Properties of robustness and nonfragility in controllers PI/PD/PID via substitutions

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I. Abstract

In this work using algebraic methods, we give a complete solution to the problem to turn rational, real, proper, stable function or unstable $H(s) = \frac{N_h(s)}{D_h(s)}$, in rational, real, proper and stable function, using the substitution of the variable s by $\alpha(s) = \frac{as+b}{cs+d}$, i.e., to make stable to $H(\alpha(s))$. These results generalize and extend previous results. , we characterized the space of parameters a, b, c, d for which the map before mentioned, preserves stability for any stable polynomial, mapping stable polynomials in stable polynomials and unstable polynomials in stable polynomials. Like a consequence, we obtain a dual result, in the sense that the robust stabilization of a plant $H(s)$ with disturbances nonlinear in its parameters, induced by the substitution of the variable s by $\alpha^{-1}(s)$, with a controller $C(s)$, implies the nonfragility of the controller $C(s)$ under the same class of disturbances, induced by the substitution of the variable s by $\alpha(s)$, in the controller, and vice versa. In the particular case when $b = 0$, the substitution $\alpha(s)$, preserves the structure of the controllers type PD/PI/PID and we give to rules of tuning for the derivative part of the controller type PD, and for the proportional and integral part of the controller type PI.

II. Introduction

Recently [3,4,5,8] has appeared in Literature a series of articles on the subject of preservation of stability for linear systems in the domain of the frequency. Some of these are based on maps that preserve the stability of stable polynomials, is to say the map that is obtained to multiply the vector of coefficients of stable polynomials by a fixed matrix, to obtain a vector of stable coefficients [5]. Nevertheless, this method does not have a complete characterization of which matrices preserve stability. Other methods to guarantee stability under substitution of a rational function in a polynomial, are based on H-domains and diagrams of Mikhailov [8], but requires that the intersection of a pair of sets which depend on the frequency and the polynomials at issue, it is empty or to calculate the norm of the rational function to replace, which in general is not easy. In [1] is used the substitution $\alpha(s) = \frac{as+b}{cs+d}$, and is proved that for positive real numbers a, b, c, d such that $ad-bc \neq 0$, this preserves stability, but it is a case very restricted. In [3, 4] the results are extend and generalize, showing that substitutions by positive strictly real functions of relative degree zero (functions SPR0), they preserve stability, and under some additional conditions powers of functions SPR0, also preserve stability, but sufficient conditions only are given. In this work using algebraic methods, we give a complete solution to the problem to turn rational, real, proper, stable function or unstable $H(s) = \frac{N_h(s)}{D_h(s)}$, in rational, real, proper and stable function, using the substitution of the variable s by $\alpha(s) = \frac{as+b}{cs+d}$, i.e., to make stable to $H(\alpha(s))$. These results generalize and extend previous results Fernández et al. [1,2,3,4], giving in addition, an answer to the open problem proposed by Djaferis [5] for the case of maps that preserve stability, obtained under the substitution of the variable s by $\alpha(s)$ in a stable polynomial. Characterizing all the maps obtained under this substitution, that preserve stability for any stable polynomial $D_h(s)$ which is mapping to stable polynomial $(cs+d)^m D_h(\alpha(s))$. That is to say, we characterized the space of parameters a, b, c, d for which the map before mentioned, preserves stability for any stable polynomial, mapping stable polynomials in stable polynomials. But also we characterized the space of parameters a, b, c, d for which the map before mentioned, mapping unstable polynomials in stable polynomials. Like a consequence, we obtain a dual result, in the sense that the robust stabilization of a plant $H(s)$ with disturbances nonlinear in its parameters, induced by the substitution of the variable s by $\alpha^{-1}(s)$, with a controller $C(s)$, implies the nonfragility of the controller $C(s)$ under the same class of disturbances, induced by the substitution of the variable s by $\alpha(s)$, in the controller, and vice versa. That is to say, the nonfragility of the controller $C(s)$ under disturbances, induced by the substitution of the variable s by $\alpha^{-1}(s)$, in the controller, implies the robust stabilization of a plant $H(s)$ with disturbances nonlinear in its parameters, induced by the substitution of the variable s by $\alpha(s)$ with a controller $C(s)$. In the particular case when $b = 0$, the substitution $\alpha(s)$, preserves the structure of the controllers type PD/PI and we give to rules of tuning for the derivative part of the controller type PD, and for the proportional and integral part of the controller type PI. Based on the resulting parametrization for this type of controller, after of the substitution of the variable s by $\alpha(s)$, taking $b = 0$. Finally, we give a result about stabilization based in passivity. To show the developed methodology, we presented an example for a controller type PI. In the controller we have a nonfragil performance and we give to tuning rules based in numerical methods of solution of the inequalities that are from the application of this methodology

III. Preliminaries

Consider a rational function $H(s) \in \mathsf{R}(s)$ in the following way

$$H(s) = \frac{N_h(s)}{D_h(s)} = k\frac{s^n + a_{n-1}s^{n-1} + \cdots + a_0}{s^m + b_{m-1}s^{m-1} + \cdots + b_0}$$

where $N_h(s)$ and $D_h(s)$ are coprime, with $m \geq n$.

Let us factorize $H(s)$ as $H(s) = H_r(s)H_c(s)$, where

$$H_r(s) = k\frac{(s - \overline{R}_1)\cdots(s - \overline{R}_{n-l_1})}{(s - R_1)\cdots(s - R_{m-j_1})}$$

has real poles and zeros, $l_1 < n$, $j_1 < m$; and

$$H_c(s) = \frac{\left[(s - \bar{r}_1)^2 + \bar{t}_1^2\right]\cdots\left[(s - \bar{r}_{l_0})^2 + \bar{t}_{l_0}^2\right]}{\left[(s - r_1)^2 + t_1^2\right]\cdots\left[(s - r_{j_0})^2 + t_{j_0}^2\right]}$$

has complex poles and zeros, $l_0 = \frac{l_1}{2}$ and $j_0 = \frac{j_1}{2}$.

Definition 1. [6,7] A rational function $H(s) \in R(s)$ of complex variable $s = \sigma + j\omega$ is positive real (PR) if

i) $p(s)$ is analytic in $\mathrm{Re}[s] > 0$,

ii) $p(s)$ is real for s real,

iii)$\mathrm{Re}[p(s)] \geq 0$ for all $\mathrm{Re}[s] > 0$.

Definition 2. [6,7] A real and rational function $p(s)$ of zero relative degree is SPR0 if the following conditions holds:

i) $p(s)$ is analytic in $\mathrm{Re}[s] \geq 0$,

ii) $\mathrm{Re}[p(j\omega)] > 0$ for all $\omega \in R$, where $j = \sqrt{-1}$.

Let us define the set $SPR0^*$:

$$SPR0^* = \{p(s) \in \mathsf{R}(s) : p(s) \text{ is SPR0}\} \cup \{s\}\,.$$

and the set as $\Gamma(a, b, c, d)$ follows

$\Gamma(a, b, c, d) = \{\alpha(s) \in \mathsf{R}(s) : \alpha(s) = \frac{as+b}{cs+d},\ ad - bc \neq 0$ and $a, b, c, d > 0\} \cup \{s\}$.

the following properties can be easily verified for the set $\Gamma_\alpha(a, b, c, d)$

1) $\lim_{(b,c)\to(0,0)} \frac{as+b}{cs+a} = s$, where $a^2 - bc > 0$;

2) if $\alpha(s)$, $\beta(s) \in \Gamma(a, b, c, d)$, then $\alpha(\beta(s))$, $\beta(\alpha(s)) \in \Gamma(a, b, c, d)$.

From the associative property of the function composition, we know that the set $\Gamma(a, b, c, d)$ is a non commutative monoid under the composition operation. Additionally, is well know that $\Gamma(a, b, c, d) \subset SPR0^*$ [1].

IV. Results

In this section we present our main results.

Theorem 1. Consider the plant $H(s) = \frac{N_h(s)}{D_h(s)}$ where $N_h(s)$ and $D_h(s)$ are polynomials satisfying $\deg D_h(s) = m \geq \deg N_h(s) = n$. Let us also define the lineal fractional transformation $\alpha(s) = \frac{as+b}{cs+d}$ where a, b, c, d are real numbers, and let us substitute the s variable by $\alpha(s)$ in $H(s)$ with $cd \neq 0$ and $ad - bc \neq 0$. Then $H_\alpha(s) \equiv H(\alpha(s))$ is stable if and only if the following conditions holds:

i) Either $R_i d - b > 0$ and $a - R_i c < 0$, or $R_i d - b < 0$ and $a - R_i c > 0$ for each $i = 1, \ldots, m - j_1$ where $R_1, \ldots, R_{m-j_1}$ are the real poles of $H(s)$;

ii) a, b, c, d satisfies

$$r_i^2 - \left(\frac{a}{c} + \frac{b}{d}\right) r_i + \frac{ab}{cd} + t_i^2 > 0$$

for $i = 1, \ldots, j_0$, where $r_i + t_i\sqrt{-1}$ are the complex poles of $H(s)$.

Note that the parameters a, b, c, d can be negative, and c and d must be different from zero.

The case where one or two of the a, b, c, d parameters are equal to zero are considered in the following result. It is clear that there exist only two cases having sense for two parameters equal to zero and anyone for more than two parameters equal to zero.

Theorem 2. Consider $H(s) = \frac{N_h(s)}{D_h(s)}$ as defined in Theorem 1, but stable, (i.e., $R_1, \ldots, R_{m-j_1} < 0$, $r_1, \ldots, r_{j_0} < 0$), and suppose that $\alpha(s)$ and $H(s)$ satisfies at least one of the following conditions:

1) $a, b, c, d > 0$ and $ad - bc \neq 0$, or $a, b, c, d < 0$ and $ad - bc \neq 0$;

2) $b, c, d > 0$, $a = 0$ or $b, c, d < 0$, $a = 0$;

3) $a, c, d > 0,\ b = 0$ or $a, c, d < 0,\ b = 0$;

4) $a, b, d > 0,\ c = 0$;

5) $a, b, c > 0,\ d = 0$, and $\max\{r_1, \ldots, r_{j_0}\} < \frac{a}{c}$;

6) $a, d > 0,\ b = c = 0$ or $a, d < 0,\ b = c = 0$;

7) $b,\ c > 0,\ a = d = 0$ or $b, c < 0,\ a = d = 0$;

8) $a,\ b > 0,\ d < 0,\ c = 0$ and $R_i d - b < 0$ for $i = 1, \ldots, m - j_1$ and $\max\{r_1, \ldots, r_{j_0}\} < \frac{b}{d}$;

9) $a,\ b < 0,\ d < 0,\ c = 0$;

10) $a < 0, b > 0, d < 0, c = 0$ and $R_i d - b > 0$ for $i = 1, \ldots, m - j_1$ and $b - dr_j > 0$ for $j = 1, \ldots, j_0$;

11) $a,\ b > 0,\ c < 0,\ d = 0,\ a - R_i c > 0$ for $i = 1, \ldots, m - j_1$ and $a - r_j c > 0$ for $j = 1, \ldots, j_0$;

12) $a, b < 0,\ c < 0,\ d = 0$;

13) $a > 0, b < 0, c < 0, d = 0$ and $a - R_i c < 0$ for $i = 1, \ldots, m - j_1$ and $a - cr_j > 0$ for $j = 1, \ldots, j_0$.

then $H(\alpha(s))$ is a stable plant except in the case 2) when $H(s)$ has real poles and the case 7) when $H(s)$ has complex poles. When $a > 0,\ b < 0,\ d < 0,$ $c = 0$ or $a < 0,\ b > 0,\ c < 0,\ d = 0$ then stability is not guarantee, and stable plants are not mapped into stable plants, unless $R_1, \ldots, R_{m-j_1} > 0$ and $r_1, \ldots, r_{j_0} > 0$. In the case when $R_1, \ldots, R_{m-j_1} > 0$ and $r_1, \ldots, r_{j_0} > 0$ the plant $H(\alpha(s))$ is a stable if at least one of the following conditions holds:

14) $a > 0, b < 0, d < 0, c = 0$ and $R_i d - b < 0$ for $i = 1, \ldots, m - j_1$ and $b - dr_j > 0$ for $j = 1, \ldots, j_0$;

15) $a < 0, b > 0, c < 0, d = 0$ and $a - R_i c < 0$ for $i = 1, \ldots, m - j_1$ and $a - cr_j > 0$ for $j = 1, \ldots, j_0$.

Note that as conditions 1), 2), 3), 6) and 7) from Theorem 2 are independent from the plant, then they generates transformations that preserves stability for stable polynomials. The other cases depends on the particular plant and then they preserves stability of only a proper subset of the set of stable polynomials. Conditions 14) and 15) give us plants completely unstable.

Theorem1 and Theorem 2 gives us a complete solution to the problem of converting a rational function, real proper and stable or unstable into a stable one, by substituting the s variable by $\alpha(s)$ in $H(s)$. These results generalizes and extend Theorem 1 [1] and for the case of functions of the type $\alpha(s)$, they generalizes and extend Theorem 1 in [2]. On the other hand, they provide an answer to the open problem proposed in Djaferis [5]., for the case of mappings preserving stability, obtained by substituting the s variable by functions of the type $\alpha(s)$ is stable polynomials. Characterizing all the maps obtained under this substitution, that preserve stability for any stable polynomial, in this case the polynomial $D_h(s)$ (stable or unstable), is mapping to the stable polynomial $(cs + d)^m D_h(\alpha(s))$. That is to say, we characterized the space of parameters a, b, c, d for which the map before mentioned, preserves stability for any stable polynomial, mapping stable polynomials in stable polynomials. But also we characterized the space of parameters a, b, c, d for which the map before mentioned, mapping unstable polynomials in stable polynomials.

By the other hand, note that the substitution of the s variable by $\alpha(s)$ fix, preserves the operations of addition, multiplication, division and inversion of real and proper rational functions as well as the value of constants. As a consequence, in the cases 1), 2), 3), 6) and 7) in Theorem 2, the substitution by $\alpha(s)$ fix, induces an homomorphism of the ring of all the real and proper functions in himself an similarity for the set of real, rational, proper and stable functions.

Proposition 3. Let us consider the plant $H(s) = \frac{N_h(s)}{D_h(s)}$ and the proper controller $C(s) = \frac{N_c(s)}{D_c(s)}$ such that it stabilizes the plant, where $N_h(s)$, $N_c(s)$, $D_c(s)$ and $D_h(s)$ are polynomials with $\deg D(s) = n \geq \deg N(s)$. Also consider the linear transformation $\alpha(s) = \frac{as+b}{cs+d}$ were a, b, c, d are real numbers, and let us substitute the s variable by $\alpha^{-1}(s) = \frac{b-ds}{cs-a}$ in $H(s)$. Then:

a) the controllers of the form $C_\alpha(s) \equiv C(\alpha(s))$ stabilizes $H(s)$, if $C(s)$ stabilizes in a robust way the plant $H_{\alpha^{-1}}(s) \equiv H(\alpha^{-1}(s))$, where the a, b, c and d parameters satisfies at least one of the conditions of Theorem 2, or the conditions of Theorem 1 for the closed loop plant:

$$\bar{P}(s) = \frac{C(s)H_{\alpha^{-1}}(s)}{1 + C(s)H_{\alpha^{-1}}(s)}.$$

b) the controller $C(s)$ stabilizes in a robust way the plants $H_\alpha(s) \equiv H(\alpha(s))$, if the controllers $C(\alpha^{-1}(s))$ stabilizes the plant $H(s)$, where the a, b, c and d parameters satisfies at least one of the conditions of Theorem 2, or the conditions of Theorem 1 for the closed loop plant:

$$\hat{P}(s) = \frac{C(\alpha^{-1}(s))H(s)}{1 + C(\alpha^{-1}(s))H(s)}.$$

This proposition can be interpreted as a dual result, in the sense that the robust stabilization of the plant $H(s)$ with non linear disturbances in the parameters, induced by the substitution of the s variable by $\alpha^{-1}(s)$, with the controller $C(s)$ implies that the controller $C(s)$ is non fragile when faced to the same kind of disturbances, induced by the substitution of the s variable by $\alpha^{-1}(s)$, for the plant $H(s)$, and vice versa.

Let us consider a PD controller of the form $C_{PD}(s) = K_p + \frac{K_D s}{s+r}$, and a PI controller of the form $C_{PI}(s) = K_p + \frac{K_I}{s}$. These controllers can be rewritten in the way: $C_{PD}(s) = \frac{(K_p+K_D)s+K_p r}{s+r}$ and $C_{PI}(s) = \frac{K_p s+K_I}{s}$. Note that $C_{PD}(s) \in \Gamma(a,b,c,d)$ if K_p, K_D, $r > 0$. We can now take the problem of the non fragile stabilization under non linear disturbances induced by the substitution $\alpha(s)$, and the problem of the non lineal tuning in the parameters of the controller, induced by the substitution $\alpha(s)$, for controllers PD and PI.

We then have the following results:

Corollary 4.

a) If the controller $C_{PD}(s) = K_p + \frac{K_D s}{s+r}$ stabilizes in a robust way the plants $H_{\alpha^{-1}}(s)$, where the parameters a, b, c and d satisfies at least one of the conditions of Theorem 2 for the closed loop system formed by $C_{PD}(s)$ and $H_{\alpha^{-1}}(s)$, then the controllers

$$C_{PD}(\alpha(s)) = \left(\frac{(K_p+K_D)a + K_p rc}{a+rc}\right) \frac{s+\frac{b+ld}{a+lc}}{s+\frac{b+rd}{a+rc}}$$

stabilizes $H(s)$ with $l = \frac{K_p r}{K_p+K_D}$. If $\alpha(s) \in \Gamma(a,b,c,d)$, and K_p, K_D, $r > 0$, then

$$C_{PD}(\alpha(s)) \in \Gamma(a,b,c,d).$$

b) If the controller $C_{PI}(s) = K_p + \frac{K_I}{s}$ stabilizes in a robust way the plants $H_{\alpha^{-1}}(s)$, where the parameters a, b, c and d satisfies at least one of the conditions of Theorem 2 for the closed loop system formed by $C_{PI}(s)$ and $H_{\alpha^{-1}}(s)$, then the controllers

$$C_{PI}(\alpha(s)) = \left(\frac{K_p a + K_I c}{a}\right) \frac{s+\frac{K_p b+K_I d}{K_p a+K_I c}}{s+\frac{b}{a}}$$

stabilizes $H(s)$. If $\alpha(s) \in \Gamma(a,b,c,d)$ and K_p, $K_I > 0$, then

$$C_{PI}(\alpha(s)) \in \Gamma(a,b,c,d).$$

Note that the controllers $C_{PD}(\alpha(s))$ and $C_{PD}(\alpha(s))$ in Corollary 4 are not PD or PI controllers (unless $b = 0$). Boot controllers are of type lead-lag networks. As $\Gamma(a,b,c,d) \subset SPR0^*$, then $C_{PD}(\alpha(s))$ and $C_{PD}(\alpha(s))$ are controllers strictly passive.. Obviously, they are also a dual version of this result.

When the substitution is $\gamma(s) = \frac{as}{cs+d}$ we then have the following result:

Corollary 5.

a) If $C_{PD}(s) = K_p + \frac{K_D s}{s+r}$ stabilizes in a robust way the family $H_{\gamma^{-1}}(s)$, then the PD controllers:

$$\hat{C}_{PD}(s) = K_p + \frac{\hat{K}_D s}{s+q}$$

with $\hat{K}_D = \frac{aK_D}{a+rc}$ and $q = \frac{rd}{a+rc}$, stabilizes $H(s)$, for any real a, b, c and d such that $a, c, d > 0$ and $b = 0$;

b) If $C_{PI}(s) = K_p + \frac{K_I}{s}$ stabilizes in a robust way the family $H_{\gamma^{-1}}(s)$. then the PI controllers:

$$\hat{C}_{PI}(s) = \hat{K}_p + \frac{\hat{K}_I}{s}$$

with $\hat{K}_p = K_p + \frac{K_I c}{a}$ and $\hat{K}_I = \frac{dK_I}{a}$, stabilizes $H(s)$, for any real a, b, c and d such that $a, c, d > 0$ and $b = 0$.

Clearly, the substitution $\gamma(s)$, preserves the structure of the PD and PI controllers. In the case of PD controllers is interesting to note that the K_p constant doesn't change. This can be interpreted in the following way: the predictive part of the PD controller can be modified following the relations:

$$\hat{K}_D = \frac{aK_D}{a+rc}$$
$$q = \frac{rd}{a+rc}$$

They can be seen as a parametrization of the derivative part. We can then give tuning rulers for the derivative part of the controller. In the same way, we can see that in the case of PI controllers the gains change following

$$\hat{K}_p = K_p + \frac{K_I c}{a}$$
$$\hat{K}_I = \frac{dK_I}{a}$$

This can be seen as a parametrization of the proportional and integral part and can be used to obtain some tuning rulers.

By using standard results on passivity, we can to give the following result.

Proposition 6. Consider the following controllers:

1) $C_1(s) = C_{PI}(s) = K_p + \frac{K_I}{s}$ where $K_p, K_I > 0$.

2) $C_2(s) = C_{PD}(s) = K_p + \frac{K_D s}{s+r}$ where $r, K_p, K_D > 0$.

3) $C_3(s) = C_{LL}(s) = K_p \frac{1+T_N s}{1+T_D s}$ where $K_p, T_D, T_N > 0$.

4) $C_4(s) = C_{PID_1}(s) = K_p + \frac{K_I}{s} + \frac{K_D s}{s+r}$ where $r, K_p, K_I, K_D > 0$.

5) $C_5(s) = C_{PID_2}(s) = K_p + \frac{K_I}{s} + K_D s$ where $K_p, K_I, K_D > 0$.

6) $C_6(s) = C_{PID_3}(s) = K_p \frac{1+T_i s}{T_i s} \frac{1+T_d s}{1+\eta T_d s}$ where $K_p > 0$, $0 < T_d < T_i$ and $0 < \eta \leq 1$.

7) $C_7(s) = C_{PID_4}(s) = K_p \beta \frac{1+T_i s}{1+\beta T_i s} \frac{1+T_d s}{1+\eta T_d s}$ where $K_p > 0$, $0 < T_d < T_i$, $1 \leq \beta$ and $0 < \eta \leq 1$.

Now the following assumption is taken:

A) Given a fixed plant $H(s)$, there exists a subset Ω of linear transformations $\alpha(s) = \frac{as+b}{cs+d}$ were

a, b, c, d are real numbers, such that $H(\alpha(s))$ is PR function for each $\alpha(s) \in \Omega$.

Then, for all SPR0 function $\nu(s)$ and for all $\alpha(s) \in \Omega$, the controller $C_j(\nu(s))$ stabilizes to the plant $H(\alpha(s))$ for $j = 1, \ldots, 7$.

Notice that the plant $H(s)$ can be unstable and non minimum phase and that the controller $C_7(\nu(s))$ stabilizes to the plant $H(\alpha(s))$ for any $\nu(s) \in SPR0^*$.

V. Example

We take a plant of the form $p_1(s) = \frac{2(s+1)}{s^2+2s-3}$ and a lead-lag controller $c_1(s) = \frac{34.745(s+1.6373)}{s+37.9063}$, which stabilizes this plant. In this case the closed-loop plant's denominator polynomial

$$\frac{C_1(s)p_1(\alpha^{-1}(s))}{1 + C_1(s)p_1(\alpha^{-1}(s))}$$

is given by

$$\begin{aligned} f(s,a,b,c,d) &= \left(-66.49c^2 - 1.0d^2 + 71.49cd\right)s^3 \\ &+ \begin{pmatrix} 2.0bd + 189.59cd - 71.49bc + 132.98ac \\ -5.7077 \times 10^{-2}c^2 - 37.906d^2 - 71.49ad \end{pmatrix} s^2 \\ &+ \begin{pmatrix} .11415ac - 189.59ad + 75.813bd - 66.49a^2 \\ -189.59bc - 1.0b^2 + 71.49ab \end{pmatrix} s \\ &+189.59ab - 37.906b^2 - 5.7077 \times 10^{-2}a^2 \end{aligned}$$

and this is stable if and only if the following inequalities are met:

$$\begin{aligned} & A_1(a,b,c,d)A_2(a,b,c,d) - A_0(c,d)A_3(a,b) > 0 \\ & A_0(c,d) = \left(-66.49c^2 - 1.0d^2 + 71.49cd\right) > 0 \\ & \begin{pmatrix} A_1(a,b,c,d) = \\ 2.0bd + 189.59cd - 71.5bc + 133ac \\ -5.7077 \times 10^{-2}c^2 - 37.9d^2 - 71.5ad \end{pmatrix} > 0 \\ & \begin{pmatrix} A_2(a,b,c,d) = \\ .11415ac - 189.59ad + 75.813bd- \\ 66.49a^2 - 189.59bc - 1.0b^2 + 71.49ab \end{pmatrix} > 0 \\ & \begin{pmatrix} A_3(a,b) = \\ 189.59ab - 37.906b^2 - 5.7077 \times 10^{-2}a^2 \end{pmatrix} > 0 \end{aligned}$$

Moreover, we require that at least one of the conditions 1), 2), 3), 6) or 7) in Theorem 2.

Now by item a) in Proposition 3, the controllers

$$c(\alpha(s)) = 1.7373 \left\{ \frac{(10000.0a+16373.c)s+10000.0b+16373.d}{(500.0a+18953.c)s+500.0b+18953.d} \right\}$$

stabilize the plant $p_1(s)$ for the set of parameters a, b, c, d that met with the last conditions. For example with $a, d \in \left[10.0^{-3}, 8\right]$, $b, c \in [0, 5]$, we get the controllers $c(\alpha(s))$ that stabilized to $p_1(s)$.

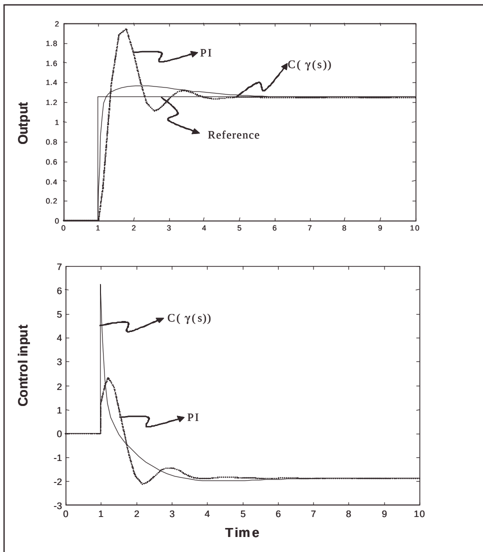

Figure 1

Figure 1 –

We take the following PI controller:

$$C(\alpha(s)) = \frac{25s + 25}{5s}$$

stabilizes the plant $p_1(s)$ and shows a better performance that a simple PI controller $1.0 + \frac{10}{s}$ as we show in Figure 1.

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