Stable Visual Servoing using Neural Network Compensation with an Uncertain Robot Jacobian Matrix

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Abstract

In this work a stable neuro visual servoing for set point control of planar robot manipulators in a fixed-camera configuration is proposed. The gravity terms and the robot Jacobian matrix are assumed unknown. Gravitational terms are approximated using Radial Basis Functions Neural Network with visual information feeding the activation functions and with on-line real-time learning. It is shown that all the closed loop signals are uniformly ultimately bounded. Experimental results in a two degrees of freedom robot are presented to evaluate the proposed controller.

Keywords: Radial basis function, neural networks, set point control, visual servoing.

1 INTRODUCTION

An approach to improve the performance of robot manipulators evolving in unstructured environments is to use visual information to guide the robot towards a target. The above philosophy is termed as visual servoing or visual servo control [1].

In several previous works concerning visual control of planar manipulators in a fixed-camera framework, the gravitational terms are assumed to be fully known [2]. The above requirement is partially overcome using adaptive techniques where the linear in the parameters property of robot manipulators is exploited. Examples of this approach are [3] and [4]. It is worth remarking that in the cited works the structure of the gravity terms must be known in order to implement those controllers. Recently, in [5] exact knowledge of the gravity regressor was relaxed in a task-space adaptive controller. Concerning the robot Jacobian matrix, its a priori knowledge is relaxed in [6], [3] and [4] assuming the existence of an upper bound on the Euclidean norm of the difference between the true and the approximate Jacobian matrix. An alternative approach is [7] where the Jacobian matrix is approximated using a Neural Network trained off-line.

In this work we propose a visual servo controller for planar robots controlled using a fixed camera configuration. The proposed scheme removes completely the requirement of the exact or partial knowledge of the gravity terms. Radial Basis Functions (RBF) Neural Network, which have been applied previously in robot joint control [8], [9], are used for approximating the gravitational terms. A feature of the proposed approach is the fact that activation functions are fed with visual information. The above feature contrasts with previous works using classic adaptive techniques where the regressor matrix containing structural information about the gravity terms, depends on robot joint measurements. The approach presented in [3], [4] for avoiding exact knowledge of the Jacobian matrix is employed here. We show that all the closed loop signals are uniformly ultimately bounded (UUB) and no off-line training phase for the Neural Network is required.

2 BACKGROUND

Throughout this paper, we use the notations $\lambda_{\min}\{\mathbf{A}\}$ and $\lambda_{\max}\{\mathbf{A}\}$ to indicate the smallest and largest eigenvalues respectively of a symmetric positive definite bounded matrix $\mathbf{A}(\mathbf{x}) \in \Re^{nxn}$, for any $\mathbf{x} \in \Re^{n}$. The Euclidean norm of $\mathbf{x}$ is defined as $\|\mathbf{x}\| = \sqrt{\mathbf{x}^T\mathbf{x}}$, the induced norm of matrix $\mathbf{A}$ is defined as $\|\mathbf{A}\| = \sqrt{\lambda_{\max}\{\mathbf{A}^T\mathbf{A}\}}$ and the Frobenius norm is defined by $\|\mathbf{A}\|_F^2 = tr\left(\mathbf{A}^T\mathbf{A}\right) = \sum_{i,j} a_{ij}^2$ with $tr(\cdot)$ the trace of a matrix. The associated inner product is $\langle \mathbf{A}, \mathbf{B}\rangle_F = tr\left(\mathbf{A}^T\mathbf{B}\right)$ where $\mathbf{B} \in \Re^{mxn}$. The Frobenius norm is compatible with the Euclidean norm so that $\|\mathbf{A}\mathbf{x}\| \leq \|\mathbf{A}\|_F \|\mathbf{x}\|$

2.1 Neural network

Consider the RBF neural network with N_2 hidden neurons and N_3 output neurons. Assume that $\mathbf{c}_j,\ j = 1 \ldots N_2$ are the RBF centers, then, the output $y_k,\ k = 1, \ldots, N_3$ is given by

$$y_k = \sum_{j=1}^{N_2} w_{kj}\sigma\left(\|\mathbf{x} - \mathbf{c}_j\|\right) + w_{ko} \qquad (1)$$

where $\mathbf{x} \in \Re^{N_1}$ is the input vector, w_{kj} is the weight connecting the hidden neuron j and the output neuron k, w_{ko} is the threshold offset of output neuron k and $\sigma(\cdot)$ is an activation function which we select it as Gaussian function $\sigma(x) = \exp\left(-\frac{\|\mathbf{x}-\mathbf{c}_j\|}{\mathbf{p}^2}\right)$ where $\mathbf{p}$ is a width parameter. Introducing the following notation $\mathbf{y} = \begin{bmatrix} y_1 & \cdots & y_{N_3} \end{bmatrix}^T$, $\boldsymbol{\sigma}(\mathbf{x}) = \begin{bmatrix} 1 & \sigma_1(x) & \sigma_2(x) & \cdots & \sigma_{N_2}(x) \end{bmatrix}^T$ with $\sigma_j(x) = \sigma_j(\|\mathbf{x} - \mathbf{c}_j\|)$ and weight matrix $\mathbf{W}^T = [w_{kj}]$ including the thresholds w_{ko} as the first column of it, we can express (1) as

$$\mathbf{y} = \mathbf{W}^T\boldsymbol{\sigma}(\mathbf{x}) \qquad (2)$$

2.2 Robot model

In the absence of friction or other disturbances, the dynamics of the 2-revolute links rigid robot manipulator can be expressed as [10]

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \boldsymbol{\tau}(t) \qquad (3)$$

where $\mathbf{q}(t) \in \Re^2$ is the joint angular displacement vector, $\dot{\mathbf{q}}(t) \in \Re^2$ is the joint velocity vector, $\mathbf{M}(\mathbf{q}) \in \Re^{2x2}$ is the inertia matrix, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \in \Re^{2x2}$ is the centrifugal and Coriolis term, $\mathbf{G}(\mathbf{q}) \in \Re^2$ is the gravity term and $\boldsymbol{\tau}(t) \in \Re^2$ is the control torque. Two important properties of robot dynamics are the following [10]

Property 1 *Matrix* $\dot{\mathbf{M}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ *is skew-symmetric, that is,*

$$\dot{\mathbf{M}}(\mathbf{q}) = \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})^T \qquad (4)$$

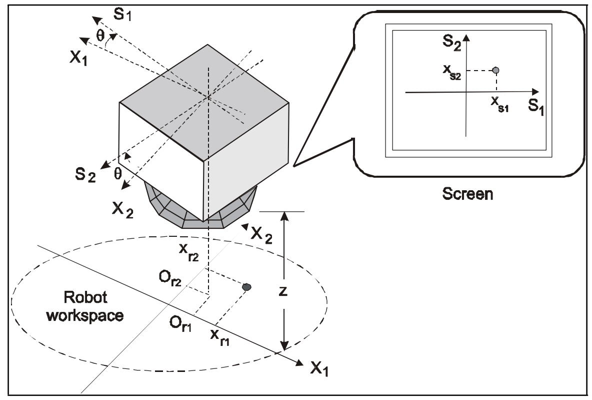

Figure 1: Coordinate frames

Property 2 *There exists a positive constant* k_c *such that*

$$\left\|\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\right\| \leq k_c \left\|\dot{\mathbf{q}}\right\|$$

The robot direct kinematics gives the position $\mathbf{x}_r = \begin{bmatrix} x_{r1} & x_{r2} \end{bmatrix}^T$ of the end-effector with respect to the robot coordinate frame in terms of the joint positions $\mathbf{q}(t) \in \Re^2$

$$\mathbf{x}_r = \mathbf{f}_x(\mathbf{q}) \qquad (5)$$

where $\mathbf{f}_x : \Re^2 \longrightarrow \Re^2$. An important property of revolute joint robots Jacobian matrix defined as $\mathbf{J}(\mathbf{q}) = \frac{\partial \mathbf{f}_x(\mathbf{q})}{\partial \mathbf{q}}$ is [10]

Property 3 *The Jacobian* $\mathbf{J}(\cdot) \in \Re^{2x2}$ *is bounded for all* $\mathbf{q} \in \Re^2$, *i.e., there exists a finite constant* b_J *such that*

$$\|\mathbf{J}(\mathbf{q})\| \leq b_J \qquad \forall \mathbf{q} \in \Re^2$$

2.3 Camera model

Consider a perspective transformation as an ideal pinhole camera model [2]. The description of a point $\mathbf{x}_r = \begin{bmatrix} x_{r1} & x_{r2} \end{bmatrix}^T$ in the robot coordinate frame is given in terms of the computer screen coordinate frame $\mathbf{x}_s = \begin{bmatrix} x_{s1} & x_{s2} \end{bmatrix}^T$ as (see Fig. 1)

$$\begin{bmatrix} x_{s1} \\ x_{s2} \end{bmatrix} = \alpha h \mathbf{R}(\theta)\left\{\begin{bmatrix} x_{r1} \\ x_{r2} \end{bmatrix} - \begin{bmatrix} O_{r1} \\ O_{r2} \end{bmatrix}\right\} + \begin{bmatrix} C_x \\ C_y \end{bmatrix} \qquad (6)$$

where $\begin{bmatrix} C_x & C_y \end{bmatrix}^T$ is the image center, $\mathbf{R}(\theta)$ is a rotation matrix defined as

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \qquad (7)$$

which is generated by clockwise rotating the camera about its optical axis by θ radians, $\begin{bmatrix} O_{r1} & O_{r2} \end{bmatrix}^T$ is the intersection between the optical axis and the $X_1 - X_2$ plane, α is the scale factor of length in pixels/m and h is the magnification factor defined as $h = \frac{\lambda}{\lambda - z}$ where λ is the focal length, z is the distance between camera and robot frame. In our application, the point $\mathbf{x}_r$ is attached to the robot arm end effector, thus, it can be considered as a function of the robot joint positions, i.e., $\mathbf{x}_r(\mathbf{q})$. Then an image point $\mathbf{x}_s$ can be defined as

$$\mathbf{x}_s = \mathbf{h}(\mathbf{x}_r(\mathbf{q})) \tag{8}$$

where $\mathbf{h} : \Re^2 \longrightarrow \Re^2$ is a function that maps robot joint configuration to image values, this map is also referred as the *perceptual kinematic map* [11].

Next, We define the visual distance between the end-effector position $\mathbf{x}_s \in \Re^2$ and the target position $\mathbf{x}_s^* \in \Re^2$ called the image position error $\widetilde{\mathbf{x}}_s \in \Re^2$ as $\widetilde{\mathbf{x}}_s = \mathbf{x}_s^* - \mathbf{x}_s$. We assume that the target is static and located inside the robot workspace, so there is at least a joint position configuration $\mathbf{q}_d$ for wich

$$\left\| \mathbf{x}_s^* (\mathbf{q}_d) - \mathbf{x}_s (\mathbf{q}) \right\| \leq \beta; \qquad \beta > 0. \tag{9}$$

using (5) and (6) the image position error $\widetilde{\mathbf{x}}_s$ is written as

$$\widetilde{\mathbf{x}}_s = \alpha h \mathbf{R}(\theta) \left[\mathbf{f}_x(\mathbf{q}_d) - \mathbf{f}_x(\mathbf{q}) \right] \tag{10}$$

3 Stability analysis

Using the neural network universal approximation property and the inverse perceptual kinematic mapping $\mathbf{q} = \mathbf{h}^{-1}(\mathbf{x}_s)$ the gravity term $\mathbf{G}(\mathbf{q})$ in (3) can be approximated by a RBF neural network as

$$\mathbf{G}(\mathbf{q}) = \mathbf{G}(\mathbf{h}^{-1}(\mathbf{x}_s)) = \mathbf{W}^T \boldsymbol{\sigma}(\mathbf{x}_s) + \varepsilon \tag{11}$$

where $\mathbf{W} \in \Re^{N_3 x N_2}$, $\boldsymbol{\sigma}(\mathbf{x}_s) \in \Re^{N_2}$, ε is the neural network approximation error. Note that in (11) visual information is used instead of joint information. For some unknown constant ideal weights $\mathbf{W}$, the reconstruction error is bounded by $\|\boldsymbol{\varepsilon}\| < k_\varepsilon$. In orden to implement neural network compensator (11) the following assumption for the ideal weights is needed [9]

Assumption 1 *The ideal weights are bounded by positive values k_w so that*

$$\|\mathbf{W}\|_F \leq k_w$$

Then, an estimate of the gravity term $\mathbf{G}(\mathbf{q})$ denoted as $\widehat{\mathbf{G}}(\mathbf{q})$ is

$$\widehat{\mathbf{G}}(\mathbf{q}) = \widehat{\mathbf{G}}(\mathbf{h}^{-1}(\mathbf{x}_s)) = \widehat{\mathbf{W}}^T \boldsymbol{\sigma}(\mathbf{x}_s) \tag{12}$$

where $\widehat{\mathbf{W}} \in \Re^{N_3 x N_2}$ are estimates of $\mathbf{W}$. We define the weight estimation error as

$$\widetilde{\mathbf{W}} = \mathbf{W} - \widehat{\mathbf{W}} \tag{13}$$

If $\mathbf{J}(\mathbf{q})$ is uncertain and only an estimate $\widehat{\mathbf{J}}(\mathbf{q})$ is available such that

$$\left\| \mathbf{J}(\mathbf{q}) - \widehat{\mathbf{J}}(\mathbf{q}) \right\| \leq k_J \tag{14}$$

where k_J is a positive constant, then, the control law is proposed as

$$\boldsymbol{\tau} = \widehat{\mathbf{J}}(\mathbf{q})^T \mathbf{R}(\theta)^T \mathbf{K}_p \widetilde{\mathbf{x}}_s - \mathbf{K}_d \dot{\mathbf{q}} + \widehat{\mathbf{W}}^T \boldsymbol{\sigma}(\mathbf{x}_s) \tag{15}$$

where $\mathbf{K}_p$ and $\mathbf{K}_d$ are the 2 x 2 symmetric positive definite diagonal proportional and derivative gain matrices. The following theorem shows how to adjust the weights of neural network (12) to guarantee closed-loop stability in spite of an uncertain Jacobian matrix and gravity terms.

Theorem 1 *Consider system (3) in closed-loop with control law (15) where the updating law for the weights of neural network (12) is given by*

$$\begin{aligned} \dot{\widehat{\mathbf{W}}} &= -\mathbf{K}_w \boldsymbol{\sigma}(\mathbf{x}_s) \left[\dot{\mathbf{q}} - \mu \widehat{\mathbf{J}}(\mathbf{q})^{-1} \mathbf{R}(\theta)^T \mathbf{f}(\widetilde{\mathbf{x}}_s) \right]^T \\ &\quad -\kappa \mathbf{K}_w \left\| \widetilde{\mathbf{x}}_s \right\| \left\| \dot{\mathbf{q}} \right\| \widehat{\mathbf{W}} \end{aligned} \tag{16}$$

where $\mathbf{K}_w$ is a positive defined matrix, κ is a positive constant and function $\mathbf{f}(\widetilde{\mathbf{x}}_s)$ is defined as

$$\mathbf{f}(\widetilde{\mathbf{x}}_s) = \beta \widetilde{\mathbf{x}}_s; \qquad \beta = \frac{1}{1 + \|\widetilde{\mathbf{x}}_s\|} \tag{17}$$

If μ is chosen such that

$$\min \left\{ \sqrt{\frac{\lambda_{min}\{\mathbf{K}_p\}}{\alpha h \lambda_{max}\{\Lambda\}}}, \frac{\lambda_{min}\{\mathbf{K}_d\}}{\sqrt{8}[b_1 \gamma_1 + b_2 \lambda_{max}\{\mathbf{M}(\mathbf{q})\} + b_1 k_c]}, \frac{2\lambda_{min}\{\mathbf{K}_p\}}{\lambda_{max}\{\Lambda_K\}} \right\} > \mu > 0 \tag{18}$$

where

$$\begin{aligned} \Lambda &= \left[\widehat{\mathbf{J}}(\mathbf{q})^{-1} \mathbf{R}(\theta)^T \right]^T \mathbf{M}(\mathbf{q}) \widehat{\mathbf{J}}(\mathbf{q})^{-1} \mathbf{R}(\theta)^T \\ \Lambda_K &= \left[\widehat{\mathbf{J}}(\mathbf{q})^{-1} \mathbf{R}(\theta)^T \right]^T \mathbf{K}_d \widehat{\mathbf{J}}(\mathbf{q})^{-1} \mathbf{R}(\theta)^T \\ \gamma_1 &= \sqrt{8} \alpha h b_J \lambda_{max} \{\mathbf{M}(\mathbf{q})\} \end{aligned} \tag{19}$$

b_1 and b_2 are positive constants, then, $\widetilde{\mathbf{x}}_s$, $\dot{\mathbf{q}}$ and $\widetilde{\mathbf{W}}$ are UUB.

Proof: Define a Lyapunov function candidate as

$$\begin{aligned} V(\widetilde{\mathbf{x}}_s,\dot{\mathbf{q}},\widetilde{\mathbf{W}}) &= \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{M}(\mathbf{q})\dot{\mathbf{q}}+\frac{1}{2\alpha h}\widetilde{\mathbf{x}}_s^T\mathbf{K}_p\widetilde{\mathbf{x}}_s \\ &\quad -\mu\mathbf{f}(\widetilde{\mathbf{x}}_s)^T\mathbf{R}(\theta)\widehat{\mathbf{J}}^{-T}(\mathbf{q})\mathbf{M}(\mathbf{q})\dot{\mathbf{q}} \\ &\quad +\frac{1}{2}tr(\widetilde{\mathbf{W}}^T\mathbf{K}_w^{-1}\widetilde{\mathbf{W}}) \end{aligned} \tag{20}$$

Equation (20) is positive definite since by hypothesis $\sqrt{\frac{\lambda_{min}\{\mathbf{K}_p\}}{\alpha h\lambda_{max}\{\Lambda\}}}>\mu$. Using (3), (4), (11), (13), (15), robot Property 1 and $\dot{\widetilde{\mathbf{x}}}_s=\frac{\partial\widetilde{\mathbf{x}}_s}{\partial\mathbf{q}}\dot{\mathbf{q}}=-\alpha h\mathbf{R}(\theta)\mathbf{J}(\mathbf{q})\dot{\mathbf{q}}$, the time derivative of (20) is

$$\begin{aligned} \dot{V}(\widetilde{\mathbf{x}}_s,\dot{\mathbf{q}},\widetilde{\mathbf{W}}) &= -\dot{\mathbf{q}}^T\mathbf{K}_d\dot{\mathbf{q}} \\ &\quad -\mu\dot{\mathbf{f}}(\widetilde{\mathbf{x}}_s)^T\mathbf{R}(\theta)\widehat{\mathbf{J}}^{-T}(\mathbf{q})\mathbf{M}(\mathbf{q})\dot{\mathbf{q}} \\ &\quad -\mu\mathbf{f}(\widetilde{\mathbf{x}}_s)^T\mathbf{R}(\theta)\dot{\widehat{\mathbf{J}}}^{-T}(\mathbf{q})\mathbf{M}(\mathbf{q})\dot{\mathbf{q}} \\ &\quad -\mu\mathbf{f}(\widetilde{\mathbf{x}}_s)^T\mathbf{K}_p\widetilde{\mathbf{x}}_s \\ &\quad +\mu\mathbf{f}(\widetilde{\mathbf{x}}_s)^T\mathbf{R}(\theta)\widehat{\mathbf{J}}^{-T}(\mathbf{q})\mathbf{K}_d\dot{\mathbf{q}} \\ &\quad -\mu\mathbf{f}(\widetilde{\mathbf{x}}_s)^T\mathbf{R}(\theta)\widehat{\mathbf{J}}^{-T}(\mathbf{q})\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})^T\dot{\mathbf{q}} \\ &\quad -\dot{\mathbf{q}}\widetilde{\mathbf{J}}(\mathbf{q})^T\mathbf{R}(\theta)^T\mathbf{K}_p\widetilde{\mathbf{x}}_s \\ &\quad -\left[\dot{\mathbf{q}}^T-\mu\mathbf{f}(\widetilde{\mathbf{x}}_s)^T\mathbf{R}(\theta)\widehat{\mathbf{J}}^{-T}(\mathbf{q})\right]\boldsymbol{\varepsilon} \\ &\quad +tr\left\{\widetilde{\mathbf{W}}^T\left[\mathbf{K}_w^{-1}\dot{\widetilde{\mathbf{W}}}-\boldsymbol{\sigma}(\mathbf{x}_s)\dot{\mathbf{q}}^T\right.\right. \\ &\quad \left.\left.+\mu\boldsymbol{\sigma}(\mathbf{x}_s)\mathbf{f}(\widetilde{\mathbf{x}}_s)^T\mathbf{R}(\theta)\mathbf{J}^*(\mathbf{q})\right]\right\} \end{aligned} \tag{21}$$

where $\widetilde{\mathbf{J}}(\mathbf{q})=\mathbf{J}(\mathbf{q})-\widehat{\mathbf{J}}(\mathbf{q})$. We now provide upper bounds on the following terms

$$\begin{aligned} &-\dot{\mathbf{q}}^T\mathbf{K}_d\dot{\mathbf{q}}\le-\tfrac{1}{2}\dot{\mathbf{q}}^T\mathbf{K}_d\dot{\mathbf{q}}-\tfrac{1}{2}\lambda_{min}\{\mathbf{K}_d\}\left\|\dot{\mathbf{q}}\right\|^2 \\ &-\mu\dot{\mathbf{f}}(\widetilde{\mathbf{x}}_s)^T\mathbf{R}(\theta)\widehat{\mathbf{J}}^{-T}(\mathbf{q})\mathbf{M}(\mathbf{q})\dot{\mathbf{q}}\le\mu\sqrt{2}b_1\gamma_1\left\|\dot{\mathbf{q}}\right\|^2 \\ &-\mu\mathbf{f}(\widetilde{\mathbf{x}}_s)^T\mathbf{R}(\theta)\dot{\widehat{\mathbf{J}}}^{-T}(\mathbf{q})\mathbf{M}(\mathbf{q})\dot{\mathbf{q}} \\ &\qquad\le\mu\sqrt{2}b_2\lambda_{max}\{\mathbf{M}(\mathbf{q})\}\left\|\dot{\mathbf{q}}\right\|^2 \\ &-\mu\mathbf{f}(\widetilde{\mathbf{x}}_s)^T\mathbf{R}(\theta)\widehat{\mathbf{J}}^{-T}(\mathbf{q})\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})^T\dot{\mathbf{q}} \\ &\qquad\le\mu\sqrt{2}b_1k_c\left\|\dot{\mathbf{q}}\right\|^2 \\ &-\dot{\mathbf{q}}^T\widetilde{\mathbf{J}}(\mathbf{q})^T\mathbf{R}(\theta)^T\mathbf{K}_p\widetilde{\mathbf{x}}_s \\ &\qquad\le\sqrt{2}k_J\lambda_{max}\{\mathbf{K}_p\}\left\|\widetilde{\mathbf{x}}_s\right\|\left\|\dot{\mathbf{q}}\right\| \end{aligned} \tag{22}$$

where b_1 denotes the norm bound for $\widehat{\mathbf{J}}^{-T}(\mathbf{q})$ and we have used Property 2, Frobenius norm of $\mathbf{R}(\theta)$ and the following inequalities

$$\begin{aligned} &\left\|\mathbf{f}(\widetilde{\mathbf{x}}_s)\right\|\le 1 \\ &\left\|\dot{\mathbf{f}}(\widetilde{\mathbf{x}}_s)\right\|\le 2\left\|\dot{\widetilde{\mathbf{x}}}_s\right\|\le\sqrt{8}\alpha hb_J\left\|\dot{\mathbf{q}}\right\| \\ &\left\|\dot{\widehat{\mathbf{J}}}^{-T}(\mathbf{q})\right\|\le b_2\left\|\dot{\mathbf{q}}\right\| \end{aligned} \tag{23}$$

Since μ satisfies (18), then, the following term

$$\begin{aligned} \gamma_2 &= \frac{1}{2}\lambda_{min}\{\mathbf{K}_d\}-\mu\left[\sqrt{2}b_1\gamma_1\right. \\ &\quad \left.+\sqrt{2}b_2\lambda_{max}\{\mathbf{M}(\mathbf{q})\}+\sqrt{2}b_1k_c\right] \end{aligned} \tag{24}$$

is a positive constant. It now follows from the above term and the inequalities (22) that the time derivative of the Lyapunov function candidate (21) satisfies

$$\begin{aligned} \dot{V}(\widetilde{\mathbf{x}}_s,\dot{\mathbf{q}},\widetilde{\mathbf{W}}) &\le -\frac{1}{2}\left[\dot{\mathbf{q}}-\mu\widehat{\mathbf{J}}(\mathbf{q})^{-1}\mathbf{R}(\theta)^T\mathbf{f}(\widetilde{\mathbf{x}}_s)\right]^T \\ &\quad \cdot\mathbf{K}_d\left[\dot{\mathbf{q}}-\mu\widehat{\mathbf{J}}(\mathbf{q})^{-1}\mathbf{R}(\theta)^T\mathbf{f}(\widetilde{\mathbf{x}}_s)\right] \\ &\quad -\gamma_2\left\|\dot{\mathbf{q}}\right\|^2-\mu\mathbf{f}(\widetilde{\mathbf{x}}_s)^T\mathbf{K}_p\widetilde{\mathbf{x}}_s \\ &\quad +\frac{1}{2}\mu^2\mathbf{f}(\widetilde{\mathbf{x}}_s)^T\left[\widehat{\mathbf{J}}(\mathbf{q})^{-1}\mathbf{R}(\theta)^T\right]^T \\ &\quad \cdot\mathbf{K}_d\widehat{\mathbf{J}}(\mathbf{q})^{-1}\mathbf{R}(\theta)^T\mathbf{f}(\widetilde{\mathbf{x}}_s) \\ &\quad +\sqrt{2}k_J\lambda_{max}\{\mathbf{K}_p\}\left\|\widetilde{\mathbf{x}}_s\right\|\left\|\dot{\mathbf{q}}\right\| \\ &\quad -\left[\dot{\mathbf{q}}^T-\mu\mathbf{f}(\widetilde{\mathbf{x}}_s)^T\mathbf{R}(\theta)\widehat{\mathbf{J}}^{-T}(\mathbf{q})\right]\boldsymbol{\varepsilon} \\ &\quad +tr\left\{\widetilde{\mathbf{W}}^T\left[\mathbf{K}_w^{-1}\dot{\widetilde{\mathbf{W}}}-\boldsymbol{\sigma}(\mathbf{x}_s)\dot{\mathbf{q}}^T\right.\right. \\ &\quad \left.\left.+\mu\boldsymbol{\sigma}(\mathbf{x}_s)\mathbf{f}(\widetilde{\mathbf{x}}_s)^T\mathbf{R}(\theta)\widehat{\mathbf{J}}^{-T}(\mathbf{q})\right]\right\} \end{aligned} \tag{25}$$

Now, note that

$$\begin{aligned} &\tfrac{1}{2}\mu^2\mathbf{f}(\widetilde{\mathbf{x}}_s)^T\left[\widehat{\mathbf{J}}(\mathbf{q})^{-1}\mathbf{R}(\theta)^T\right]^T\mathbf{K}_d\widehat{\mathbf{J}}(\mathbf{q})^{-1}\mathbf{R}(\theta)^T\mathbf{f}(\widetilde{\mathbf{x}}_s) \\ &\qquad\le\tfrac{1}{2}\mu^2\beta\lambda_{max}\{\Lambda_K\}\left\|\widetilde{\mathbf{x}}_s\right\|^2 \\ &-\mu\mathbf{f}(\widetilde{\mathbf{x}}_s)^T\mathbf{K}_p\widetilde{\mathbf{x}}_s\le-\mu\beta\lambda_{min}\{\mathbf{K}_p\}\left\|\widetilde{\mathbf{x}}_s\right\|^2 \\ &-\dot{\mathbf{q}}^T\boldsymbol{\varepsilon}\le k_\varepsilon\left\|\dot{\mathbf{q}}\right\| \\ &\mu\mathbf{f}(\widetilde{\mathbf{x}}_s)^T\mathbf{R}(\theta)\widehat{\mathbf{J}}^{-T}(\mathbf{q})\boldsymbol{\varepsilon}\le\mu\beta\sqrt{2}b_1k_\varepsilon\left\|\widetilde{\mathbf{x}}_s\right\| \end{aligned} \tag{26}$$

where we have used the upper bound of the reconstruction error k_ε and definitions (17), (19). Since μ satisfies (18), then, the term

$$\gamma_3 = \lambda_{min}\left\{\mathbf{K}_p\right\} - \frac{1}{2}\mu\lambda_{max}\left\{\Lambda_K\right\} \tag{27}$$

is a positive constant. Due to the inequalities (26), using neural network weight update law (16), the inequality $tr\{\widetilde{\mathbf{W}}^T(\mathbf{W}-\widetilde{\mathbf{W}})\} \leq \left\|\widetilde{\mathbf{W}}\right\|_F \left(k_w - \left\|\widetilde{\mathbf{W}}\right\|_F\right)$ and completing the squere (25) becomes

$$\begin{aligned}
\dot{V}(\widetilde{\mathbf{x}}_s, \dot{\mathbf{q}}, \widetilde{\mathbf{W}}) \leq & -\frac{1}{2}\left[\dot{\mathbf{q}} - \mu\widehat{\mathbf{J}}(\mathbf{q})^{-1}\mathbf{R}(\theta)^T\mathbf{f}(\widetilde{\mathbf{x}}_s)\right]^T \\
& \cdot\mathbf{K}_d\left[\dot{\mathbf{q}} - \mu\widehat{\mathbf{J}}(\mathbf{q})^{-1}\mathbf{R}(\theta)^T\mathbf{f}(\widetilde{\mathbf{x}}_s)\right] \\
& - \left[\gamma_2\left\|\dot{\mathbf{q}}\right\| - k_\varepsilon\right]\left\|\dot{\mathbf{q}}\right\| \qquad (28) \\
& - \left[\gamma_3\left\|\widetilde{\mathbf{x}}_s\right\| - \sqrt{2}b_1 k_\varepsilon\right]\mu\beta\left\|\widetilde{\mathbf{x}}_s\right\| \\
& - \left[\kappa\left(\left\|\widetilde{\mathbf{W}}\right\|_F - \frac{k_w}{2}\right)^2 - \kappa\frac{k_w^2}{4}\right. \\
& \left. -\sqrt{2}k_J\lambda_{max}\left\{\mathbf{K}_p\right\}\right]\left\|\widetilde{\mathbf{x}}_s\right\|\left\|\dot{\mathbf{q}}\right\|
\end{aligned}$$

Then, the time derivative of Lyapunov function (20) is guaranteed to be negative as long as the following conditions hold

$$\begin{aligned}
\left\|\widetilde{\mathbf{x}}_s\right\| &> \frac{\sqrt{2}b_1 k_\varepsilon}{\gamma_3}, \qquad \left\|\dot{\mathbf{q}}\right\| > \frac{k_\varepsilon}{\gamma_2}, \qquad (29) \\
\left\|\widetilde{\mathbf{W}}\right\|_F &> \frac{k_w}{2} + \sqrt{\frac{\sqrt{2}k_J\lambda_{max}\left\{\mathbf{K}_p\right\}}{\kappa} + \frac{k_w^2}{4}}
\end{aligned}$$

where the right-hand sides of (29) are the convergence regions and practical bounds for the signals $\widetilde{\mathbf{x}}_s$, $\dot{\mathbf{q}}$ and $\widetilde{\mathbf{W}}$ in the sense that excursions beyond these bounds will be very small. Therefore, $\dot{V}(\widetilde{\mathbf{x}}_s, \dot{\mathbf{q}}, \widetilde{\mathbf{W}})$ is negative outside a compact set. According to the standard Lyapunov theory extension [9], the above demostrates that $\widetilde{\mathbf{x}}_s$, $\dot{\mathbf{q}}$ and $\widetilde{\mathbf{W}}$ are UUB. ∎

Remark 1 *The controller (15), (16) does not need knowledge of the joint positions* $\mathbf{q}(t)$ *to compensate the robot gravity term, only visual information is required and stalling at a singular position can be avoided by chosing* $\widehat{\mathbf{J}}(\mathbf{q})$ *to be full rank where* $\mathbf{J}(\mathbf{q})$ *lose rank. As ilustrated in [6], there are many possibilities in designing* $\widehat{\mathbf{J}}(\mathbf{q})$.

Remark 2 *As show in (24) and (27) lower bounds for* $\left\|\widetilde{\mathbf{x}}_s\right\|$ *and* $\left\|\dot{\mathbf{q}}\right\|$ *in (29) can be made smaller increasing* $\mathbf{K}_d$ *and* $\mathbf{K}_p$ *or reducing the constant* μ.

4 EXPERIMENTAL RESULTS

The experimental setup used to show the performance features of the controller (15) corresponds to robot arm having two degree of freedom moving in the vertical plane and a Pulnix camera, model 9710. Complete information regarding this visual servoing system can be found in [12]. In the experimental set-up, the center of the robot first axis coincides with the origin of the image plane and we assume that the camera is perfectly aligned, so that $\theta = 0$, then, $\mathbf{R}(\theta)$ is the identity matrix. The estimated robot Jacobian matrix used is

$$\widehat{\mathbf{J}}(\mathbf{q}) = \begin{bmatrix} \widehat{l}_1\cos(q_1) & \widehat{l}_2\cos(q_2) \\ \widehat{l}_1\sin\ (q_1) & \widehat{l}_2\sin\ (q_2) \end{bmatrix} \tag{30}$$

where $\widehat{l}_1$, $\widehat{l}_2$ are the estimated lenghts of the first and second links respectively, the exact values for the manipulator lenghts are 0.21 m and the estimated values used in the experiment are 0.105 m. The RBF neural network was formed with 8 neurons, their centers and their widths are assumed to be fixed. We choose the centers evenly spaced between $[100, 60]^T$ to $[-100, -60]^T$, their widths were set to $p = [30, 20]$ and all the initial weights values were set to zero. The initial image position of the end-effector was $\mathbf{x}_s = \begin{bmatrix} 80 & 15 \end{bmatrix}^T$ *pixels.* In the experiments the visual sampling rate was 50 hz (20 ms). This value is a function of the time required to image acquisition (16.7 ms), processing (2.3 ms) and the time required for sending data (1 ms). Joint velocity was computed from position measurements through a simple high-pass filter. Sampling frequency at the joint level was 1 *Khz.*

Two experiments were performed, in both the proportional and derivative matrices were maintained at relatively low values in order to appreciate the effect of the compensations and were set to $\mathbf{K}_p = diag\left\{0.2, 0.2\right\}$ and $\mathbf{K}_d = diag\left\{0.3, 0.3\right\}$ and the RBF neural network parameters were chosen as $\mathbf{K}_w = diag\{1.3\}$, $\kappa = 0.0001$ and $\mu = 0.035$. In the first experiment, the desired image position was set to $\mathbf{x}_s^* = \begin{bmatrix} x_{s1}^* & 15 \end{bmatrix}^T$ *pixels*, where x_{s1}^* was a square wave signal of 15 *pixels* of amplitude centered at 80 *pixels* and at a frequency of 0.15 *Hertz.* In the second experiment the reference was set to $\mathbf{x}_s^* = \begin{bmatrix} 80 & x_{s2}^* \end{bmatrix}^T$ *pixels*, where x_{s2}^* was a square wave signal of 15 *pixels* of amplitude centered at 15 *pixels* and at a frequency of 0.15 *Hertz.* Figures (2) and (3) depict the experimental results for the first and second experiments respectively.

5 Conclusion

In this paper we presented theoretical and experimental results in visual servoing of robot manipulators in a fixed camera configuration using a radial basis function neural network for compensating the gravitational force wich does not need off-line phase training. In contrast with other approaches, the controller does not need any knowledge of the gravity term structure and use visual information for compensating the gravitational torques. The main contribution of this paper is the fact that we prove that the closed-loop system with visual information as feedback and neuro visual compensator in spite of uncertain Jacobian matrix are uniform ultimate bounded.

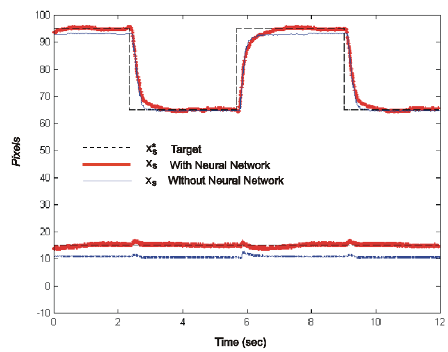

Figure 2: First experiment

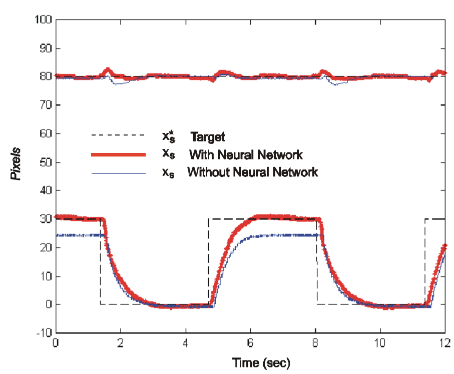

Figure 3: Second experiment

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