A Global Fuzzy Regulator with Bounded Torques for Robot Manipulators *

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Abstract—*In this paper, we take advantage of properties of the so called sectorial fuzzy controllers to cope with the regulation problem of robot manipulators with bounded torques. To this end we propose a control law structure given by a sectorial fuzzy controller plus a term of gravity compensation. This controller deals with physical constraints. When friction is considered, we prove, via Lyapunov theory, that the steady state position errors owing to static friction are inside of a global attractor, which can be arbitrarily reduced. In case of absence of friction, the closed loop system becomes globally asymptotic stable. For both cases, independently of the initial positions, the controller, in a natural way, delivers bounded torques, in agreement with the actuator torque capabilities.*

Index Terms—*Fuzzy control, robot control, stability analysis, friction, saturation.*

1 Introduction

In recent years the use of fuzzy techniques to control the motion of robot manipulators has grown considerably [1]– [4]. This is due, mainly, to some new advances in fuzzy systems stability theory [5] that have formally guaranteed the fulfilment of the motion control aim —global asymptotic tracking or positioning of robot's joints—, united with the excellent performance shown in practice [3]–[4].

On the one hand, passivity properties of a class of the so called sectorial fuzzy controllers has been reported in [6] and [7]. This class of fuzzy controllers has two inputs and one output and it can be characterized from an input–output point of view as a nonlinear static mapping. In [6] was proven that the most of fuzzy control applications use a general class of fuzzy controllers having specific sectorial properties of their input output mapping.

On the other hand, inherent physical constraints, as saturation nonlinearities of actuators and friction phenomena are present in the real dynamics of manipulators, limiting the system performance and stability. In the motion control problem, friction will cause a bounded steady state tracking error [9] and saturation will lead to a lack of stability guarantee. Some efforts, into the conventional control, have been conducted to deal with this subject; in regulation problem [10]–[13] as well as in tracking control problem [14]–[16].

In this paper, we exploit the properties of sectorial fuzzy controllers to face up to regulation problem of robot manipulators holding the torques delivered by actuators inside prescribed capabilities. To this end we propose a control law structure given by a sectorial fuzzy controller plus a term of gravity compensation. When friction is considered, we prove, via Lyapunov theory, that the steady state position errors owing to static friction are inside of a global attractor, which can be arbitrarily reduced. In case of absence of friction, this global attractor becomes the origin of the state space; that means, the closed loop system is now globally asymptotic stable, and hence, the regulation goal is achieved. For both cases, independently of the initial positions, the controller, in a natural way, delivers bounded torques.

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2 Fuzzy Logic Controller

In this paper, two–inputs one–output rules will be used in the formulation of the knowledge base. The IF–THEN rules ($R^{l_1 l_2}$) are of the following form:

$$\text{IF } x_1 \ is \ A_1^{l_1} \text{ AND } x_2 \ is \ A_2^{l_2} \text{ THEN } y \ is \ B^{l_1 l_2}, \quad (1)$$

where $[x_1 \ \ x_2]^T = \boldsymbol{x} \in U = U_1 \times U_2 \subset \mathbb{R}^2$ and $y \in V \subset \mathbb{R}$. For each input fuzzy set $A_j^{l_j}$ in $x_j \subset U_j$ and output fuzzy set $B^{l_1 l_2}$ in $y \subset V$ exists an input membership function $\mu_{A_j^{l_j}}(x_j)$ and output membership function $\mu_{B^{l_1 l_2}}(y)$, respectively, with $l_j = -\frac{N_j-1}{2} \cdots, \frac{N_j-1}{2}$; $j = 1, 2$; N_j being an odd number of membership functions associated to the input j. The total number of rules M is defined by the number of membership functions of each input $M = N_1 N_2$.

The output variable of a fuzzy logic controller (FLC) can have associated an odd number, say N, of membership functions $\mu_{B^l}(y)$, with $l = -\frac{N-1}{2}, \cdots, \frac{N-1}{2}$, which are associated to each consequent $\mu_{B^{l_1 l_2}}(y)$ of the rule base, that is,

$$\mu_{B^{l_1 l_2}}(y) \in \{\mu_{B^{-\frac{N-1}{2}}}(y), \cdots, \mu_{B^{\frac{N-1}{2}}}(y)\}. \quad (2)$$

The particular choice of each $\mu_{B^{l_1 l_2}}(y)$ will depend on the heuristic knowledge of the expert.

In the remainder of this paper we consider the so called Sectorial Fuzzy Controllers (SFC) studied in [6] and [7], where we have selected the following specifications: Singleton fuzzifier; N_j (odd) triangular membership functions for each input, with $j = 1, 2$ (see Figure 1); N (odd) singleton membership functions for the output (see Figure 2); rule base $R^{l_1 l_2}$ defined by (1) for two inputs; product or minimum inference and, center average defuzzifier.

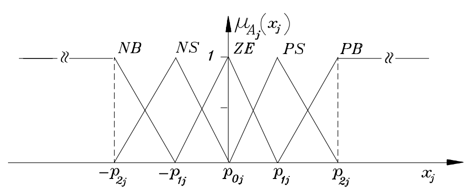

Figure 1: Input membership functions.

For this class of fuzzy controller we will denote the output y of the corresponding input–output mapping, in terms of the two inputs x_1, and x_2, as $y = \phi(x_1, x_2)$, which can be computed as [5] and [6]:

$$y(\boldsymbol{x}) = \phi(x_1, x_2)$$

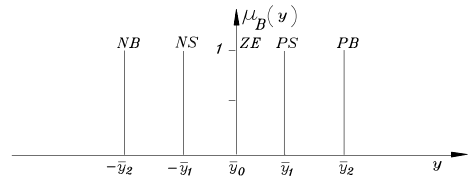

Figure 2: Output membership functions.

$$= \frac{\sum_{l_1=-\frac{N_1-1}{2}}^{\frac{N_1-1}{2}} \sum_{l_2=-\frac{N_2-1}{2}}^{\frac{N_2-1}{2}} \bar{y}^{l_1 l_2} \left(\bigcap_{j=1}^{2} \mu_{A_j^{l_j}}(x_j) \right)}{\sum_{l_1=-\frac{N_1-1}{2}}^{\frac{N_1-1}{2}} \sum_{l_2=-\frac{N_2-1}{2}}^{\frac{N_2-1}{2}} \left(\bigcap_{j=1}^{2} \mu_{A_j^{l_j}}(x_j) \right)} \quad (3)$$

where $\bar{y}^{l_1 l_2}$ is the point in V at which $\mu_{B^{l_1 l_2}}(y)$ achieves its maximum value 1, and $\bigcap$ denotes the intersection operator which can be a product or minimum operator.

Passivity properties of this class of SFC were reported in [6] and [7]. Below we recall and present novel properties of this input–output mapping $\phi(x_1, x_2)$ for $x_1, x_2 \in \mathbb{R}$. The proofs for the properties 1 to 4 are given in [6]. The proofs for the remaining properties are in [8].

Property 1. $\phi(0, 0) = 0$
Property 2. $\phi(x_1, x_2) = -\phi(-x_1, -x_2)$
Property 3. There exist $\lambda, \gamma > 0$ such that

$$0 < x_1[\phi(x_1, x_2) - \phi_i(0, x_2)] \leq \lambda {x_1}^2 \quad \forall \ x_1 \neq 0.$$

$$0 \leq x_2[\phi(x_1, x_2) - \phi(x_1, 0)] \leq \gamma {x_2}^2.$$

Property 4. $\phi(x_1, 0) = 0 \Rightarrow x_1 = 0$.
Property 5. Equation (3) can be simplified to

$$\phi(x_1, x_2) = \sum_{l_1=-\frac{N_1-1}{2}}^{\frac{N_1-1}{2}} \sum_{l_2=-\frac{N_2-1}{2}}^{\frac{N_2-1}{2}} \bar{y}^{l_1 l_2} \left(\prod_{j=1}^{2} \mu_{A_j^{l_j}}(x_j) \right) \quad (4)$$

provided that the product inference method is used. $\prod$ denotes the product operator.
Property 6. $|\phi(x_1, x_2)| \leq \delta := \max_{l_1 l_2} \bar{y}^{l_1 l_2}$.
Property 7. Around $x_1 = 0$,

$$0 \leq |\phi(x_1, 0)| \leq \bar{y}^{1\,0}$$

where $\bar{y}^{1\,0}$ is the point in V at which $\mu_{B^{1\,0}}(y)$ achieves its maximum value 1.

3 Dynamics of robot manipulatores and control problem formulation

The dynamics of a serial n-link robot can be written as [17]:

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{f}(\dot{\boldsymbol{q}},\boldsymbol{\tau}) = \boldsymbol{\tau} \tag{5}$$

where $\boldsymbol{q}$ is the $n \times 1$ vector of joint displacements, $\dot{\boldsymbol{q}}$ is the $n \times 1$ vector of joint velocities, $\boldsymbol{\tau}$ is the $n \times 1$ vector of applied torque inputs, $M(\boldsymbol{q})$ is the $n \times n$ symmetric positive definite manipulator inertia matrix, $C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$ is the $n \times 1$ vector of centripetal and Coriolis torques, $\boldsymbol{g}(\boldsymbol{q})$ is the $n \times 1$ vector of gravitational torques obtained as the gradient of the robot potential energy $\mathcal{U}(\boldsymbol{q})$ due to gravity, i.e.:

$$\boldsymbol{g}(\boldsymbol{q}) = \frac{\partial \mathcal{U}(\boldsymbol{q})}{\partial \boldsymbol{q}}, \tag{6}$$

and $\boldsymbol{f}(\dot{\boldsymbol{q}},\boldsymbol{\tau})$ stands for the $n \times 1$ vector of friction torques. The friction torque $\boldsymbol{f}(\dot{\boldsymbol{q}},\boldsymbol{\tau})$ is decentralized in the sense that $f_i(\dot{\boldsymbol{q}},\boldsymbol{\tau})$ depends only on $\dot{q}_i$ and τ_i, that is,

$$\boldsymbol{f}(\dot{\boldsymbol{q}},\boldsymbol{\tau}) = \begin{bmatrix} f_1(\dot{q}_1,\tau_1) \\ f_2(\dot{q}_2,\tau_2) \\ \vdots \\ f_n(\dot{q}_n,\tau_n) \end{bmatrix}.$$

The friction torque $\boldsymbol{f}(\dot{\boldsymbol{q}},\boldsymbol{\tau})$ is assumed to dissipate energy at all non–zero velocities, and therefore its entries are bounded within the first and third quadrants. This feature allows to consider the common Coulomb, viscous and static friction models [9]:

$$f_i(\dot{q}_i,\tau_i) = b_i\dot{q}_i + f_{ci}\,\mathrm{sgn}(\dot{q}_i) + [1 - |\mathrm{sgn}(\dot{q}_i)|\,]\,\mathrm{sat}(\tau_i; f_{si}) \tag{7}$$

where b_i, f_{ci} and f_{si} denote the coefficients of the viscous, Coulomb and static friction, respectively, with $i = 1,\ldots,n$, and $|\cdot|$ denotes absolute value. The $\mathrm{sgn}(\cdot)$ and $\mathrm{sat}(\cdot;\cdot)$ functions are defined as follow

$$\mathrm{sgn}(\dot{q}_i) = \begin{cases} 1 & \text{if } \dot{q}_i > 0 \\ 0 & \text{if } \dot{q}_i = 0 \\ -1 & \text{if } \dot{q}_i < 0 \end{cases}$$

$$\mathrm{sat}(\tau; f_{si}) = \begin{cases} f_{si} & \text{if } \tau_i > f_{si} \\ \tau_i & \text{if } -f_{si} \le \tau_i \le f_{si} \\ -f_{si} & \text{if } \tau_i < -f_{si} \end{cases}$$

At zero velocities, only static friction (stiction) is present satisfying —from (5) and (7)—:

$$f_i(0,\tau_i) = \tau_i - g_i(\boldsymbol{q}) \qquad \text{for} \quad -f_{si} \le \tau_i - g_i(\boldsymbol{q}) \le f_{si}. \tag{8}$$

We assume the robot links are joined together with revolute joints. Three important properties are the following:

Property 8. (See e.g. [18]) The matrix $C(\boldsymbol{q},\dot{\boldsymbol{q}})$ and the time derivative $\dot{M}(\boldsymbol{q})$ of the inertia matrix satisfy:

$$\dot{\boldsymbol{q}}^T \left[\frac{1}{2}\dot{M}(\boldsymbol{q}) - C(\boldsymbol{q},\dot{\boldsymbol{q}})\right]\dot{\boldsymbol{q}} = \boldsymbol{0} \qquad \forall\ \boldsymbol{q},\dot{\boldsymbol{q}} \in \mathbb{R}^n.$$

Property 9. The friction torque vector $\boldsymbol{f}(\dot{\boldsymbol{q}},\boldsymbol{\tau})$ satisfies

$$\dot{\boldsymbol{q}}^T \boldsymbol{f}(\dot{\boldsymbol{q}},\boldsymbol{\tau}) > 0 \qquad \forall\ \boldsymbol{\tau} \in \mathbb{R}^n, \dot{\boldsymbol{q}} \ne \boldsymbol{0} \in \mathbb{R}^n.$$

Property 10. (See e.g. [19]). The gravitational torque vector $\boldsymbol{g}(\boldsymbol{q})$ is bounded for all $\boldsymbol{q} \in \mathbb{R}^n$. This means that there exist finite constants $\bar{g}_i \ge 0$ such that

$$\sup_{\boldsymbol{q}\in\mathbb{R}^n} \{|g_i(\boldsymbol{q})|\} \le \bar{g}_i \qquad i = 1,\cdots,n, \tag{9}$$

where $g_i(\boldsymbol{q})$ stands for the elements of $\boldsymbol{g}(\boldsymbol{q})$.

We are now in position to formulate the regulation problem under actuator torque constraints. Consider the robot dynamic model (5). Assume that each joint actuator is able to supply a known maximum torque $\tau_i^{\max}$ so that:

$$|\tau_i| \le \tau_i^{\max}, \qquad i = 1,\cdots,n \tag{10}$$

where τ_i stands for the i–entry of vector $\boldsymbol{\tau}$. We also assume that the maximum torque $\tau_i^{\max}$ of each actuator satisfies the following condition

$$\tau_i^{\max} > \bar{g}_i + f_{si} \tag{11}$$

where $\bar{g}_i$ was defined in property 10. This assumption implies that the robot actuators are able to supply torques in order to hold the robot at rest for all desired joint position $\boldsymbol{q}_d \in \mathbb{R}^n$. Let us define the position error as $\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}$. The regulation goal addressed in this paper is to drive the position error $\tilde{\boldsymbol{q}}$ toward the inside of an arbitrary small region around zero maintaining the torques within the constraints (10).

4 Proposed controller

This Section shows that, thanks to Properties 1 to 7 of the SFC systems, they are effective for global positioning of robot manipulators taking into account natural phenomena such as the torque constraints of the actuators and the inherent presence of the friction in the joints.

More specifically, it is shown that above kind of fuzzy controllers provided with gravity compensation drive the position error $\tilde{\boldsymbol{q}}$ toward the inside of an arbitrary small region around zero maintaining the torques within the constraints (10). In absence of friction, the closed–loop system turns out to be a globally asymptotically stable system.

The structure of the proposed set–point fuzzy controller is captured by the following control law

$$\boldsymbol{\tau} = \Phi(\tilde{\boldsymbol{q}}, \dot{\tilde{\boldsymbol{q}}}) + \boldsymbol{g}(\boldsymbol{q}) \tag{12}$$

where $\tilde{\boldsymbol{q}} = \boldsymbol{q}_d - \boldsymbol{q}$ denotes the $n \times 1$ joint position error, while $\dot{\tilde{\boldsymbol{q}}} = \dot{\boldsymbol{q}}_d - \dot{\boldsymbol{q}}$ stands for the $n \times 1$ vector of velocity error, $\boldsymbol{q}_d$ and $\dot{\boldsymbol{q}}_d$ are the $n \times 1$ vectors of desired position and velocity respectively, and $\Phi(\tilde{\boldsymbol{q}}, \dot{\tilde{\boldsymbol{q}}})$ is a $n \times 1$ vector whose entries $\phi_i(\tilde{q}_i, \dot{\tilde{q}}_i)$ are the real input–output mappings of the SFC whose properties were established in [6] and that we listed on the previous Section. $\Phi(\tilde{\boldsymbol{q}}, \dot{\tilde{\boldsymbol{q}}})$ is a decoupled nonlinear mapping (in the sense that $\phi_i(\tilde{\boldsymbol{q}}, \dot{\tilde{\boldsymbol{q}}})$ depends only on $\tilde{q}_i, \dot{\tilde{q}}_i$) of the form

$$\Phi(\tilde{\boldsymbol{q}}, \dot{\tilde{\boldsymbol{q}}}) = \begin{bmatrix} \phi_1(\tilde{q}_1, \dot{\tilde{q}}_1) \\ \phi_2(\tilde{q}_2, \dot{\tilde{q}}_2) \\ \vdots \\ \phi_n(\tilde{q}_n, \dot{\tilde{q}}_n) \end{bmatrix} \tag{13}$$

where $\tilde{q}_i, \dot{\tilde{q}}_i$ can be seen as the crisp inputs x_1, x_2 respectively of the i–*th* FLC $\phi_i(\tilde{q}_i, \dot{\tilde{q}}_i)$. In our case the desired position $\boldsymbol{q}_d$ is constant, hence $\dot{\tilde{\boldsymbol{q}}} = -\dot{\boldsymbol{q}}$.

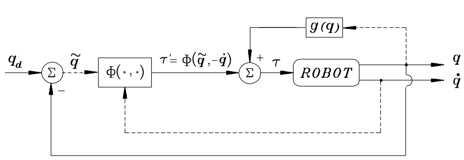

Figure 3: Closed–loop system.

Control law (12) can be seen as a direct FLC with a gravity compensation term (see Figure 3), that is, a SFC with gravity compensation.

At rest, when each $\dot{q}_i = 0$, the static friction $f_i(0, \tau_i)$ opposes all motion as long as the torque τ_i satisfies $-f_{si} \leq \tau_i - g_i(\boldsymbol{q}) \leq f_{si}$. From (12), we have $\phi_i(\tilde{q}_i, -\dot{q}_i) = \tau_i - g_i(\boldsymbol{q})$, and using (8) we can write

$$\phi_i(\tilde{q}_i, 0) = f_i(0, \tau_i) = \tau_i - g_i(\boldsymbol{q}) \tag{14}$$

and hence,

$$-f_{si} \leq \phi_i(\tilde{q}_i, 0) \leq f_{si}. \tag{15}$$

In order to avoid the robot joints remain stuck at rest due to static friction, in agreement with Properties 6 and 7, we assume that

$$\delta_i > \bar{y}_i^{1\,0} > f_{si} \tag{16}$$

where $\bar{y}_i^{1\,0}$ was defined in Property 7.

In addition, in this paper we assume that each of the saturating limits, given in Property 6, δ_i satisfy

$$\delta_i \leq \tau_i^{\max} - \bar{g}_i \quad \text{for} \quad i = 1, \ldots, n. \tag{17}$$

This assumption guarantee that the applied torques, computed by the control law (12), remain bounded within the prescribed limits (10). This is shown by the following arguments: Notice from (11) and (16) that it is always possible to find suitable δ_i in agreement with (17). On the other hand, from (12) the absolute value of the joint torque τ_i supplied by control law leads to $|\tau_i| \leq |\delta_i| + |g_i(\boldsymbol{q})|$. Using Property 10, we have $|\tau_i| \leq \delta_i + \bar{g}_i$. Finally, taking into account the selection procedure of δ_i given in (17) we get $|\tau_i| \leq \tau_i^{\max}$.

4.1 Stability analysis

The closed-loop system is obtained by combining the robot dynamic model (5) with the control law (12). This can be written as:

$$\frac{d}{dt}\begin{bmatrix} \tilde{\boldsymbol{q}} \\ \dot{\boldsymbol{q}} \end{bmatrix} = \begin{bmatrix} -\dot{\boldsymbol{q}} \\ M^{-1}(\boldsymbol{q})[\Phi(\tilde{\boldsymbol{q}}, -\dot{\boldsymbol{q}}) - C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{f}(\dot{\boldsymbol{q}}, \Phi(\tilde{\boldsymbol{q}}, -\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}))] \end{bmatrix} \tag{18}$$

where we have used (12) into $\boldsymbol{f}(\dot{\boldsymbol{q}}, \boldsymbol{\tau})$. Equation (18) is an autonomous nonlinear differential equation whose equilibria are given by the set

$$\begin{aligned} \mathcal{E} \;=\; & \{\dot{\boldsymbol{q}} = \boldsymbol{0} \in \mathbb{R}^n \text{ and } \tilde{\boldsymbol{q}} \in \mathbb{R}^n : \\ & \Phi(\tilde{\boldsymbol{q}}, \boldsymbol{0}) - \boldsymbol{f}(\boldsymbol{0}, \Phi(\tilde{\boldsymbol{q}}, \boldsymbol{0}) + \boldsymbol{g}(\boldsymbol{q})) = \boldsymbol{0}\}. \end{aligned} \tag{19}$$

In other words, taking into account (14), the equilibria of the closed loop are the state vectors with $\dot{\boldsymbol{q}} = \boldsymbol{0}$ and $\tilde{q}_i$ satisfying $-f_{si} \leq \phi_i(\tilde{q}_i, 0) \leq f_{si}$. Due to assumption (16) and the Property 7, inside this region, the equation (4) gives

$$\phi_i(\tilde{q}_i, 0) = \begin{cases} \bar{y}_i^{1\,0} - \bar{y}_i^{1\,0}\mu_{A_1^0}(\tilde{q}_i) & \text{if } \tilde{q}_i \geq 0 \\ -\bar{y}_i^{1\,0} + \bar{y}_i^{1\,0}\mu_{A_1^0}(\tilde{q}_i) & \text{if } \tilde{q}_i < 0 \end{cases}$$

Therefore the position errors $\tilde{q}_i$ satisfying $-f_{si} \leq \phi_i(\tilde{q}_i, 0) \leq f_{si}$ are:

$$|\tilde{q}_i| : \mu_{A_1^0}(\tilde{q}_i) \geq 1 - \frac{f_{si}}{\bar{y}_i^{1\,0}}.$$

For the particular case of this paper, in which we have chosen triangular input membership functions, we have

$$\mu_{A_1^0}(\tilde{q}_i) = \begin{cases} 1 - \frac{|\tilde{q}_i|}{p_{11}} & \text{if } |\tilde{q}_i| \leq p_{11} \\ 0 & \text{otherwise} \end{cases}$$

where p_{11} is a positive parameter denoting the half of the base of the triangular membership function $\mu_{A_1^0}(\tilde{q}_i)$, hence, the equilibria are the state vectors with $\dot{\boldsymbol{q}} = \boldsymbol{0}$ and $\tilde{q}_i$ satisfying

$$|\tilde{q}_i| \leq f_{s_i} \frac{p_{11}}{\bar{y}_i^{1\,0}}.$$

That means, that the equilibria set (19), for the particular case of triangular input membership functions, becomes

$$\mathcal{E} = \left\{ \dot{\boldsymbol{q}} = \boldsymbol{0} \in \mathbb{R}^n \text{ and } \tilde{\boldsymbol{q}} \in \mathbb{R}^n : |\tilde{q}_i| \leq f_{s_i} \frac{p_{11}}{\bar{y}_i^{1\,0}},\ i = 1, \ldots, n \right\}. \quad (20)$$

In the case of absence of friction ($f_{si} = 0$), the origin of the state space $[\tilde{\boldsymbol{q}}^T\ \dot{\boldsymbol{q}}^T]^T = \boldsymbol{0}$ is the unique equilibrium point.

To carry out the stability analysis we propose the following Lyapunov function candidate:

$$V(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}}) = \frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} + \sum_{i=1}^{n} \int_0^{\tilde{q}_i} \phi_i(\xi_i, 0)\, d\xi_i. \quad (21)$$

The first term of $V(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}})$ is a positive definite function with respect to $\dot{\boldsymbol{q}}$ because the positive definiteness of the inertia matrix $M(\boldsymbol{q})$. For (21) qualifies as a Lyapunov function candidate, it remains to show that its second term is a positive definite function with respect to $\tilde{\boldsymbol{q}}$. To this end, notice that from Properties 8 and 10 of $\phi_i(\tilde{q}_i, \dot{\tilde{q}}_i)$, it results that $0 < \tilde{q}_i \phi_i(\tilde{q}_i, 0) \leq \lambda \tilde{q}_i^2$, for all $\tilde{q}_i \neq 0$, which means that $\phi(\tilde{q}_i, 0)$ belongs to the sector $(0, \lambda]$ and hence it is clear that, $\int_0^{\tilde{q}_i} \phi_i(\xi_i, 0)\, d\xi_i > 0\ \ \forall\ \tilde{q}_i \neq 0$ and $\int_0^{\tilde{q}_i} \phi_i(\xi_i, 0)\, d\xi_i \to \infty$ as $\tilde{q}_i \to \infty$, so that, $V(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}})$ is a globally positive definite and radially unbounded function.

The time derivative of the Lyapunov function candidate is given by

$$\begin{aligned} \dot{V}(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}}) &= \dot{\boldsymbol{q}}^T M(\boldsymbol{q})\ddot{\boldsymbol{q}} + \frac{1}{2}\dot{\boldsymbol{q}}^T \dot{M}(\boldsymbol{q})\dot{\boldsymbol{q}} \\ &\quad + \sum_{i=1}^{n} \frac{\partial}{\partial \tilde{q}_i} \left[\int_0^{\tilde{q}_i} \phi_i(\xi_i, 0)\, d\xi_i \right] \dot{\tilde{q}}_i \end{aligned} \quad (22)$$

where we have used the Leibnitz' rule for differentiation of integrals. By using Property 8, the time derivative of the Lyapunov function candidate along of the closed–loop system trajectories yields

$$\dot{V}(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}}) = \dot{\boldsymbol{q}}^T \left[\Phi(\tilde{\boldsymbol{q}}, -\dot{\boldsymbol{q}}) - \Phi(\tilde{\boldsymbol{q}}, \boldsymbol{0})\right] - \dot{\boldsymbol{q}}^T \boldsymbol{f}(\dot{\boldsymbol{q}}, \boldsymbol{\tau}).$$

Since $\Phi(\tilde{\boldsymbol{q}}, -\dot{\boldsymbol{q}})$ is a decoupled nonlinearity of the form (13), we can use Property 10 of $\phi_i(\tilde{q}_i, \dot{\tilde{q}}_i)$ to conclude that $\dot{\boldsymbol{q}}^T \left[\Phi(\tilde{\boldsymbol{q}}, -\dot{\boldsymbol{q}}) - \Phi(\tilde{\boldsymbol{q}}, \boldsymbol{0})\right] \leq 0$, and hence, using Property 9 of the friction torque, we have $\dot{V}(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}})$ is a globally negative semidefinite function. Thus by invoking the Lyapunov's direct method [20] we conclude stability of the closed–loop system.

In order to prove the existence of a global attractor [20] we exploit the autonomous nature of the closed–loop system (18) to apply the LaSalle's theorem [20]. In the region

$$\Omega = \left\{ \begin{bmatrix} \tilde{\boldsymbol{q}} \\ \dot{\boldsymbol{q}} \end{bmatrix} : \dot{V}(\tilde{\boldsymbol{q}}, \dot{\boldsymbol{q}}) = 0 \right\} = \left\{ \begin{bmatrix} \tilde{\boldsymbol{q}} \\ \dot{\boldsymbol{q}} \end{bmatrix} = \begin{bmatrix} \tilde{\boldsymbol{q}} \\ \boldsymbol{0} \end{bmatrix} \in \mathbb{R}^{2n} \right\}$$

the largest invariant set in Ω is $\mathcal{E}$ defined in (20). Therefore, invoking the LaSalle's theorem we conclude that the equilibria set $\mathcal{E}$ is a global attractor.

So, we have proven the following:

Proposition. Consider the robot dynamics (5) in closed loop with the control law (12) using triangular input membership functions. The closed–loop global attractor is given by $\dot{q}_i = 0$ and $\tilde{q}_i$ such that

$$|\tilde{q}_i| \leq f_{s_i} \frac{p_{11}}{\bar{y}_i^{1\,0}} \qquad \text{for} \quad i = 1, \ldots, n. \quad (23)$$

Furthermore, the applied torques are bounded by

$$|\tau_i| \leq \tau_i^{\max} \text{ for } \quad i = 1, \ldots, n. \quad (24)$$

$\nabla\nabla\nabla$

Remark. The presence of static friction in the robot dynamics causes the closed–loop system have a set of equilibria. This may produce a non–zero steady state position error; however, from (23) we see that the bounds on the steady state position errors can be decreased arbitrarily, and still keeping the applied torques inside the prescribed limits, by decreasing the parameter p_{11} of the input membership function

$\mu_{A_1^0}(\tilde{q}_i)$, or increasing the parameter $\bar{y}_i^{1\,0}$ of the respective output membership function $\mu_{B^{1\,0}}(\tilde{q}_i)$. The output membership function parameter that limits the torques delivered by the control action τ_i, according to Property 6, is $\max_{l_1\,l_2} \bar{y}^{l_1\,l_2}$.

The following useful result in case of absence of friction arises from Proposition 1.

Corollary. Consider the robot dynamics (5) without friction together with the control law (12). Then the overall closed–loop system is globally asymptotically stable.

$\nabla\nabla\nabla$

Proof. Since we have assumed the robot is free from friction, then we have $f_{si} = 0$. Therefore, the equilibria set $\mathcal{E}$, given in (20) becomes the origin $[\tilde{\boldsymbol{q}}^T \quad \dot{\boldsymbol{q}}^T]^T = \mathbf{0}$ of the state space. According with the above Proposition, this equilibrium is globally attractive. To demonstrate that this is globally asymptotically stable, it remains to show that it is also a stable equilibrium. This is proven by invoking the Lyapunov's direct method with function (21) which qualifies as a Lyapunov function.

□□□

5 Conclusions

In base on the interesting input/output properties of the so called sectorial fuzzy controllers, we have presented a novel set–point fuzzy controller for robot manipulators which, when friction is present, assures that the steady state position errors are inside a region which can be arbitrarially reduced closed to zero. In case of absence of friction such a controller produces a global asymptotic stable closed–loop system. For both cases, it is always guaranteed that the torques delivered by the actuators be inside prescribed limits given in agreement with actuators torque capabilities. Experimental tests (no reported in this paper) conducted on a robotic arm confirm the theoretical outcomes.

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