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$$

are drawn from [2] (cf. also that of Berghuis and Nijmeijer [3] for the friction-free robot manipulator model).

In the simulation, performed with MATLAB, the manipulator was required to move from the origin $q^T(0) = [0, 0]$ to the desired endpoint $q_d^T = [0.1, 0.3]$(rad). The desired endpoint has been prespecified close enough to the origin to ensure that the initial state deviation $x_i(0) = q_{di} - q_i(0) = q_{di},\ i = 1, 2$ is within the attraction domain of the $\mathcal{H}_\infty$-regulator to be constructed. The initial velocity $\dot{q}(0)$, Dahl friction force $F_d(0)$, and compensator state $\xi(0)$ were equivalent to zero for all simulations.

The control goal was achieved by simulating the $\mathcal{H}_\infty$-regulator (20), (21) and (27) with the weight parameter $\rho = 1.45$. By iterating on γ we found the infimal achievable level $\gamma^* \simeq 100$. However, in the subsequel simulation $\gamma = 350$ was selected to avoid an undesirable high-gain controller design that would appear for a value of γ close to the optimum. With $\gamma = 350$ we obtained that for $\varepsilon = 0.8$ the corresponding Riccati equations (13), (14) have the following positive definite solutions:

$$P_\varepsilon =$$

$$\begin{bmatrix} 6.129 & 0.0 & & 0.743 & 0.0 & 0.0 \\ 0.0 & 9.93 & 2.977 & 0.0 & 1.952 & 0.758 \\ 0.0 & 2.977 & 1.466 & 0.0 & 0.795 & 0.352 \\ 0.743 & 0.0 & 0.0 & 0.182 & 0.0 & 0.0 \\ 0.0 & 1.952 & 0.795 & 0.0 & 0.495 & 0.203 \\ 0.0 & 0.758 & 0.352 & 0.0 & 0.203 & 0.088 \end{bmatrix},$$

$$Z_\varepsilon =$$

$$\begin{bmatrix} 40.55 & 0.0 & 0.0 & 139.69 & 0.0 & 0.0 \\ 0.0 & 890.83 & 234.12 & 0.0 & 313.53 & 92.61 \\ 0.0 & 234.12 & 152.61 & 0.0 & 51.22 & 95.02 \\ 139.69 & 0.0 & 0.0 & 504.28 & 0.0 & 0.0 \\ 0.0 & 313.53 & 51.22 & 0.0 & 132.20 & -13.57 \\ 0.0 & 92.61 & 95.02 & 0.0 & -13.57 & 117.59 \end{bmatrix}.$$

By Theorem 1, these solutions result in the control law (20)-(21) solving the regulation problem. We simulated two cases of the $\mathcal{H}_\infty$-regulator thus constructed, without disturbance and with both permanent disturbances $w_{ij} = 0.1,\ i = 1, 2,\ j = 0, 1, 2$ and 5% parameter variations. The resulting trajectories are depicted in Figure 1. This figure demonstrates that the regulator stabilizes the disturbance-free system motion around the desired position and attenuates permanent external disturbances as well as parametric uncertainties.

4. Conclusions

A Nonlinear $\mathcal{H}_\infty$-synthesis is developed for a class of nonsmooth control systems. In contrast to the standard techniques (cf. that of Basar and Bernhard, [1]; Isidori and Astolfi [7]; Van der Schaft, [10]), the corresponding Hamilton-Jacobi-Isaacs expressions are now required to be negative definite rather than semidefinite. This feature has allowed us to develop an $\mathcal{H}_\infty$-design procedure with no apriori-imposed stabilizability-detectability conditions on the control system (note that in the nonlinear case, verification of these conditions becomes a formiddable problem in itself). Although the design procedure results in an infinite-dimensional problem we have circumvented this difficulty by solving the problem locally. Implementation of the controller thus constructed remains of the same level of simplicity as that in the linear case.

The afore-mentioned design procedure has been shown to be eminently suited to solving position regulation problem for friction mechanical systems. To facilitate exposition, the friction model chosen for treatment has been confined to the Dahl model augmented with viscous friction. Effectiveness of the design procedure has been supported by the simulations made for a two degrees-of-freedom robot manipulator.

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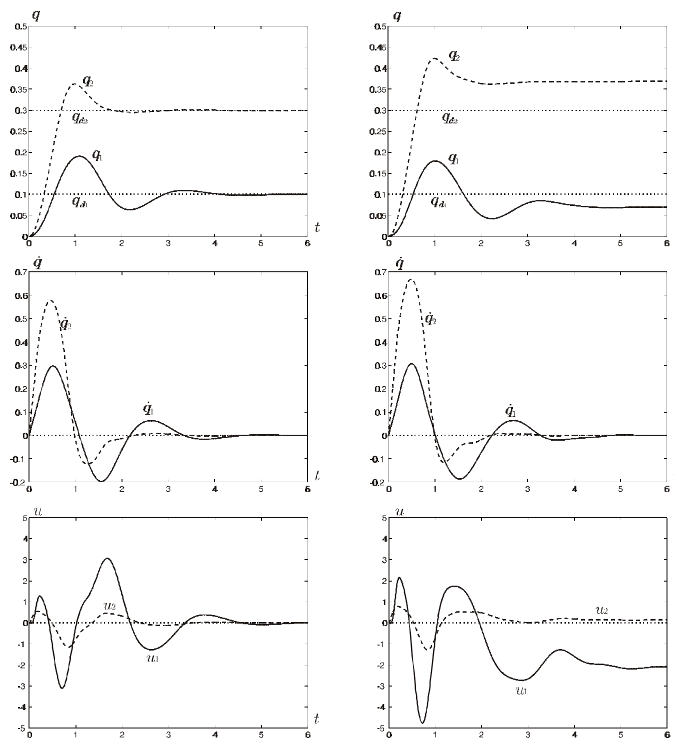

Figure 1: Robustness of the $\mathcal{H}_\infty$-regulator against external disturbances and parametric variations: left column for the no disturbance case, right column for both the permanent disturbance and parametric variations.