New Structural Results on Fault Detection and Identification.

B. del-Muro-Cuéllar[♮], E. Orduña-Reyes[♮◊] and J.C. Martínez García.
[♮] Programa de Investigación en Matemáticas Aplicadas y Computación
Instituto Mexicano del Petróleo
Eje Central Lázaro Cárdenas 152
Col. San Bartolo Atepehuacan, 07730 México D.F. México.
Tel.:+52 55 30037571; Fax: +52 55 3003 6277
e-mail: *bdelmuro@imp.mx; eorduna@imp.mx; martinez@ctrl.cinvestav.mx*

[◊]PHD student at Depto. de Control Automático, CINVESTAV-IPN, México

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Abstract

The aim of this paper is to present structural solutions to the so-called Fault Detection and Isolation Beard and Jones Filter Problem FDIBJP (also called Restricted Diagonal Detection Filter Problem). We give necessary and sufficient conditions for the existence of a solution which insures the existence of a fault detection and identification filter. The structural results are obtained as a direct dualization of the results corresponding to input-output decoupling problem.

1. Introduction

To prevent industrial processes against catastrophic failures with possible consequences for the man and the environment, it is fundamental to implement fault tolerant control systems. This is the main reason why monitoring is always present in modern technological systems. Failures in actuators, sensors, and components, are normal sources of risk in technological systems, and because of this, fault detection and isolation systems are common in monitoring processes. One approach to achieve fault tolerant control systems consist in first detecting and identifying the failure. Model-based Failure Detection and Identification constitutes now an important field of research. In this paper we are interested in model-based fault detection and isolation techniques. We tackle here the deterministic linear time-invariant case.

Many results have been obtained concerning the residual generation. See for instance the survey paper by Willsky [7].Using the Geometric Approach (see [8] and [1]), Massoumia [4], uses the so-called Beard and Jones filter to give geometric conditions to solve the problem of residual generation. In [5], Massoumia states a variation of the Beard and Jones filter as a dual version of the well known Disturbance Decoupling Problem as sated in [8] and solved in [6]. In this paper we will serve us of this dualization to obtain some new results for the Fault Detection problem directly obtained from structural results concerning the Disturbance Decoupling problem (see [3]).

The paper is organized as follows:

In Section 2 we introduce the notation and we recall the definition of some useful invariant subspaces implied in input-output decoupling and fault detection and isolation. We also recall the notion of infinite zero structure of linear time-invariant systems. Section 3 is dedicated to the definition of the input-output decoupling problem. We recall the geometric solvability condition in order to remark the dual nature of both problems. We also recall the structural solvability conditions. Section 4 concerns the formulation of the fault detection and isolation problem and in Section 5 we state our main results. Finally, we present some concluding remarks.

2. Preliminaries

2.1. Some useful subspaces

Consider a linear time-invariant system (A, B, C) described by:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t), \end{cases} \tag{2.1}$$

where: $x(t) \in \mathcal{X} \simeq \mathbb{R}^n$ denotes the state; $u(t) \in \mathcal{U} \simeq \mathbb{R}^m$ denotes the input, and $y(t) \in \mathcal{Y} \simeq \mathbb{R}^p$ denotes the output. It is considered here that $A : \mathcal{X} \to \mathcal{X}$, $B : \mathcal{U} \to \mathcal{X}$, and $C : \mathcal{X} \to \mathcal{Y}$, are linear maps represented by real constant matrices.

Consider a subspace $\mathcal{K} \subseteq \mathcal{X}$ be given. The notation:

- $\mathcal{V}_{(\mathcal{K})}$ indicates that the subspace $\mathcal{V}_{(\mathcal{K})}$ is $(A,\ \mathcal{K})$-invariant,i.e., $A\mathcal{V}_{(\mathcal{K})} \subseteq \mathcal{V}_{(\mathcal{K})} + \mathcal{K}$. We note $\mathcal{F}\left(\mathcal{V}_{(\mathcal{K})}\right)$ the family of maps $F : \mathcal{X} \to \mathcal{U}$ (state feedbacks) such that $(A + BF)\,\mathcal{V}_{(\mathcal{K})} \subseteq \mathcal{V}_{(\mathcal{K})}$.

- $\mathcal{S}_{(\mathcal{K})}$ indicates that the subspace $\mathcal{S}_{(\mathcal{K})}$is $(\mathcal{K},\ A)$-invariant,i.e., $A\left(\mathcal{K} \cap \mathcal{S}_{(\mathcal{K})}\right) \subseteq \mathcal{S}_{(\mathcal{K})}$. We note $\mathcal{D}\left(\mathcal{S}_{(\mathcal{K})}\right)$ the family of maps $D : \mathcal{Y} \to \mathcal{X}$ (output injections) such that $(A + DC)\,\mathcal{S}_{(\mathcal{K})} \subseteq \mathcal{S}_{(\mathcal{K})}$.

If there exists a couple of maps $F : \mathcal{X} \to \mathcal{U}$ and $G : \mathcal{U} \to \mathcal{U}$ such that the subspace $\mathcal{R}_{(\operatorname{Im} B)}$ satisfies $\mathcal{R}_{(\operatorname{Im} B)} = \langle A + BF \mid \operatorname{Im}(BG)\rangle := \operatorname{Im}(BG) + (A + BF)\operatorname{Im}(BG) + \cdots + (A + BF)^{n-1}\operatorname{Im}(BG)$,we say that $\mathcal{R}_{(\operatorname{Im} B)}$ is an $(A, \operatorname{Im} B)$ controllability subspace.

If there exists a couple of maps $D : \mathcal{Y} \to \mathcal{X}$ and $H : \mathcal{Y} \to \mathcal{Y}$ such that the subspace $\mathcal{S}_{(\operatorname{Ker} C)}$ satisfies $\mathcal{S}_{(\operatorname{Ker} C)} = \langle \operatorname{Ker}(HC) \mid A + DC\rangle := \operatorname{Ker}(HC) \cap (A + DC)^{-1}\operatorname{Ker}(HC) \cap \cdots \cap (A + DC)^{-n+1}\operatorname{Ker}(HC)$, we say that $\mathcal{S}_{(\operatorname{Ker} C)}$ is $(\ker C, A)$-complementary observability (or simply $(\ker C, A)$-unobservability) subspace.

Given two subspaces $\mathcal{K}$ and $\mathcal{L} \subseteq \mathcal{X}$, we shall note (see [8, Wonham, 1985] and [1, Basile and Marro, 1992]):

- $\mathcal{V}^*_{(\mathcal{K},\mathcal{L})}$:=the supremal $(A, \mathcal{K})$-invariant ($\mathcal{V}_{(\mathcal{K})}$)contained in $\mathcal{L}$;

- $\mathcal{S}^*_{(\mathcal{L},\mathcal{K})}$:=the infimal $(\mathcal{L}, A)$-invariant subspace ($\mathcal{S}_{(\mathcal{L})}$) containing $\mathcal{K}$;

- $\mathcal{R}^*_{(\mathcal{K},\mathcal{L})}$:= the supremal $(A, \mathcal{K})$-controllability subspace ($\mathcal{R}_{(\mathcal{K})}$) contained in $\mathcal{L}$ ($\mathcal{R}^*_{(\mathcal{K},\mathcal{L})} = \mathcal{V}^*_{(\mathcal{K},\mathcal{L})} \cap \mathcal{S}^*_{(\mathcal{L},\mathcal{K})}$);

- $\mathcal{N}^*_{(\mathcal{L},\mathcal{K})}$:=the infimal $(\mathcal{L}, A)$-un ob ser - va bi li ty subspace $\mathcal{N}_{(\mathcal{L})}$ containing $\mathcal{K}$ ($\mathcal{N}^*_{(\mathcal{L},\mathcal{K})} = \mathcal{V}^*_{(\mathcal{K},\mathcal{L})} + \mathcal{S}^*_{(\mathcal{L},\mathcal{K})}$).

2.2. Infinite Zero Structure

Consider a linear time-invariant system (A, B, C) described by 2.1 or equivalently by its strictly proper pxm transfer function matrix $T(s) = C\left(sI_n - A\right)^{-1}B$. The structure at infinity of the system is described by the multiplicity orders of its zeros at infinity, say $\{n_1, n_2, ..n_r\}$, where $r = rank[T(s)]$.This structure can be derived from the so called Smith-McMillan form at infinity of $T(s)$, noted T_∞ and defined as follows: there exist bipropes matrices $B_1(s)$ and $B(s)$ such that:

$$B_1(s)T(s)B_2(s) = T_\infty = \begin{bmatrix} \Delta_\infty & 0 \\ 0 & 0 \end{bmatrix}$$

ou $\Delta_\infty = diag\{s^{-n_1}, s^{-n_2}, ..., s^{-n_r}\}$. We shall use the notion of content at infinity of a given system (A, B, C), noted $C_\infty(A, B, C)$, which is simply the sum of the orders of its zeros at infinity. From the geometric characterization of the integers n_i (see [2]), it can easily be shown that:

$$C_\infty(A, B, C) := \Sigma_{i=1}^{r} n_i$$

$$= \dim(\mathcal{N}^*_{(\ker C, \mathrm{Im}\, B)} / \mathcal{V}^*_{(\mathrm{Im}\, B, \ker C)})$$

3. Input-Output Decoupling (IOD) problem.

Let us note $\overline{\ker C_i} = \cap_{j \neq i} \ker C_j$,with C_i the ith row of C and $T_i(s)$ the ith row of $T(s) = C\left(sI_n - A\right)^{-1}B$.

Definition 3.1 (IOD). *Consider a given system (2.1) square, invertible and (A, B) controllable. Find a regular static state feedback:*

$$u(t) = Fx(t) + Gv(t),$$

with $F : \mathcal{X} \to \mathcal{U}$ and $G : \mathcal{U} \to \mathcal{U}$ (G invertible), such that:

$$T_{(F,G)}(s) := C\left(sI_n - (A + BF)\right)^{-1}BG$$

$$= \begin{bmatrix} \epsilon_1(s) & 0 & \cdots & 0 \\ 0 & \epsilon_2(s) & \cdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \epsilon_p(s) \end{bmatrix},$$

where $\epsilon_i(s) := C_i\left(sI_n - (A + BF)\right)^{-1}BG$, for $i \in \{1, 2, \ldots, p\}$ ($C = \begin{bmatrix} C_1^\top & C_2^\top & \cdots & C_p^\top \end{bmatrix}^\top$), is a strictly proper transfer function.

Remark 1. *This problem is also known as the Restricted Decoupling Problem (see [8], Section 9.2). It is called restricted because only state feedback is used, with no augmentation of system dynamic order.*

Remark 2. *IOD is established in geometric terms as follows (see [6, Wonham and Morse, 1970]): Find a family of controllability subspaces $\left\{\mathcal{R}_{(\mathrm{Im}\,B)_1}, \mathcal{R}_{(\mathrm{Im}\,B)_2}, \ldots, \mathcal{R}_{(\mathrm{Im}\,B)_p}\right\}$ satisfying:*

$$\mathcal{R}_{(\mathrm{Im}\,B)_i} \;:\; = \langle A + BF \mid \mathrm{Im}\,(BG_i)\rangle \quad (3.1)$$

$$\text{for } i \in \{1, 2, \ldots, p\}, \quad (3.2)$$

for a couple of maps $F : \mathcal{X} \to \mathcal{U}$ and $G := \begin{bmatrix} G_1 & G_2 & \cdots & G_p \end{bmatrix}$ invertible, and such that:

$$\mathcal{R}_{(\mathrm{Im}\,B)_i} \subseteq KerC_j, \forall j \neq i. \quad (3.3)$$

$$C_i\mathcal{R}_{(\mathrm{Im}\,B)_i} = \mathcal{Y}_i, \quad (3.4)$$

$$\text{for } i \in \{1, 2, \ldots, p\}, \quad (3.5)$$

with $\mathcal{Y}_i$ denoting the i-th output space. Since (A, B) is controllable, (3.4) is equivalent to:

$$KerC_i + \mathcal{R}_{(\mathrm{Im}\,B)_i} = \mathcal{X}. \quad (3.6)$$

The condition (3.1) require the same F for all the subspaces: this is the so-called compatible condition.

The solvability conditions for this problem are well known:

Theorem 3.2. *The IOD problem as a solution iff any of the following equivalent statements hold:*

1. *[6, Wonham and Morse, 1970]*

$$\operatorname{Im} B = \sum_{i=1}^{p} \operatorname{Im} B \cap \mathcal{R}^*_{\left(\operatorname{Im} B, \overline{\ker e_i}\right)} \qquad (3.7)$$

   (in this condition, the system (2.1) can be only right invertible).

2. *[3] The orders of the infinite zeros of $T(s)$ are respectively equal to the orders of infinite zeros of $T_i(s), i \in p$*

4. Fault Detection and Identification: problem formulation.

Consider a continuous linear time-invariant system which includes an actuator failures model $(A,\ B,\ C,\ \left[\begin{array}{cccc} L_1 & L_2 & \cdots & L_l \end{array}\right])$ described by:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + \sum_{i=1}^{l} L_i m_i(t), \\ y(t) = Cx(t), \end{cases} \qquad (4.1)$$

where: $m_i(t) \in \mathcal{M}_i \simeq \mathbb{R}$ denotes the i-th actuator failure mode and $L_i : \mathcal{M}_i \to \mathcal{X}$ denotes the i-th actuator failure signature. We shall note $\overline{\operatorname{Im} L_i} := \Sigma_{j \neq i} \operatorname{Im} L_j$.

Remark 3. *The unknown i-th actuator failure mode $m_i(t)$ has the following property:*

$$m_i(t) \neq 0$$

when the i-th actuator is failing, if it is not the case $m_i(t) = 0$.

Let system (4.1) be given and consider a full-order observer (residual generator) of the form:

$$\begin{cases} \dot{w}(t) = (A + DC) w(t) - Dy(t) + Bu(t), \\ r_i(t) = H_i [Cw(t) - y(t)], \quad i \in \{1, 2, \ldots, l\}, \end{cases} \qquad (4.2)$$

where: $r_i(t)$, for $i \in \{1, 2, \ldots, l\}$, are the residuals; D corresponds to the observer gain, i.e., $D : \mathcal{Y} \to \mathcal{X}$ is an output injection map, and $H_i : \mathcal{Y} \to \mathcal{Y}$, for $i \in \{1, 2, \ldots, l\}$, are the measurement mixing maps.

Let $e(t) := w(t) - x(t)$ be the vector error. Using (4.1) and (4.2), we have:

$$\begin{cases} \dot{e}(t) = (A + DC) e(t) - \sum_{i=1}^{l} L_i m_i(t), \\ r_i(t) = H_i C e(t), \ i \in \{1, 2, \ldots, l\}. \end{cases} \qquad (4.3)$$

Definition 4.1 (FDIBJP). *The residual generator must be such that a nonzero $m_i(t)$ should only have a nonzero effect on $r_i(t)$ and none of the other residuals $r_j(t)$, $j \neq i$. More precisely we would like the system relating $m_i(t)$ to $r_i(t)$, i.e., $(A + DC, L_i, H_i C)$, to be input observable. Since the $m_i(t)$ are scalars, this corresponds to the left invertibility of the transfer matrix relating $m_i(t)$ to $r_i(t)$, and hence any failure mode will show up in the corresponding residual.*

Remark 4. *In geometric terms the Exact Fault Detection and Isolation Beard and Jones Filter Problem FDIBJP, (called Restricted Diagonal Detection Filter Problem in [5]) is then defined as follows: Find an output injection map $D : \mathcal{Y} \to \mathcal{X}$ and a family of compatible unobservability subspaces $\mathcal{N}_{(\ker H_i C)}$ (i.e. $\mathcal{N}_{(\ker H_i C)} := \langle \ker H_i C | A + DC \rangle$), such that the following conditions hold:*

$$\mathcal{N}_{(\ker H_i C)} = \left\langle \ker C + \mathcal{N}_{(\ker H_i C)} | A + DC \right\rangle \qquad (4.4)$$

$$i \in \{1, 2, \ldots, l\} \qquad (4.5)$$

$$\overline{\operatorname{Im} L_i} \subset \mathcal{N}_{(\ker H_i C, A + DC)}, \qquad (4.6)$$

$$i \in \{1, 2, \ldots, l\}$$

$$\operatorname{Im} L_i \cap \mathcal{N}_{(\ker H_i C)} = 0, \qquad (4.7)$$

$$i \in \{1, 2, \ldots, l\}. \qquad (4.8)$$

Remark 5. *Condition (4.4) ask for a compatible set of unobservability subspaces. Condition (4.6) means that nonzero* $m_i(t)$ *should not affect* $r_j(t)$ $(j \neq i)$*, and condition (4.7) ask for the system relating* $m_i(t)$ *to* $r_i(t)$ *to be input observable, i.e, left invertible (since the failure modes are scalars).*

Remark 6. *In input-output terms the maps* $D : \mathcal{Y} \to \mathcal{X}$*,* $H_i : \mathcal{Y} \to \mathcal{Y}$*, for* $i \in \{1, 2, \dots, l\})$ *solve FDIBJP if we have:*

$$\begin{bmatrix} r_1(t) \\ r_2(t) \\ \vdots \\ r_l(t) \end{bmatrix} = T_{mr}(s) \begin{bmatrix} m_1(t) \\ m_2(t) \\ \vdots \\ m_l(t) \end{bmatrix} \tag{4.9}$$

with:

$$T_{mr}(s) \ := \ \operatorname{diag}\left\{ T_{mr_1}, \ T_{mr_2}, \ \dots, \ T_{mr_l} \right\}$$

$$= \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_k \end{bmatrix}^{\top} (sI_n - (A + DC))^{-1} \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_k \end{bmatrix}^{\top}$$

In order to achieve a practical solution, the following condition must hold [5, Massoumnia, 1986]:

$$\ker HC = \ker C \tag{4.10}$$

Then the geometrical solution to FDIBJP in is given by the following theorem:

Theorem 4.2. *[5, Massoumnia, 1986] Consider a system (A, B, C,* $\begin{bmatrix} L_1 & L_2 & \cdots & L_l \end{bmatrix}$*) be given. Assume that (C, A) is observable and that* $e(0) = 0$*. FDIBJP restricted to (4.10) has a solution if and only if:*

$$KerC = \cap_{i=1}^{l} \left(\mathcal{N}^{*}_{(\ker C, \overline{\operatorname{Im} L_i})} + KerC \right). \tag{4.11}$$

5. New results on fault detection and identification obtained by dualization.

It is quite obvious that the IOD problem is a dualization of the FDIBJP problem. Indeed, the geometric conditions (4.4), (4.6), and (4.7), can be obtained from the dualization of the geometric conditions (3.1), (3.3), and (3.6), respectively. The solvability condition (4.11) is a direct dualization of (3.7). Then, from a direct dualization of Theorem 5.2, items 1 and 2 we can obtain the following result:

Lemma 5.1. *The FDIBJP problem has as a solution iff the orders of the infinite zeros of the transfer function matrix* $\begin{bmatrix} L_1' & L_2' & \cdots & L_l' \end{bmatrix}'(sI - A')C'$ *are respectively equal to the orders of infinite zeros of each row.*

See the proof of the dual problem in [3].
The above result can also be expressed in the following way:

Theorem 5.2. *The FDIBJP problem has as a solution iff the orders of the infinite zeros of the transfer function matrix* $C(sI - A)\begin{bmatrix} L_1 & L_2 & \cdots & L_l \end{bmatrix}$ *are respectively equal to the orders of infinite zeros of each colon.*

This condition states that for a $(A, \begin{bmatrix} L_1 & L_2 & \cdots & L_l \end{bmatrix}, C)$ system the FDIBJP problem as has a solution iff its infinite zero orders are equal to the orders of the infinite zeros of each subsystem (A, L_i, C).
See the proof of the dual problem in [3].

6. Concluding Remarks.

In this paper we has present a new results relating the infinite zero structure of a system and the FDIBJP problem. The results are stated as a straightforward dualization of the input-output decoupling problem results. This result make evident how the viability of detecting and identificating faults only depends on the infinite zero structure of the system. This fact is not surprising as similar results exist not only for the dual (decoupling) problem, but also for the disturbance rejection problem.

Note that an equivalent result can be obtained for the discrete case were the infinite zero structure corresponds with a time delay structure.

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